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# Variants of Newton's Method for the Solution of System of Nonlinear Equations

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**ABSTRACT:** In this paper, we discuss the way of applying variants of Newton's method which are normally used for a nonlinear equation of a single variable, for the solution of system of nonlinear equations. Few numerical are examples are considered to illustrate these methods.

**KEYWORDS:**

System of nonlinear equations, Gauss-Seidel method, Adomian decomposition method

## I. INTRODUCTION

We consider a system of 'n' nonlinear equations in 'n' variables having a unique solution, of the form

$$\left. \begin{aligned} f_1(x_1, x_2, \dots, x_n) &= 0 \\ f_2(x_1, x_2, \dots, x_n) &= 0 \\ \cdot & \\ \cdot & \\ \cdot & \\ f_n(x_1, x_2, \dots, x_n) &= 0 \end{aligned} \right\} \dots\dots\dots(1.1)$$

This system (1.1) can be briefly described as

$$F(X) = 0 \dots\dots\dots(1.2)$$

where

$$F = (f_1, f_2, \dots, f_n)^T$$

and  $X = (x_1, x_2, \dots, x_n)^T$

The Adomian decomposition method discussed by E.Babolian et.al [2], is efficient and effective one for the solution of (1.2) by generating a rapidly convergent series. A new techniques is discussed by J.Biazar and B.Ghanbary [3] for the solution of (1.2) by considering a method which is generally used to find a simple root of  $f(x) = 0$ .



**II. VARIANTS OF NEWTON’S METHOD**

The well-known Newton's method for obtaining (n+1)<sup>th</sup> approximation to a root x\* of

$$f(x) = 0 \dots\dots\dots (2.1)$$

is given by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \dots\dots\dots (2.2)$$

(n = 0,1,2,.....)

starting with an initial guess x<sub>0</sub> to x\* .

V.B.Kumar,Vatti [8] developed Extrapolated Newton Raphson (ENR) method by extrapolating the method (2.2), is given by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n) - \frac{f(x_n)f''(x_n)}{2f'(x_n)}} \dots\dots\dots (2.3)$$

which is same as Halley's method a third order convergent one.

An eighteenth order convergent method (EOCM) a 3 step method, for solving (2.1) developed by

V.B.Kumar, Vatti et.al [9] is given by

$$x_{n+1} = y_n - \frac{2f(y_n)}{f'(y_n)} \left[ \frac{1}{1 + \sqrt{1 - 2\rho_n}} \right] \dots\dots\dots (2.4)$$

where

$$\rho_n = \frac{f(y_n)f''(y_n)}{[f'(y_n)]^2} \dots\dots\dots (2.5)$$

$$y_n = w_n - \frac{f(w_n)}{f'(w_n)} \left[ \frac{1}{1 - \frac{\rho_n}{2}} \right] \dots\dots\dots (2.6)$$



$$\hat{\rho}_n = \frac{f(w_n)f''(w_n)}{[f'(w_n)]^2} \dots\dots\dots (2.7)$$

and  $w_n$  is same as that of  $x_{n+1}$  in (2.2).

Treating  $x_2, x_3, \dots, x_n$  as independent variables in  $f_1(x_1, x_2, \dots, x_n) = 0$  of (1.1),  $x_1, x_3, \dots, x_n$  as independent variables in  $f_2(x_1, x_2, \dots, x_n)$  of (1.1) and so on, we will be left with 'n' equations in n single variables as

$$\left. \begin{aligned} \hat{f}_1(x_1) &= f_1(x_1, x_2, \dots, x_n) = 0 \\ \hat{f}_2(x_2) &= f_2(x_1, x_2, \dots, x_n) = 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \hat{f}_n(x_n) &= f_n(x_1, x_2, \dots, x_n) = 0 \end{aligned} \right\} \dots\dots\dots(2.8)$$

Now, the Gauss-Seidel process for obtaining the solution of (1.1) using (2.3) or (2.4) with (1.2) is

$$\left. \begin{aligned} \hat{f}_1(x_1^{k+1}) &= f_1(x_1^k, x_2^k, \dots, x_n^k) = 0 \\ \hat{f}_2(x_2^{k+1}) &= f_2(x_1^{k+1}, x_2^k, \dots, x_n^k) = 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \hat{f}_n(x_n^{k+1}) &= f_n(x_1^{k+1}, x_2^{k+1}, \dots, x_{n-1}^{k+1}, x_n^k) = 0 \end{aligned} \right\} \dots\dots\dots(2.9)$$

(k = 0, 1, 2, \dots)

by choosing a proper initial guess  $(x_1^{(0)}, x_2^{(0)}, \dots, x_n^{(0)})$ .



**III. NUMERICAL EXAMPLES**

Here, we consider some examples as considered by E.Babolian et. al [2] and compare the results discussed in this paper.

**Example3.1:**

For the non-linear system

$$\left. \begin{aligned} f_1(x_1, x_2) &= x_1^2 - 10x_1 + x_2^2 + 8 = 0 \\ f_2(x_1, x_2) &= x_1x_2^2 + x_1 - 10x_2 + 8 = 0 \end{aligned} \right\} \dots\dots\dots(3.1)$$

whose exact solution is (1, 1), the Gauss-Seidel method for its solution is given by

$$\left. \begin{aligned} \hat{f}_1(x_1^{k+1}) &= x_1^{(k)2} - 10x_1^{(k)} + x_2^{(k)2} + 8 \\ \hat{f}_2(x_2^{k+1}) &= x_1^{(k+1)}x_2^{(k)2} + x_1^{k+1} - 10x_2^{(k)} + 8 \end{aligned} \right\} \dots\dots\dots(3.2)$$

(k = 0, 1, 2, .....

Taking  $x_1^{(0)} = x_2^{(0)} = 0.8$ , the solution of (3.1) is obtained using (3.2) by the formulae (2.3) and (2.4) up to six significant digits .The iterative values and the absolute errors are tabulated below for all methods discussed in this paper.

Iteration	Method	Approximations	
		X <sub>1</sub>	X <sub>2</sub>
1	3 Newton's Method 2 ENR Method ECOM	0.95525031	0.98895007
		0.98166470	0.99171726
		0.92800002	0.93120001
		0.9551963	0.9551963
		0.95525032	0.98895077
2	3 Newton's Method 2 ENR Method ECOM	0.99725389	0.99931400
		0.99901628	0.99994708
		0.9694202	0.96898561
		0.98906435	0.98906435
		0.9972539	0.999314



3	3 Newton's Method 2 ENR Method ECOM	0.9998256 0.99999999 0.98551298 0.99728195 0.99982856	0.99995714 0.99999999 0.98462127 0.99728195 0.99995714
4	3 Newton's Method 2 ENR Method ECOM	0.99998928 1 0.99263422 0.99932147 0.99998929	0.99999732 1 0.99191131 0.99932147 0.99999732
5	3 Newton's Method 2 ENR Method ECOM	0.9999993 1 0.99607595 0.99983043 0.99999933	0.9999998 1 0.99556087 0.99983043 0.99999983

Table (3.1)

Iteration	Method	Absolute errors in each of the iterations	
		A <sub>1</sub>	A <sub>2</sub>
1	3 Newton's Method 2 ENR Method ECOM	0.04474969 0.0183353 0.07199998 0.0448037 0.04474968	0.01104993 0.00828274 0.06879999 0.0448037 0.01104923
2	3 Newton's Method 2 ENR Method ECOM	0.00274611 0.00098372 0.0305798 0.0193565 0.0027461	0.000686 0.00005292 0.03101439 0.01093565 0.000686
3	3 Newton's Method 2 ENR Method ECOM	0.0001744 0.00000001 0.01448702 0.00271805 0.00017144	0.00004286 0.00000001 0.01537873 0.00271805 0.00004286



4	3	0.00001072	0.00000268
	Newton's Method	0	0
	2	0.00736578	0.00808869
	ENR Method	0.00067854	0.00067854
	ECOM	0.00001071	0.00000268
5	3	0.0000007	0.0000002
	Newton's Method	0	0
	2	0.00392405	0.00443913
	ENR Method	0.00016957	0.00016957
	ECOM	0.00000067	0.000000017

Table (3.2)

**Example 3.2:**

For the non-linear system

$$f_1(x_1, x_2, x_3) = 15x_1^2 + x_2^2 - 4x_3 - 13 = 0$$

$$f_2(x_1, x_2, x_3) = x_1^2 + 10x_2 - e^{-x_3} - 11 = 0$$

$$f_3(x_1, x_2, x_3) = x_2^3 - 25x_3 + 22 = 0$$

whose exact solution as mentioned by E.Babolian et.al [ 2 ] is X= (1.04214966, 1.03109169, 0.92384809)

We tabulate the computed results as done in previous example3.1

Iteration	Method	Approximations		
		X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>
1	3	1.02066666	1.03730224	0.92464532
	Newton's Method	1.11364025	1.011779832	1.01887013
	2	1.02066667	1.06636720	0.93324000
	ENR Method	1.02066667	1.03730225	0.92464532
	ECOM	1.02066666	1.03730225	0.92464532
2	3	1.04150568	1.03119379	0.92386123
	Newton's Method	1.04305725	1.10314838	0.91687004
	2	1.03979682	1.02246038	0.92835651
	ENR Method	1.04150569	1.0311938	0.92386124
	ECOM	1.04150569	1.0311938	0.92386124



3	3 Newton's Method 2 ENR Method ECOM	1.04213895	1.03109296	0.9238437
		1.04230666	1.03103551	0.92440087
		1.04485749	1.02748608	0.92202779
		1.04213895	1.03109296	0.92384837
		1.04213895	1.03109296	0.92384837
4	3 Newton's Method 2 ENR Method ECOM	1.04214938	1.03109130	0.92384815
		1.04221301	1.03109806	0.92377882
		1.04223720	1.01570328	0.92314446
		1.04214939	1.0310913	0.92384816
		1.04214939	1.0310913	0.92384816
5	3 Newton's Method 2 ENR Method ECOM	1.04214955	1.03109127	0.92384815
		1.04215194	1.03109043	0.92385665
		1.04223716	1.01568782	0.92314446
		1.04214956	1.03109127	0.92384815
		1.04214956	1.03109127	0.92384815

Table(3.3)

Iteration	Method	Absolute errors in each of the iterations		
		A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>
1	3 Newton's Method 2 ENR Method ECOM	0.021483	0.00621055	0.00079723
		0.07149059	0.019311858	0.09502204
		0.02148299	0.03527551	0.00939191
		0.021483	0.00621056	0.00079723
		0.21483	0.00621056	0.00079723
2	3 Newton's Method 2 ENR Method ECOM	0.00064398	0.0061021	0.00001314
		0.00090759	0.0003921	0.00697805
		0.00235284	0.00863131	0.00450842
		0.00064397	0.00010211	0.00001315
		0.00064397	0.00010211	0.00001315





3	3 Newton's Method 2 ENR Method ECOM	0.00001071	0.00000127	0.00000439
		0.000157	0.00005618	0.00055278
		0.00270783	0.00360561	0.0018203
		0.00001071	0.00000127	0.00000028
		0.00001071	0.00000127	0.00000028
4	3 Newton's Method 2 ENR Method ECOM	0.00000028	0.00000039	0.00000006
		0.00006335	0.000000637	0.00006927
		0.00008754	0.01538841	0.00070363
		0.00000027	0.00000039	0.00000007
		0.00000027	0.00000039	0.00000007
5	3 Newton's Method 2 ENR Method ECOM	0.00000011	0.00000042	0.00000006
		0.00000228	0.00000126	0.00000856
		0.0000875	0.01540387	0.00070363
		0.0000001	0.00000042	0.00000006
		0.0000001	0.00000042	0.00000006

Table (3.4)

#### IV. CONCLUSION

It can be seen from the above-tabulated results that all the methods yield almost same results except Newton's method which converges rapidly in the case of less variables whereas, its efficiency decreases in the case of more variables as it requires more computations and almost all the methods have more are near same absolute errors irrespective of their convergent orders.

#### REFERENCES

- [1] J.H. He, Non-perturbative method for strongly nonlinear equations, dissertation. de-Verlag im Internet GmbH, Berlin, 2006.
- [2] E. Babolian, J. Biazar, A.R. Vahidi, Solution of a system of nonlinear equations by Adomian decomposition method, Applied Mathematics and Computation.150(2004) 847-854.
- [3] J. Biazar and B. Ghanbary , A New Technique for Solving Systems of Nonlinear Equations, Applied Mathematical Science, Vol. 2, 2008, no.55, 2690-2703
- [4] K. Abbaoui, Y. Cherruault, Convergence of Adomian's method applied to non-linear equations, Math. Comput. Modell. Vol. 20 (9) pp no:69-73, 1994.
- [5] E. Babolian, J. Biazar, A.R. Vahidi, On the decomposition method for system of linear equations and system of linear Volterra integral equations, Appl. Math. Comput., in press.
- [6] K. Abbaoui, Y. Cherruault, V. Seng, Practical formula for the calculus of multivariable Adomian polynomials, Math. Comput. Modell., Vol 22 (1), pp no: 89-93.1995.
- [7] Y. Cherruault, Y. Saccomandi, B. Some, New results for convergence of Adomians method applied to integral equations, Math. Comp. Modell., Vol: 16 (2), pp no: 83-93, 1992.
- [8] V.B.K. Vatti , Numerical Analysis - Iterative Methods , ISBN:9789385909009, I.K International Publishing House Pvt.Ltd. 2006.
- [9] V.B.K. Vatti , M S Kumar Mylapalli "Eighteenth Order Convergent Method for Solving Non-Linear Equations" in ISSN:0974-6471 Orient.J. Comp.Sci.and Technol Vol:(1) pp no:144-150.



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