

On Near Perfect Number and k-Hyper Perfect Number

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ABSTRACT: In this paper we shall see another generalization of a near perfect number with a fixed redundant divisor and also establish results concerning the number of positive divisors of certain forms of near perfect numbers and hyper perfect numbers.

KEYWORDS: Perfect number, Near perfect number, k-Near perfect number, Hyper perfect number, k-Hyper perfect number.

I. INTRODUCTION

A positive integer n is said to be a perfect number if

$$\sigma(n) = 2n$$

where the function $\sigma(n)$ is the sum of all positive divisors of n . Many have studied the properties and results concerning perfect numbers. In the meanwhile P.Pollack and V. Shevlev[1] introduced the concept of Near perfect numbers and defined a positive integer n is a Near perfect number if

$$\sigma(n) = 2n + d$$

where d is called the redundant divisor and the function $\sigma(n)$ is the sum of all positive divisors of n . Further study led them to define k-Near perfect number as follows. A positive integer n is said to be a k-Near perfect number, if

$$\sigma(n) = 2n + \sum_{i=1}^k d_i$$

where d_i 's are the proper divisors of n and are called redundant divisors of n and the function $\sigma(n)$ is the sum of all positive divisors of n . Manoli and Bear [7] defined n as a k-hyper perfect number, if

$$\sigma(n) = \frac{k+1}{k}n + \frac{k-1}{k}$$

where the function $\sigma(n)$ is the sum of all positive divisors of n .

II. NOTATIONS

Notation 1: The function $\sigma(n)$ denotes the sum of all positive divisors of n .

Notation 2: The function $\tau(n)$ is the number of positive divisors of n .

Notation 3: (a, b) is the greatest common factor of the two positive integers a and b .

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III.MAIN RESULTS

Theorem 1: If p and $2^p - 1$ are primes then $2^p(2^p - 1)$ is a Near Perfect Number with redundant divisor 2^p .

Proof:

$$\begin{aligned} n &= 2^p(2^p - 1) \\ \sigma(n) &= \sigma(2^p(2^p - 1)) \\ &= \sigma(2^p)\sigma(2^p - 1) \\ &= (2^{p+1} - 1)[(2^p - 1) + 1] \\ &= (2^{p+1} - 1)2^p \end{aligned}$$

By the definition of near perfect number we have

$$\begin{aligned} d &= \sigma(n) - 2n \\ &= (2^{p+1} - 1)2^p - (2^p - 1)2^{p+1} \\ &= 2^p(2^{p+1} - 1 - 2(2^p - 1)) \\ &= 2^p(2^{p+1} - 1 - 2^{p+1} + 2) \\ &= 2^p \end{aligned}$$

Hence $2^p(2^p - 1)$ is a Near Perfect Number with redundant divisor 2^p .

Theorem 2: If $n = 2^p(2^p - 1)$ is a Near Perfect Number then $\tau(n) = 2(p + 1)$.

Proof:

$$\begin{aligned} n &= 2^p(2^p - 1) \\ \tau(n) &= \tau(2^p(2^p - 1)) \end{aligned}$$

Since $2^p - 1$ is a prime and $p > 1$ we have $(2^p, 2^p - 1) = 1$ hence

$$\begin{aligned} \tau(n) &= \tau(2^p)\tau(2^p - 1) \\ &= 2(p + 1) \end{aligned}$$

Hence the proof.

Lemma 3: If p and $2^p - 1$ are primes then $(2^{p-1}, (2^p - 1)^2) = 1$.

Proof: On contrary assume that $(2^{p-1}, (2^p - 1)^2) = d > 1$

$$\begin{aligned} &\Rightarrow d \mid 2^{p-1} \text{ and } d \mid (2^p - 1)^2 \\ d \mid 2^{p-1} &\Rightarrow d \in \{2, 2^2, 2^3, \dots, 2^{p-1}\} = A \\ d \mid (2^p - 1)^2 &\Rightarrow d \in \{(2^p - 1), (2^p - 1)^2\} = B \\ &\Rightarrow d \in A \text{ and } d \in B \\ &\Rightarrow d \in A \cap B \end{aligned}$$

This contradicts the fact $A \cap B = \emptyset$ hence the proof.

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Theorem 4: If both p and $2^p - 1$ are primes then $n = 2^{p-1}(2^p - 1)^2$ is a Near Perfect Number with redundant divisor $2^p - 1$.

Proof: See [1]

Theorem 5: If $n = 2^{p-1}(2^p - 1)^2$ is a Near Perfect Number then $\tau(n) = 3p$.

Proof:

$$n = 2^{p-1}(2^p - 1)^2$$

$$\tau(n) = \tau(2^{p-1}(2^p - 1)^2)$$

By lemma 3 we have $(2^{p-1}, (2^p - 1)^2) = 1$ hence

$$\tau(n) = \tau(2^{p-1})\tau(2^p - 1)^2$$

Since $2^p - 1$ is a prime $\tau(2^p - 1)^2 = 3$ hence

$$\tau(n) = 3p$$

Hence the proof.

Corollary 6: If $n = 2^{p-1}(2^p - 1)^2$ is a Near Perfect Number then $\tau(\tau(n)) = \begin{cases} 3 & \text{if } p = 3 \\ 4 & \text{if } p \neq 3 \end{cases}$

Proof: From previous theorem we have $\tau(n) = 3p$

Case 1: $p = 3$,

$$\tau(\tau(n)) = \tau(9)$$

$$= 3$$

Case 2: $p \neq 3$

In this case since p is a prime we have $\tau(p) = 2$ and $(3, p) = 1$

$$\tau(\tau(n)) = \tau(3p)$$

$$= \tau(3)\tau(p)$$

$$= 2(2)$$

$$= 4$$

Hence the proof.

Lemma 7: If $k + 1 = p$ and $p^q - k$ are primes then $(p^{q-1}, p^q - k) = 1$.

Proof:

$$k + 1 = p \tag{1}$$

On contrary assume that $(p^{q-1}, p^q - k) = d > 1$

$$\Rightarrow d \mid p^{q-1} \text{ and } d \mid p^q - k$$

$$d \mid p^{q-1} \Rightarrow d \in \{p, p^2, p^3, \dots, p^{q-1}\} = A$$

$$d \mid p^q - k \Rightarrow d \in \{p^q - k\} = B$$

Since the element $p^q - k \in B$ is a prime it can at most be equal to $p \in A$. Hence assume that $p = p^q - k$

$$p + k = p^q \tag{2}$$

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But

$$p \equiv 0 \pmod{p}$$

$$\Rightarrow p^q \equiv 0 \pmod{p} \text{ and } p+k \equiv k \pmod{p}$$

Substituting in (2) we get

$$k \equiv 0 \pmod{p}$$

Substituting in (1) we get

$$1 \equiv 0 \pmod{p}$$

This contradicts the fact that p is a prime.

Hence $p \neq p^q - k$. Since

$$d \in A \text{ and } d \in B$$

$$\Rightarrow d \in A \cap B$$

This contradicts the fact that $A \cap B = \emptyset$. Hence $(p^{q-1}, p^q - k) = 1$.

Theorem 8: [2] If $k+1 = p$ and $p^q - k$ are primes then the number of the form $n = p^{q-1}(p^q - k)$ is a k -hyper perfect number.

Proof: See [2].

Theorem 9: If n is a k -hyper perfect number of the form $n = p^{q-1}(p^q - k)$ then $\tau(n) = 2q$.

Proof:

$$n = p^{q-1}(p^q - k)$$

$$\tau(n) = \tau(p^{q-1}(p^q - k))$$

By lemma 7 we have $(p^{q-1}, p^q - k) = 1$

$$\Rightarrow \tau(n) = \tau(p^{q-1})\tau(p^q - k)$$

Since $p^q - k$ is a prime $\tau(p^q - k) = 2$

$$\Rightarrow \tau(n) = 2q$$

Hence the proof.

Corollary 10: If n is a k -hyper perfect number of the form $n = p^{q-1}(p^q - k)$ then $\tau(\tau(n)) = \begin{cases} 3 & \text{if } q = 2 \\ 4 & \text{if } q \neq 2 \end{cases}$

Proof: From previous theorem we have $\tau(n) = 2q$

Case 1: $q = 2$

$$\tau(\tau(n)) = \tau(2q)$$

$$= \tau(4)$$

$$= 3$$

Case 2: $q \neq 2$

In this case since q is prime we have $(2, q) = 1$ and $\tau(q) = 2$

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$$\begin{aligned}\tau(\tau(n)) &= \tau(2q) \\ &= \tau(2)\tau(q) \\ &= 2(2) \\ &= 4\end{aligned}$$

Hence the proof.

IV. CONCLUSION

In this paper we have generalized near perfect number with a fixed redundant divisor and also we have proved some results concerning the number of positive divisors of certain Near perfect numbers and Hyper perfect numbers.

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