

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: www.ijirset.com

Vol. 8, Issue 1, January 2019

On Double Representation of q-k-normal Matrices

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ABSTRACT: In this paper, the conjugate transpose of double representation of quaternion matrix is introduced. Also some basic results have been proved.

AMS Classifications : 15A09, 15A57, 15A24, 15A33, 15A15

KEYWORDS : q-k-normal matrices, double representation matrix, permutation matrix, Moore-Penrose inverse

I. INTRODUCTION

Wu.J.L and Zhang.P introduced bicomplex representation method for quaternion matrix is 2011[5]. The multiplication $A * B$ was defined as $A_0B_0 + A_1B_1j$ where $A_0 + A_1j = A$, $B_0 + B_1j = B$, A_0, B_0, A_1, B_1 are complex matrices with this concept, the conjugate transpose of bicomplex method developed and applied for the q-k-normal matrices. Also elementary results are discussed.

II. SOME DEFINITIONS AND THEOREMS

Definition 2.1

A quaternion matrix $A \in H_{n \times n}$ is represented as $A_0 + A_1j$, where A_0 and A_1 are in $C_{n \times n}$, known as double representation of quaternion matrix.

Definition 2.2

The product of double representation of A and B in $H_{n \times n}$ is defined as $AB = A_0B_0 + A_1B_1j$ with $A_0, B_0, A_1, B_1 \in C_{n \times n}$.

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Remark 2.3

If $A \in H_{n \times n}$ then $A^* = (A_0 + A_1 j)^* = A_0^* - A_1^* j$

Remark 2.4

From the definition of q-k-Hermitian or q-k-normal matrix $KA^*K = KA_0^*K - KA_1^*Kj$ is easily understood by one.

Theorem 2.5

If A is q-k-normal then A_0 and A_1 are k-normal with K.

Proof

Since A is q-k-normal. So $(A_0 + A_1 j)(KA_0^*K - KA_1^*Kj) = (KA_0^*K - KA_1^*Kj)(A_0 + A_1 j)$.

This implies that $A_0KA_0^*K - A_1KA_1^*Kj = KA_0^*KA_0 - KA_1^*KA_1j$

Equate the corresponding terms,

We have $A_0KA_0^*K = KA_0^*KA_0$ and $A_1KA_1^*K = KA_1^*KA_1$. Thus A_0 and A_1 are q-k-normal with respect to the permutation matrix K.

Hence proved.

Remark 2.6

The converse of the theorem (2.5) is also true. The proof is left to the reader.

Theorem 2.7

If A is q-k-normal then $KA^{-1}K = KA_0^{-1}K - KA_1^{-1}Kj$

Proof

If A^{-1} is q-k-normal. [by 3, theorem 2.4]

That is $A^{-1}K(A^{-1})^*K = K(A^{-1})^*KA^{-1}$.

Since $A^{-1} = A_0^{-1} - A_1^{-1}j$

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Therefore , $KA^{-1}K = KA_0^{-1}K - KA_1^{-1}Kj$

Hence proved.

Theorem 2.8

If A is q - k -normal then A_0^T and A_1^T are k -normal.

Proof

If A^\dagger is q - k -normal [by 3, theorem 2.5]

That is $K(A^T)^*KA = AK(A^T)^*K$

Since $A^T = A_0^T + A_1^Tj$

So, $KA^TK = KA_0^TK + KA_1^TKj$.

Thus,

$$\begin{aligned} K(A^T)^*KA^T &= (K(A_0^T)^*K - K(A_1^T)^*Kj)(A_0^T + A_1^Tj) \\ &= K(A_0^T)^*KA_0^T - K(A_1^T)^*KA_1^Tj \end{aligned} \tag{2.9}$$

$$\begin{aligned} A^TK(A^T)^*K &= (A_0^T + A_1^Tj)(K(A_0^T)^*K - K(A_1^T)^*Kj) \\ &= A_0^TK(A_0^T)^*K - A_1^TK(A_1^T)^*Kj \end{aligned} \tag{2.10}$$

Since A^T is q - k -normal so equation (2.9) and equation (2.10) are equal.

Therefore , $A_0^TK(A_0^T)^*K = K(A_0^T)^*KA_0^T$ and $A_1^TK(A_1^T)^*K = K(A_1^T)^*KA_1^T$

This gives as A_0^T and A_1^T are k -normal.

Hence proved.

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Theorem 2.11

If A is q - k -normal then A_0^* and A_1^* are k -normal.

Proof

If A^* is q - k -normal.

[by 3, theorem 2.6]

Now, $A^* = A_0^* - A_1^* j$

$$\begin{aligned}
 A^* K(A^*)^* K &= A^* KAK \\
 &= (A_0^* - A_1^* j)(KA_0 K - KA_1 K) \\
 &= A_0^* KA_0 K - A_1^* KA_1 K j \\
 &= A_0^* K(A_0^*)^* K - A_1^* K(A_1^*)^* K j
 \end{aligned} \tag{2.12}$$

$$\begin{aligned}
 K(A^*)^* KA &= KAKA^* \\
 &= (KA_0 K + KA_1 K j)(A_0^* - A_1^* j) \\
 &= KA_0 KA_0^* - KA_1 KA_1^* j \\
 &= K(A_0^*)^* KA_0^* - K(A_1^*)^* KA_1^* j
 \end{aligned} \tag{2.13}$$

Since A^* is q - k -normal. So equations (2.12) and (2.13) shows that

$$A_0^* K(A_0^*)^* K - A_1^* K(A_1^*)^* K j = K(A_0^*)^* KA_0^* - K(A_1^*)^* KA_1^* j$$

Therefore, $A_0^* K(A_0^*)^* K = K(A_0^*)^* KA_0^*$ and $A_1^* K(A_1^*)^* K = K(A_1^*)^* KA_1^*$.

This implies that A_0^* and A_1^* are k -normal.

Hence proved.

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Remark 2.14

If $A \in H_{n \times n}$ then $A^m = (A_0 + A_1j)^m$, $m \geq 1$, $m \in \mathbb{Z}$.

Since $A = A_0 + A_1j$ So $A^m = (A_0 + A_1j)(A_0 + A_1j) \dots m$ times.

Thus $A^m = A_0^m + A_1^m j$ because $AB = A_0B_0 + A_1B_1j$ where $A = A_0 + A_1j$ and $B = B_0 + B_1j$ for all A_0, A_1, B_0, B_1 in $C_{n \times n}$.

Theorem 2.15

Let $A \in H_{n \times n}$ be a q - k -normal in the form of $A = A_0 + A_1j$ then A_0 and A_1 are k -normal.

Proof

Since A is q - k -normal then A_0 and A_1 are k -normal (by theorem 2.5)

$$\begin{aligned} \text{Now, } A^m [K(A^m)^* K] &= (A_0 + A_1j)^m [K((A_0 + A_1j)^m)^* Kj] \\ &= (A_0^m + A_1^m j) [K(A_0^m)^* K - K(A_1^m)^* Kj] \\ &= A_0^m K(A_0^m)^* K - A_1^m K(A_1^m)^* Kj \end{aligned} \tag{2.16}$$

$$\begin{aligned} [K(A^m)^* K] A^m &= [K((A_0 + A_1j)^m)^* Kj] (A_0 + A_1j)^m \\ &= [K(A_0^m)^* K - K(A_1^m)^* Kj] (A_0^m + A_1^m j) \\ &= K(A_0^m)^* K A_0^m - K(A_1^m)^* K A_1^m j \end{aligned} \tag{2.17}$$

Since A_0 and A_1 are k -normal. So equations (2.16) and (2.17) shows that

$$A_0^m K(A_0^m)^* K - A_1^m K(A_1^m)^* Kj = K(A_0^m)^* K A_0^m - K(A_1^m)^* K A_1^m j$$

Therefore, $A_0^m K(A_0^m)^* K = K(A_0^m)^* K A_0^m$ and $A_1^m K(A_1^m)^* Kj = K(A_1^m)^* K A_1^m j$

This implies that A_0^m and A_1^m are k -normal

Hence proved.

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Remark 2.18

In the case $A \in H_{n \times n}$ is singular then we can find the Moore-Penrose inverse A^\dagger satisfying the equation $AXA = A$, $XAX = X$, $(AX)^* = AX$ and $(XA)^* = XA$. From the following theorem will be proved.

Theorem 2.19

If A is q - k -normal then A^\dagger is also q - k -normal if A_0^\dagger and A_1^\dagger are existing and k -normal.

Proof

Since A is q - k -normal

Therefore, $A(KA^*K) = (KA^*K)A$

$$\begin{aligned}
 \text{Now, } A^\dagger(K(A^\dagger)^*K) &= (A_0^\dagger - A_1^\dagger j)(KA_0^\dagger K - KA_1^\dagger K j) \\
 &= (A_0^\dagger - A_1^\dagger j)(K(A_0^\dagger)^*K + K(A_1^\dagger)^*K j) \\
 &= A_0^\dagger K(A_0^\dagger)^*K - A_1^\dagger K(A_1^\dagger)^*K j \\
 &= K(A_0^\dagger K)^*A_0^\dagger - K(A_1^\dagger)^*KA_1^\dagger K j \quad [\text{since } A_0^\dagger \text{ and } A_1^\dagger \text{ are } q\text{-}k\text{-normal}] \\
 &= (K(A_0^\dagger)^*K + K(A_1^\dagger)^*K j)(A_0^\dagger - A_1^\dagger j) \\
 &= K[(A_0^\dagger)^* + (A_1^\dagger)^* j]K(A_0 + A_1 j)^\dagger \\
 &= K(A_0^\dagger - A_1^\dagger j)^*K(A_0 + A_1 j)^\dagger \\
 &= K((A_0 + A_1 j)^\dagger)^*KA^\dagger \\
 &= K(A^\dagger)^*KA^\dagger
 \end{aligned}$$

Hence Proved.

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Theorem 2.20

If A is q - k -normal then αA_0 and αA_1 are k -normal for any scalar α .

Proof

Since A is q - k -normal having the double representation $A = A_0 + A_1j$, where A_0 and $A_1 \in C_{n \times n}$.

So, $A(KA^*K) = (KA^*K)A$, αA is q - k -normal

This implies that, $\alpha AK(\alpha A)^*K = K(\alpha A)^*KA$

Now $\alpha A = \alpha A_0 + \alpha A_1j$ and $(\alpha A)^* = \overline{\alpha A_0}^* - \overline{\alpha A_1}^*j$

This implies that, $K(\alpha A)^*K = \overline{\alpha}(KA_0^*K) - \overline{\alpha}(KA_1^*K)j$

$$\begin{aligned} (\alpha A)K(\alpha A)^*K &= (\alpha A_0 + \alpha A_1j)(\overline{\alpha}KA_0^*K - \overline{\alpha}KA_1^*Kj) \\ &= \overline{\alpha}\alpha A_0(KA_0^*K) - \overline{\alpha}\alpha A_1(KA_1^*K)j \\ &= |\alpha|^2 [A_0(KA_0^*K) - A_1(KA_1^*K)j] \\ &= |\alpha|^2 A(KA^*K) \\ &= |\alpha|^2 A(KA^*K) \end{aligned}$$

This implies that, $|\alpha|^2 A_0$ and $|\alpha|^2 A_1$ are k -normal (by theorem 2.15)

Hence proved.

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