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Sorts of Colour Classes and Sorts of Families of Disjoint Sets

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ABSTRACT: In this paper we prove some results in graph theory using colouring of families of disjoint sets technique. We introduced the concept of set labeled by families of disjoint sets and the degree of this set, as generalization to the degree of intersection and proved some results related to this concept. We introduce sorts of families of disjoint sets (sorts of colour classes) as a concept related to set labeled by families of disjoint sets, also we compare between familiar and unfamiliar sorts of families of disjoint sets and explain this by some examples.

KEYWORDS: colouring of families of disjoint technique, set labeled by families of disjoint sets, sorts of colouring, types of colouring.

I. INTRODUCTION

In paper [1] we introduced the concept of colouring of families of disjoint sets technique as a topic of set theory. We also introduce definitions of degree of intersection, meeting and standard of meeting, minimum number of families of disjoint sets and maximum number of families of disjoint sets. In paper [2] we modify some definitions and examples of paper [1]. In paper [3] we introduce some concepts in graph theory to introduce new method for colouring edges and vertices of graph. This method has two types first related to vertices of the graph and second related to cycles. This method depend on the theory in our paper "Family of disjoint sets and Its Applications", with suitable changes. In paper [4] we prove minimum number of families of disjoint sets with known degree of intersection (minimum number of colours) and we introduce another proof for maximum number of families of disjoint sets (maximum number of colours). Also in that paper we introduce the concept types of families of disjoint sets (types of colouring) and prove some results related to this concept, and we introduce the concept restricted families of disjoint sets (restricted colouring), restricted colouring is colouring in which we add a condition. Suppose we have small identical cubes, we want to colour these cubes such that any two adjacent small cubes coloured by two different colours, and any big cube consists of twenty seven small identical cubes coloured by twenty seven different colours, how many colours will be used. This problem of cubes can be solved by using restricted colouring. In paper[5] we assume both sets and graphs are finite. Results in that paper use colouring of families of disjoint sets technique for finite sets and finite graphs. Using results in that paper we can compare between colouring of families of disjoint sets technique and colouring of graphs in graph theory. In this paper we prove some results in graph theory using colouring of families of disjoint sets technique in section three. In section four we introduced the concept of set labeled by families of disjoint sets and the degree of this set as generalization to the degree of intersection and proved some results related to this concept. In section five we introduce sorts of families of disjoint sets (sorts of color classes) as a concept related to set labeled by families of disjoint sets, also we compare between familiar and unfamiliar sorts of families of disjoint sets and explain this by some examples.

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II. RELATED WORK

In this section we recall some definitions and results related to colouring in graph theory and colouring of families of disjoint sets technique.

Definition 2.1. The degree of a vertex v of graph G is the number of edges incident with v , and denoted by $d(v)$ or $\deg(v)$. A vertex of degree 0 is called isolated vertex and a vertex of degree 1 is called end-vertex [7]. **Definition 2.2.** We say that two vertices v and w of a graph G are adjacent if there is an edge vw joining them, and the vertices v and w are then incident with such an edge. Similarly, two distinct edges e and f are adjacent if they have a vertex in common [7].

Definition 2.3. If G is a graph without loops, then G is called k – colourable if we can assign one of k colours to each vertex so that adjacent vertices have different colours. If G is k – colourable, but not $(k - 1)$ – colourable, then we say that G is k – chromatic, or we say that the chromatic number of G is k , and we write $\chi(G) = k$ [7].

Definition 2.4. A planar graph is a graph that can be drawn in the plane without crossings, so that no two edges intersect geometrically except at a vertex to which both are incident [7].

Proposition 2.5. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, $X_1, X_2, X_3, \dots, X_w$ are nonempty different intersections each of degree n , and t_i number of times the family of disjoint sets χ_i appear at these w intersections, then we have

$$\sum_{i=1}^m t_i = wn \quad [1].$$

Definition 2.6. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets and $X_{i_1}, X_{i_2}, X_{i_3}, \dots, X_{i_n}$ be n sets such that any two sets belong to two different families of disjoint sets, and X is nonempty set, $X = X_{i_1} \cap X_{i_2} \cap X_{i_3} \cap \dots \cap X_{i_n}$, $X = \{x : x \in X_{ij}, 1 \leq i \leq r, 1 \leq j \leq n, n \leq r, X_{ij} \in \chi_i\}$ then the positive integer n is called degree of intersection X [1].

Definition 2.7. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, $X_1, X_2, X_3, \dots, X_w$ are nonempty different intersections of these m families of disjoint sets, each intersection of degree n . The family of disjoint sets χ_i is said meets family of disjoint sets χ_j at these w intersections, if whatever we exchange families of disjoint sets at these w intersections, there exists at least one intersection the two families of disjoint sets χ_i and χ_j appear at this intersection [1].

Proposition 2.8. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_r$ be r families of disjoint sets and $X_1, X_2, X_3, \dots, X_w$ be w nonempty intersections, the family of disjoint sets χ_i appear t_i times at w intersections and the family of disjoint sets χ_j appear t_j time at w intersections, then we say the family of disjoint sets χ_i meet the family of disjoint sets χ_j at these w intersections if and only if $w + 1 \leq t_i + t_j$ [1].

Definition 2.9. Let $\bigcup_i S_i$ be infinite sets partition into families $\chi_1, \chi_2, \chi_3, \dots, \chi_{m_1}$ with degree of intersection n_1 and also

$\bigcup_i S_i$ partition into families $\chi_1, \chi_2, \chi_3, \dots, \chi_{m_2}$ with degree of intersection n_2 then we call $\chi_1, \chi_2, \chi_3, \dots, \chi_{m_1}$ and $\chi_1, \chi_2, \chi_3, \dots, \chi_{m_2}$ are two different types of families of disjoint sets [4].

Definition 2.10. Let $\bigcup_i S_i$ be infinite sets partition into families $\chi_1, \chi_2, \chi_3, \dots, \chi_{m_1}$ with degree of colouring n_1 and also

$\bigcup_i S_i$ partition into families $\chi_1, \chi_2, \chi_3, \dots, \chi_{m_2}$ with degree of colouring n_2 then we call $\chi_1, \chi_2, \chi_3, \dots, \chi_{m_1}$ and $\chi_1, \chi_2, \chi_3, \dots, \chi_{m_2}$ are two different types of colouring [4].

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Proposition 2.11. If we have infinite number of sets with finite degree of intersection n and there is no restriction any of degree of intersection so that any value of $2,3,4,\dots,n-1,n$ can represent a type of families of disjoint sets (type of colourings), then we have $n - 1$ types of families of disjoint sets (types of colourings).

Proposition 2.12. For any type of families of disjoint sets, values of number of families of disjoint sets can not exceed $n - 1$ values.

Remark 2.13. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets and $X_1, X_2, X_3, \dots, X_w$ be w nonempty intersections each of degree n , any two of m family of disjoint sets meet each other at these w intersections, then maximum value of m equal $2n - 1$, and maximum value of w equal $2n - 1$.

Remark 2.14. The colouring which we introduced in paper [1] we called it colouring of families of disjoint sets technique, for this technique of colouring we used the terminology sort of colouring and type of colouring. A sort of colouring is colouring expected not defined and need to be defined, also you can't determine edges or vertices adjacent or not adjacent unless adjacency is defined. Sort of colouring depend on degree of intersection or degree of colouring, different degrees determine different types of colouring, this means when sort of colouring is well defined, it is easy to determine many types using different degrees of intersection or different degrees of colouring [5].

Remark 2.15. In this paper we mean by different families of disjoint sets, different values of m from relation $\sum_{i=1}^m t_i = wn$,

where n is constant and both w and t_i are variables, and we mean by different types of families of disjoint sets, different values of m from relation $\sum_{i=1}^m t_i = wn$, where n is variable, both w and t_i can be variables or constant [4].

Definition 2.16. Colouring edges at vertex is defined by assignment one of m colours to each edge so that adjacent edges have not same colour [3].

Definition 2.17. Degree of colouring edges at vertex is maximum number of edges have different colours at vertex [3].

Proposition 2.18. The number of colour classes used to colour edges of connected graph that is regular of degree two (cycle graph) with even number of vertices, is trivial colouring [5].

Proposition 2.19. The number of colour classes used to colour edges of connected graph that is regular of degree two (cycle graph) with odd number of vertices, is nontrivial colouring [5].

Proposition 2.20. The number of colour classes used to colour vertices of connected graph that is regular of degree two (cycle graph) with even number of vertices is trivial colouring, and with odd number of vertices is nontrivial colouring [5].

III. SOME RESULTS OF FAMILIES OF DISJOINT SETS

In this section we will prove some results related to families of disjoint sets.

Proposition 3.1. If $n \leq wn - m \left(\frac{w+1}{2}\right)$ where m is the number of families of disjoint sets, w and is number of intersections each of degree n , then $(m+1)\left(\frac{w+1}{2}\right) \leq nw$.

Proof: From $n \leq wn - m \left(\frac{w+1}{2}\right)$ we have $n + m \left(\frac{w+1}{2}\right) \leq nw$, from Remark 2.13. we have $\frac{w+1}{2} \leq n$, then we can write $\left(\frac{w+1}{2}\right) + m \left(\frac{w+1}{2}\right) \leq n + m \left(\frac{w+1}{2}\right) \leq nw$, and $(m+1)\left(\frac{w+1}{2}\right) \leq nw$.

Example 3.2. Let $\chi_1, \chi_2, \chi_3, \chi_4$ be four families of disjoint sets, X_1, X_2, X_3, X_4, X_5 be five intersections, with degree of intersection equal three. $X_1 = (1,2,3)$, $X_2 = (1,4)$, $X_3 = (2,4)$, $X_4 = (3,4)$, $X_5 = (1,2,4)$. From $n \leq wn - m \left(\frac{w+1}{2}\right)$ we have $3 \leq 5(3) - 4 \left(\frac{5+1}{2}\right)$. Let $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5$ be five families of disjoint sets, Y_1, Y_2, Y_3, Y_4, Y_5 be five intersections, with

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degree of intersection equal three. $Y_1 = (1,2,3)$, $Y_2 = (1,2,5)$, $Y_3 = (1,3,4)$, $Y_4 = (2,4,5)$, $Y_5 = (3,4,5)$. Then from $(m + 1)\left(\frac{w + 1}{2}\right) \leq nw$, we have $(4 + 1)\left(\frac{5 + 1}{2}\right) \leq 3(5)$.

Proposition 3.3. Let $\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3, \dots, \mathcal{X}_m$ be m families of disjoint sets, X is an intersection with degree of intersection n , labeled by n different elements of the set $\{1, 2, 3, \dots, m\}$, if we want to label any intersection Y with degree of intersection n , by n different elements of the set $\{1, 2, 3, \dots, m\}$, such that Y can only be labeled by the same labels of X , then number of families of disjoint sets is trivial.

Proof: Let X be intersection with degree of intersection n , since any intersection Y with degree of intersection n , is the same as X , then number of families of disjoint sets is trivial.

Example 3.4. When we colouring edges of a graph, any colour class represented by number, two adjacent edges labeled by two different numbers. We label the three sides (edges) of the triangle by 1, 2, 3, where 1, 2, 3, are colour classes. The three vertices will be labeled by (1, 2), (1, 3) and (2, 3). The edge labeled by 1, vertices labeled twice by 1, the edge labeled by 2, at vertices labeled twice by 2, and the edge labeled by 3, at vertices labeled twice by 3. When we colouring edges of rectangle we label each of the two sides represent the length by 1, we label each of the two sides represent the width by 2, and we label each of the two diagonal lines by 3, where 1, 2, 3, are colour classes. The four vertices will be labeled by (1, 2, 3), (1, 2, 3), (1, 2, 3) and (1, 2, 3), it is trivial colouring. We have two edges labeled by 1, vertices labeled four times by 1, we have two edges labeled by 2, vertices labeled four times by 2, and we have two edges labeled by 3, vertices labeled four times by 3.

Example 3.5. When we colouring edges of regular graph of degree six, each vertex of degree five, we label the six sides (edges) by 1, 2, 1, 2, 1, and 2, where 1, 2, are colour classes. At each vertex we have three edges connect this vertex with the three opposite vertices, we label these three edges by 3, 4, and 5. Each vertex of the six vertices will be labeled by (1, 2, 3, 4, 5). We have three edges labeled by 1, vertices labeled six times by 1, we have three edges labeled by 2, vertices labeled six times by 2, same thing for edges labeled by 3, 4, and 5.

From Example 3.4. and Example 3.5. we can state and prove the following proposition.

Proposition 3.6. In finite graph (colouring of edges) number of labels at all vertices when divided by two equal number of labels for all edges.

Proof: Summation of all degrees of vertices divided by two equal number of all edges, summation of all degrees of vertices equal number of labels at all vertices, therefore number of labels at all vertices divided by two equal number of all edges.

Proposition 3.7. If the number of colour classes used to colour edges of the regular graph of degree k , is trivial where $k = 1, 2, 3, \dots$ then each colour class appears $\frac{k}{2}$ times at edges.

Proof: Suppose the number of colour classes used to colour edges equal m , and since the colouring is trivial, then each vertex can be labeled by (1, 2, 3, ..., m). The number of edges equal $\frac{mk}{2}$, the number of labels for edges equal $\frac{mk}{2}$, since each label of 1, 2, 3, ..., m appear equally, therefore each colour class appears $\frac{k}{2}$ times edges.

Proposition 3.8. For the regular graph has k vertices (and k edges) where k is even number then colouring of edges is trivial colouring.

Proof: Let k be the number of circumference's edges, where k is integer, $2 \leq k$, suppose any two adjacent edges of circumference labeled by 1 or 2 where 1, 2 are colour classes, therefore all vertices can be labeled by (1, 2). We labeled an edge connect two non-adjacent vertices by 3 such that there is no vertex with degree greater than three, therefore all vertices can be of degree three, and there is no vertex of degree two since number of vertices is even,

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therefore all vertices can be labeled by $(1,2,3)$. We labeled an edge connect two non-adjacent vertices by 4 such that there is no vertex with degree greater than four, therefore all vertices can be of degree four, and there is no vertex of degree three since number of vertices is even, therefore all vertices can be labeled by $(1,2,3,4)$. We continue in this process till all vertices of degree $2k - 1$ and any two vertices are adjacent, so all vertices can be labeled by $(1,2,3,\dots, k - 1)$. Therefore this edge colouring is trivial colouring. The result holds for all by $m = 2,3,\dots, k - 1$.

Proposition 3.9. For the regular graph has k vertices (and k edges) where k is odd number then colouring of edges is non trivial colouring.

Proof: Suppose the number of colour classes used to colour edges of the regular graph with odd number of vertices, equal k , is trivial colouring. Let the number of colour classes used to colour edges equal m . Since the graph is regular has k edges and k vertices, where k is odd, since we suppose the colouring of edges of the regular graph is trivial colouring, therefore all vertices labeled by $(1,2,3,\dots, m)$, where any one of $1,2,3,\dots, m$ represent a colour class, if m is odd, it is a contradiction, the graph can not be odd number of vertices and each vertex with odd degree, therefore all vertices can't be labeled by $(1,2,3,\dots, m)$. Suppose m is even, from Proposition 3.7. each colour class appears $\frac{k}{2}$ times at edges, it is

a contradiction since k is odd, therefore all vertices can't be labeled by $(1,2,3,\dots, m)$.

Corollary 3.10. The number of colour classes used to colour edges of the graph K_{2n} equal $2n - 1$.

Proof: Since K_{2n} is regular graph each vertex of degree equal $2n - 1$, using Proposition 3.8. colouring of edges is trivial colouring with degree of colouring equal $2n - 1$, therefore the number of colour classes used to colour edges of the graph K_{2n} equal $2n - 1$.

Corollary 3.11. The number of colour classes used to colour edges of the graph K_{2n-1} equal $2n - 1$.

Proof: Since K_{2n-1} is regular graph each vertex of degree equal $2n - 2$, using Proposition 3.9. colouring of edges is non trivial colouring with degree of colouring equal $2n - 2$, therefore the minimum number of colour classes used to colour edges of the graph K_{2n-1} equal $2n - 1$, and the maximum number of colour classes used to colour edges of the graph K_{2n-1} equal $2(2n - 1) - 1 = 4n - 3$.

IV. SET LABELED BY FAMILIES OF DISJOINT SETS

All results of paper [1] depend directly or indirectly on concept of degree of intersection denoted by n , these results related to this concept only with number n , so each set with degree of intersection n means existence of n families of disjoint sets at least, in this section we generalized this concept.

Definition 4.1. If we have m families of disjoint sets and we have a set all element of this set represent some of these m families of disjoint sets, such that any two elements of this set represent two different families of disjoint sets and there is a relation between these elements of the set, then we call this set, set labeled by families of disjoint sets.

Definition 4.2. Suppose we have m families of disjoint sets and many sets labeled by families of disjoint sets, if maximum number of elements for each set labeled by families of disjoint sets is n where $n < m$, then n is called the degree of set labeled by families of disjoint sets. If $n = m$ then the degree is called trivial degree or trivial case.

Example 4.3. The degree of intersection in Definition 2.6. is an example of degree of set labeled by families of disjoint sets, the relation here is intersection of n sets belong to n families of disjoint sets.

Example 4.4. In graph theory vertex is an example of set labeled by families of disjoint sets, and the degree of vertex is an example of degree of set labeled by families of disjoint sets, the relation: any two edges are adjacent. If the degree of vertex v is n , it means there are n edges incident with vertex v . Each edge represent a color class (families of disjoint sets), the set labeled by families of disjoint sets contain n elements, any element represents a color class, the degree

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offset labeled by families of disjoint sets equal degree of vertex v , the relation is any two edges are adjacent. In this example the degree of set labeled by families of disjoint sets equal $n = 2, 3, 4, \dots$

Remark 4.5. From Example 4.4. the degree of vertex is an example of degree of set labeled by families of disjoint sets, it is easy to discover relation: any two edges are adjacent. At vertex v_1 , $d(v_1) = n - 1$ there are $n - 1$ vertices $v_2, v_3, v_4, \dots, v_n$ adjacent to the vertex v_1 , the condition: every two of $v_2, v_3, v_4, \dots, v_n$ to be adjacent.

Example 4.6. If the degree of vertex v_1 is $n - 1$, means there are $n - 1$ vertices $v_2, v_3, v_4, \dots, v_n$ adjacent to the vertex v_1 . Each vertex represent a color class (families of disjoint sets), the set labeled by families of disjoint sets represent n elements, any element represents a color class, the degree of set labeled by families of disjoint sets equal degree of vertex $d(v) + 1$, the relation: any two vertices are adjacent. The condition for any two vertices to be adjacent $d(v_i) = n$, for all i where $1 \leq i \leq n$.

Example 4.7. K_5 is simple graph on 5 vertices, v_1, v_2, v_3, v_4, v_5 , each pair of distinct vertices are adjacent. Let the color class represented by v_1 is labeled by 1, the color class represented by v_2 is labeled by 2, and so on. Each vertex labeled by $(1, 2, 3, 4, 5)$, here vertex is an example of set labeled by families of disjoint sets, and the degree of vertex is an example of degree of set labeled by families of disjoint sets, the relation: any two vertices are adjacent. The degree of each vertex is four, any vertex labeled by five color classes, (the degree of set labeled by families of disjoint sets equal five).

Definition 4.8. Suppose we have m families of disjoint sets and we have w sets labeled by these families of disjoint sets, and n is the degree of these sets labeled by families of disjoint sets, then we say these m families of disjoint sets meet each other at w sets labeled by families of disjoint sets if any families of disjoint sets appear at least $\frac{w+1}{2}$ times if w is odd or appear $\frac{w}{2} + 1$ times if w is even.

Example 4.9. Let $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5$ be 5 families of disjoint sets and X_1, X_2, X_3 , be three sets labeled by families of disjoint sets, each with degree equal $n = 4$. X_1 represented by $\chi_1, \chi_2, \chi_3, \chi_4$, we can write $X_1 = (\chi_1, \chi_2, \chi_3, \chi_4)$ or $X_1 = (1, 2, 3, 4)$, X_2 represented by $\chi_1, \chi_2, \chi_3, \chi_5$, we write $X_2 = (1, 2, 3, 5)$, X_3 represented by $\chi_1, \chi_2, \chi_4, \chi_5$, we write $X_3 = (1, 2, 4, 5)$, $w = 3$, any family of disjoint sets appear at least twice at X_1, X_2, X_3 , therefore these 5 families of disjoint sets meet each other at X_1, X_2, X_3 .

Definition 4.10. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, $X_1, X_2, X_3, \dots, X_w$ be w sets labeled by families of disjoint sets, each of degree n , such that any two families of disjoint sets meet each other at these w sets labeled by families of disjoint sets, then these w sets called standard of meeting for these m families of disjoint sets.

Proposition 4.11. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, $X_1, X_2, X_3, \dots, X_w$ be w sets labeled by families of disjoint sets, each of degree n , each X_i labeled by n elements of the set $\{1, 2, 3, \dots, m\}$, these w sets are standard of meeting for the m families of disjoint sets. If Y is a set labeled by families of disjoint sets with degree n , Y not equal any one of $X_1, X_2, X_3, \dots, X_w$ then Y can be one of w sets labeled by families of disjoint sets, each labeled by n is element of the set $\{1, 2, 3, \dots, m\}$ and these w sets are standard of meeting for these m families of disjoint sets.

Proof: Let Y labeled by n elements $Y = Y_1 = (a_1, a_2, a_3, \dots, a_n)$, a_i is an element of the set $\{1, 2, 3, \dots, m\}$, where $1 \leq i \leq n$ and $f : X_1 \rightarrow Y_1$ such that f is one to one and onto, then any two of the set $\{a_1, a_2, a_3, \dots, a_n\}$ are not equal. let

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$Y_j = f(X_j)$, where $2 \leq j \leq w$, since f is one to one and onto, then any two elements labeled the set Y_j not equal, then any two elements labeled the set Y_j not equal for all j , such that $1 \leq j \leq w$ is Since $f : X_1 \rightarrow Y_1$ is one to one and onto, then $f : \{1,2,3,\dots,m\} \rightarrow \{1,2,3,\dots,m\}$ is one to one and onto, where $b_i \in \{1,2,3,\dots,m\}$ and $f(a_i) = b_i$ is such that $Y_j = (b_1, b_2, b_3, \dots, b_n)$. Any a_i appear at least $\frac{w+1}{2}$ times if w is odd at the sets $X_1, X_2, X_3, \dots, X_w$ then any b_i appear at least $\frac{w+1}{2}$ times if w is odd at the sets $Y_1, Y_2, Y_3, \dots, Y_w$ since f is one to one and onto, and for any a_i appear at least $\frac{w}{2} + 1$ times if w is even at the sets $X_1, X_2, X_3, \dots, X_w$ then any b_i appear at least $\frac{w}{2} + 1$ times if w is even at the sets $Y_1, Y_2, Y_3, \dots, Y_w$ since f is one to one and onto. Therefore the sets $Y_1, Y_2, Y_3, \dots, Y_w$ are standard of meeting for these m families of disjoint sets.

Example 4.12. Let $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5$ be 5 families of disjoint sets and X_1, X_2, X_3 be three sets labeled by families of disjoint sets, each with degree equal $n = 4$. X_1 represent by $\chi_1, \chi_2, \chi_3, \chi_4$, we can write $X_1 = (\chi_1, \chi_2, \chi_3, \chi_4)$ or $X_1 = (1,2,3,4)$, X_2 represent by $\chi_1, \chi_2, \chi_3, \chi_5$, we write $X_2 = (1,2,3,5)$, X_3 represent by $\chi_1, \chi_2, \chi_4, \chi_5$, we write $X_3 = (1,2,4,5)$, $w = 3$, any families of disjoint sets appear at least by twice, therefore these 5 families of disjoint sets meet each other at X_1, X_2, X_3 (sets of families of disjoint sets). Given Y_1 labeled by $(1,3,4,5)$ we want to find Y_2 and Y_3 such that Y_1, Y_2, Y_3 is standard of meeting for these 5 families of disjoint sets. We can write $Y_1 = (1,3,4,5)$, let $f : \{1,2,3,4,5\} \rightarrow \{1,2,3,4,5\}$ and let $f(X_1) = Y_1$ so we can defined the function f by $f(1) = 3, f(2) = 4, f(3) = 5, f(4) = 1, f(5) = 2$, if $f(X_2) = Y_2$, and if $f(X_3) = Y_3$, then $Y_2 = (2,3,4,5)$, and $Y_3 = (1,2,3,4)$. Therefore Y_1, Y_2, Y_3 is standard of meeting for these five families of disjoint sets $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5$.

Remark 4.13. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, $X_1, X_2, X_3, \dots, X_w$ be w sets labeled by families of disjoint sets, each of degree n , any two of these m families of disjoint sets meet each other at these w sets, from Proposition 4.11. any set labeled by families of disjoint sets, each of degree n , and not belong to these w sets, there exist another w sets $Y_1, Y_2, Y_3, \dots, Y_w$ each of degree n , labeled by m families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be families of disjoint sets, each of degree n , any two families of disjoint sets meet each other at these w sets.

V. FAMILIAR AND UNFAMILIAR SORTS OF FAMILIES OF DISJOINT SETS

Sorts of families of disjoint sets and types of families of disjoint sets related to coloring of families of disjoint sets technique, in paper [4] we introduce some examples and results about types of families of disjoint sets, in this section we introduce sort of families of disjoint sets, as a concept related to set labeled by families of disjoint sets with some examples. In this section we give examples of familiar and unfamiliar sorts of families of disjoint sets and unfamiliar sorts of colour classes.

Definition 5.1. Let S_1, S_2, S_3, \dots be sets partitioned into m families of disjoint sets, and we used a definition to determine any two sets belong to one family of disjoint sets or belong to two families of disjoint sets, then these m families of disjoint sets called a sort of families of disjoint sets.

Remark 5.2. It is same thing Definition 5.1. can be restate as following definition.

Definition 5.3. Let S_1, S_2, S_3, \dots be sets, partitioned into m colour classes, and we used a definition to determine any two sets belong to one colour class or belong to two colour classes, then these m colour classes called a sort of colour classes.

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Remark 5.4. By familiar sortfamily of disjoint sets and familiar sort of colour class we mean the base upon which sets partitioned into m families of disjoint sets is familiar, e.g. two adjacent sets will be two families of disjoint sets and two adjacent vertices, each belongs to one of two different colour classes.

Remark 5.5. By unfamiliar sortfamilies of disjoint sets and unfamiliar sort of color classes we mean the base upon which families of disjoint sets partitioned into m families of disjoint sets is not familiar, e.g. in map of countries any two adjacent countries will be in two different colors, this is familiar colouring, for a country and its k neighbors to be colored by $k + 1$ even there are two of its neighbors not adjacent, this is unfamiliar coloring, for this colouring of maps the degree of set labeled by families of disjoint sets (degree of intersection) equal $k + 1$. In familiar colouring of maps the degree of set labeled by families of disjoint sets (degree of intersection) equal three.

Example 5.6. Suppose we have small identical cubes, we want to colour these cubes such that any two adjacent small cubes coloured by two different colors, and any big cube consists of twenty seven small identical cubes coloured by twenty seven different colours, this is unfamiliar colouring, it consider unfamiliar sortfamilies of disjoint sets and unfamiliar sort of colour classes.

Example 5.7. Let G be infinite graph with maximum degree of vertex three, any edge e_i (in this graph) maximum can be adjacent to four edges and have four edges adjacent to adjacent edges, these nine edges include e_i coloured by nine colours, how many colours we need to colour the graph G , e.g. e_1 adjacent only to the edges e_2, e_3, e_4, e_5 , and e_2 adjacent only to the edges e_1, e_3, e_6, e_7 , and e_3 adjacent only to the edges e_1, e_2, e_8, e_9 , we want to colour edges of the graph G such that these nine edges coloured by nine colours and any nine edges have the same property coloured by nine colours, here the degree of set labeled by families of disjoint sets (degree of intersection) equal nine.

Remark 5.8. We see the difference between familiar sort and unfamiliar sort of families of disjoint sets (colour classes) in familiar sort it is easy to determine degree of set labeled by families of disjoint sets (degree of intersection), but for unfamiliar sort it is not easy to determine degree of set labeled by families of disjoint sets (degree of intersection).

Remark 5.9. In graph theory colouring of vertices is unfamiliar sort colouring, because we can't directly determine the degree of set labeled by colour classes (set labeled by families of disjoint sets), but colouring of edges directly we can determine the degree of set labeled by colour classes from degree of vertex.

Example 5.10. For colouring of vertices we can find a graph with vertex of degree five and degree of set labeled by colour classes equal two as in trees, because maximum number of vertices such that any two are adjacent are two, and we can find a graph with vertex of degree five and degree of set labeled by colour classes equal six as in complete graph K_6 , because maximum number of vertices such that any two are adjacent are six.

Remark 5.11. For all sorts of colouring familiar sorts or unfamiliar sorts we used the same results in paper [1] to find local minimum families of disjoint sets (local minimum colour classes) and local maximum families of disjoint sets (local maximum colour classes).

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