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Applications of Laplace Transform in Engineering

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ABSTRACT: $L[\alpha f(t) + \beta g(t)] = \alpha L[f(t)] + \beta L[g(t)]$ In this paper, we will discuss about various applications of Laplace Transform in different engineering fields. We also discuss about the way to solve Differential Equations by using Laplace Transform. The way to find transfer function of system, use Laplace Transform in Atomic Physics also as Automation Engineering, Control Engineering and Digital Signal Processing.

KEYWORDS: Laplace Transform, Differential Equation, Inverse Laplace Transform, Linearity, Convolution Theorem.

I. INTRODUCTION

The Laplace Transform is one of the widely used Integral Transform in mathematics with many applications in Science and Engineering. The Laplace Transform is often interpreted as a change from time domain where inputs and outputs are functions of your time to the frequency domain where inputs and outputs are functions of complex angular frequency. Laplace Transform methods have a key role to play within the modern approach to the analysis and style of engineering system. The concepts of Laplace Transforms are applied within the area of Science and Technology like Circuit Analysis, Communication engineering, Control Engineering and Atomic Physics etc.

1.1 Definition and important properties of Laplace Transform:

The definition and some useful properties of Laplace Transform which we have to use further for solving problems related to Laplace Transform in different Engineering fields are listed as follows.

Definition:

Let $f(t)$ be a function of t ($t > 0$), then the integral

is called Laplace Transform of $f(t)$.

We denote it as $L[f(t)]$ or $F(s)$.

$$\text{i.e. } L[f(t)] = \int_0^{\infty} e^{-st} f(t) dt = F(s)$$

1.2 Properties of Laplace Transform:

Linearity Property: If $f(t)$ and $g(t)$ are any two functions of t and α, β are any two constants then,

$$\int_0^{\infty} e^{-st} (\alpha f(t) + \beta g(t)) dt$$



First Shifting Property:

If $L[f(t)] = F(s)$, then $L[e^{at} f(t)] = F(s - a)$.

Multiplication by t^n Property:

$L[f(t)] = F(s)$, then

$$L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} [F(s)]$$

Laplace Transform of Derivative:

If $L[f(t)] = F(s)$, then

$$L[f^n(t)] = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - s^{n-3} f''(0) - \dots - f^{n-1}(0).$$

Laplace Transform of Bessel's function:

$L[J_0(t)] = \frac{1}{\sqrt{s^2 + 1}}$, where

$J_0(t) = \sum_{k=0}^{\infty} (-1)^k \frac{(\frac{1}{2}t^2)^k}{(k!)^2}$ is called Bessel's function.

Inverse Laplace

$L[f(t)] = F(s)$, then

is called inverse Laplace Transform of

$$L^{-1}[F(s)] = f(t)$$

Inverse Laplace Transform by Convolution Theorem:

If ;

then, $L^{-1}[\phi_1(s)] = f_1(t)$ $L^{-1}[\phi_2(s)] = f_2(t)$

$$L^{-1}[\phi_1(s) \cdot \phi_2(s)] = \int_0^t f_1(u) \cdot f_2(t - u) du$$

II. APPLICATIONS OF LAPLACE TRANSFORM IN SCIENCE AND ENGINEERING FIELDS

This section describes the applications of Laplace Transform in the area of science and engineering. The Laplace Transform is widely used in following science and engineering field.

1. Analysis of electronic circuits:

Laplace Transform is widely used by electronic engineers to solve quickly differential equations occurring in the analysis of Transient Response of Electronic Circuits.

2. System modeling:

Laplace Transform is used to simplify calculations in system modeling, where large number of differential equations are used.

3. Digital signal processing:

One can not imagine solving Digital Signal Processing Problems without employing Laplace Transform.

4. Nuclear Physics:

In order to get the true form of Radioactive Decay, Laplace Transform is used. It makes possible & easy to study analytic part of Nuclear physics.

5. Process Control:

Laplace Transform is used for process controls. It helps to analyze the variables which when altered, produce desired manipulations in the result.



Some of the examples in science and engineering fields in which Laplace Transforms are used to solve the differential equations, occurred in this fields. The following examples highlight the importance of Laplace Transform in different engineering fields.

2.1 Laplace Transform to solve Differential Equation:

Ordinary differential equation can be easily solved by the Laplace Transform method without finding the general solution and the arbitrary constants. The method is illustrated by following example,

Differential equation is

$$x \frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$$

given $y(0) = 2, y'(0) = 0$

Taking Laplace Transform on both sides, we get

$$L[xy''] + L[y'] + L[xy] = 0$$

$$\therefore -\frac{d}{ds}L[y''] + L[y'] - \frac{d}{ds}L[y] = 0$$

$$y(0) = 2 \text{ and } y'(0) = 0$$

$$\therefore -\frac{d}{ds}\{s^2\bar{y} - 2s - 0\} + \{s\bar{y} - 2\} - \frac{d\bar{y}}{ds} = 0$$

$$\therefore -s^2 \frac{d\bar{y}}{ds} - 2s\bar{y} + 2 + s\bar{y} - 2 - \frac{d\bar{y}}{ds} = 0$$

$$\therefore (s^2 + 1) \frac{d\bar{y}}{ds} + s\bar{y} = 0$$

$$\frac{d\bar{y}}{\bar{y}} + \frac{s ds}{s^2 + 1} = 0$$

Separating the variables, we get

∴ Integrating both sides, we get

$$\log \bar{y} + \frac{1}{2} \log(s^2 + 1) = \log C$$

$$\therefore \log \bar{y} = \log C - \log(\sqrt{s^2 + 1})$$

$$\therefore \bar{y} = \frac{C}{\sqrt{s^2 + 1}}$$



Taking Inverse Laplace Transform, we get

$$y = L^{-1}\left[\frac{C}{\sqrt{s^2+1}}\right]$$

i.e. $y = C J_0(x)$

At $x = 0$, $y(0) = C J_0(0)$ Putting $y(0) = 2$ and $J_0(0) = 1$, we get, $C = 2$

∴ Required value of y is,

$$y = 2 J_0(x)$$

2.2 Laplace Transform in Simple Electric Circuits:

Consider an electric circuit consisting of a resistance R , inductance L , a condenser of capacity C and electromotive power of voltage E in a series. A switch is also connected in the circuit.

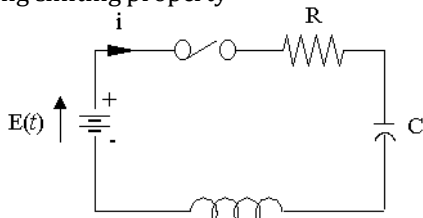
Then by Kirchoff's voltage law, we get

$$L \frac{di}{dt} + Ri + \frac{Q}{C} = E$$

$$\therefore -\frac{d}{ds}\{s^2\bar{y} - sy(0) - y'(0)\} + \{s\bar{y} - y(0)\} - \frac{d}{ds}(\bar{y}) = 0$$

Putting boundary conditions,

Using shifting property



$$Q = 6 - 6e^{-4t} \cos 3t - 8e^{-4t} \sin 3t$$

And $I = \frac{dQ}{dt} = 50e^{-4t} \sin 3t$

Example: An inductance of 3 henry, a resistor of 16 ohms and a capacitor of 0.02 farad are connected in series with an emf of 300 volts. At $t = 0$, the charge on the capacitor and current in the circuit is zero. Find the charge and current at any time $t > 0$.

Solution:

Let Q and I be instantaneous charge and current respectively at time t ,
Then by Kirchoff's voltage law

$$L \frac{di}{dt} + Ri + \frac{Q}{C} = E$$

$$2 \frac{d^2Q}{dt^2} + 16 \frac{dQ}{dt} + 50Q = E \dots \because I = \frac{dQ}{dt}$$

$$\therefore \frac{d^2Q}{dt^2} + 8 \frac{dQ}{dt} + 25Q = 150$$

Applying Laplace Transform on both sides,



$$L \left[\frac{d^2 Q}{dt^2} \right] + 8L \left[\frac{dQ}{dt} \right] + 25L[Q] = L[150]$$

$$\begin{aligned} \therefore \{s^2 L[Q] - sQ(0) - Q'(0)\} + 8\{sL[Q] - Q(0)\} + 25L[Q] &= 150L[1] \\ \therefore s^2 L[Q] + 8sL[Q] + 25L[Q] &= \frac{150}{s} \end{aligned} \quad \therefore \text{error} = \theta(t) - at$$

This is required expression for charge and current at anytime $t > 0$.

2.3 Theory of Automatic Control:

A mechanism, whether it involves electrical, mechanical or other principles, designed to accomplish such automatic control is called a servomechanism.

Suppose that a missile Mistrackingdownenemyaircraft.If at time t the enemy turns through some angle $\phi(t)$, the M must also turn through this angle, if it is to be catch with and destroy it. If a mass was aboard M, he couldoperate.

Some steering mechanism to produce required turns, but since the missile is must be accomplished automatically. To do for a man eyes, such as a radar beam which will indicate or point to the direction which must be taken by M, we also need something providing a substitute for a man’s hands which will turn a shaft through some angle in order to produce the desirereturn.

In this application, let us assume that the desired angle of turn as indicated by the radar beam is $\theta(t)$. Also let αt denotes the angle if turn of the shaft at time t . Because the things are happening so fast we must expect to have error between two. i.e. The existence of the error must be signaled back to the shaft, so that a compensating turning efforts or torque be

$\therefore (s^2 + 8s + 25)L[Q] = \frac{150}{s}$ produced. If the error is large the torque needed will be large. If the error is small the torque needed will be small. So that requires torque is proportional to the error

$$\therefore L[Q] = \frac{150}{s(s^2 + 8s + 25)}$$

Taking Inverse Laplace Transform on both sides,

$$\text{Torque} = I \frac{d^2 q}{dt^2}$$

acceleration.

i.e. M.I. multiplied by angular



$$\therefore Q = L^{-1} \left[\frac{150}{s(s^2 + 8s + 25)} \right]$$

By method of Partial Fraction

$$Q = L^{-1} \left[\frac{6}{s} - \frac{6s + 48}{(s^2 + 8s + 25)} \right]$$

Since torque is directly proportional to error,

$$\therefore I \frac{d^2 q}{dt^2} \propto [\theta(t) - at]$$

$$\therefore I \frac{d^2 q}{dt^2} = -k[\theta(t) - at] \dots (i)$$

where

$$k > 0$$

$$\therefore Q = 6L^{-1} \left[\frac{1}{s} \right] - L^{-1} \left[\frac{6(s + 4)}{(s + 4)^2 + 9} \right] - L^{-1} \left[\frac{24}{(s + 4)^2 + 9} \right]$$

The minus sign is used because if the error is positive then the torque must be opposite it. While the error is negative the torque must be positive.

Assuming that the initial angle and angular velocity are zero as possible conditions.

i.e. $\theta(0) = 0$, and $\theta'(0) = 0$
from equation (i)

$$\begin{aligned} \theta &= \alpha \sqrt{\frac{k}{I}} \left\{ \frac{t}{\sqrt{\frac{k}{I}}} + \left[\frac{I}{k} \sin \sqrt{\frac{k}{I}} (t - u) \right]_0^t \right\} \\ \therefore \theta &= \alpha \sqrt{\frac{k}{I}} \left\{ \sqrt{\frac{I}{k}} t - \frac{I}{k} \sin \sqrt{\frac{k}{I}} t \right\} \\ \therefore \theta(t) &= at - \alpha \sqrt{\frac{I}{k}} \sin \sqrt{\frac{k}{I}} t \end{aligned}$$

$$I \frac{d^2 q}{dt^2} = -k\theta(t) + kat$$

$$\Rightarrow \frac{d^2 q}{dt^2} + \frac{k}{I} \theta(t) = \frac{k}{I} at$$

Taking Laplace Transform on both sides,

$$[\theta(t)] \frac{d^2 q}{dt^2} + \frac{k}{I} L[\theta(t)]$$

$$\Rightarrow s^2 L[\theta] - s\theta(0) - \theta'(0) + \frac{k}{I} L[\theta] = \frac{k}{I} \alpha \frac{1}{s^2}$$

$$\Rightarrow \left(s^2 + \frac{k}{I} \right) L[\theta] = \frac{k}{I} \alpha \frac{1}{s^2}$$

... $\theta(0) = 0$, and $\theta'(0) = 0$

$$\Rightarrow L[\theta] = \frac{k}{I} \left[\frac{\alpha}{s^2 (s^2 + \frac{k}{I})} \right]$$



Taking Inverse Laplace Transform on bothsides,

$$\theta = \frac{k}{I} L^{-1} \left[\frac{\alpha}{s^2(s^2 + \frac{k}{I})} \right]$$

$$\therefore \theta = \frac{k\alpha}{I} [L^{-1} \left(\frac{1}{s^2} \right) * L^{-1} \left(\frac{1}{s^2 + \frac{k}{I}} \right)]$$

By convolution theorem,

$$\theta = \frac{k\alpha}{I} \int_0^t u \frac{1}{\sqrt{k/I}} \sin \sqrt{\frac{k}{I}}(t-u) du$$

This is the required turn at any time.

2.4 Laplace Transform in NuclearPhysics:

The following example is based on concepts from nuclear physics. Consider the following first order linear differential equation

$$\frac{dN}{dt} = -\lambda N$$

This equation is the fundamental relationship describing radio active decay,where $N = N(t)$ represents the number of undecayed atoms remaining in a sample of a radioactive isotope at time t and λ is the decay constant.

We can use the Laplace Transform to solve this equation. Rearranging the above equation, we get

$$\frac{dN}{dt} + \lambda N = 0$$

Taking Laplace Transform on both sides,

$$sL[N] - N(0) + \lambda L[N] = 0$$

$$\therefore s\bar{N} - N_0 + \lambda\bar{N} = 0$$

$$\text{here } L[N] = \bar{N} \quad N(0) = N_0$$

$$\therefore \bar{N} = \frac{N_0}{s+\lambda}$$

$$\therefore \theta = \alpha \int_0^t u \sin \sqrt{\frac{k}{I}}(t-u) du$$

$$\therefore \theta = \alpha \int_0^t \left\{ \left[u \frac{1}{\sqrt{k/I}} \cos \sqrt{\frac{k}{I}}(t-u) \right]_0^t - \int_0^t \frac{1}{\sqrt{k/I}} \cos \sqrt{\frac{k}{I}}(t-u) du \right\}$$



..... { Integration by parts rule

Now, Taking Inverse Laplace Transform on both sides, we get

$$N(t) = N_0 e^{-\lambda t}$$

Which is indeed the correct form for radioactive decay.

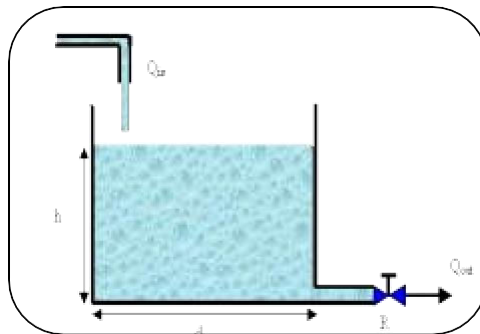
2.5 Laplace Transform in Control Engineering: Mechanical Engineering:

In Mechanical engineering field Laplace Transform is widely used to solve differential equations occurring in mathematical modeling of mechanical system to find transfer function of that particular system. Following example describes how to use Laplace Transform to find transfer function.

Example: The tank shown in figure is initially empty ($t = 0$). A constant rate of flow Q_i is added for $t > 0$. The rate at which flow leaves the tank is $Q_o = CH$. The cross

$$Q_o = CH.$$

Sectional Area of the tank is A . Determine the differential equation for the head H . Identify the time constant and find the transfer function of system.



Solution:

Given that $Q_o = CH$ Let, M = Mass of fluid e = Density of fluid

$$\therefore \text{Mass} = M = \text{Volume} \times \text{density}$$

$$= AH \times e$$

$$\therefore \text{Mass flow rate} = \dot{M} = \frac{dM}{dt}$$

$$= \frac{d}{dt} (AH \times e) = eA \times \frac{dH}{dt}$$

We know that,

Mass flow rate into tank = Mass in flow rate - Mass out flow rate.

$$\therefore eA \frac{dH}{dt} = eQ_i - eQ_o$$

$$\therefore A \frac{dH}{dt} = Q_i - CH \dots \{ Q_o = CH$$



$$\begin{aligned} \therefore A \frac{dH}{dt} &= Q_i - Q_o \\ \therefore Q_i &= A \frac{dH}{dt} + CH \end{aligned}$$

This equation represents the differential equation for head H

Now,

Taking Laplace Transform on both sides,

$$\begin{aligned} L[Q_i] &= A_s L\left[\frac{dH}{dt}\right] + C_s L[H] \\ \therefore Q_i(s) &= A_s\{sH(s) - H(0)\} + C_s H(s) \\ \therefore Q_i(s) &= A_s \cdot sH(s) + C_s H(s) \\ \\ \therefore Q_i(s) &= (sA_s + C_s) H(s) \end{aligned}$$

.....{ $\because H(0) = 0$

$$\therefore \frac{H(s)}{Q_i(s)} = \frac{1}{sA_s + C_s} \dots\dots(i)$$

But $Q_o = CH$

Taking Laplace Transform, we get

$$\begin{aligned} Q_o(s) &= C_s H(s) \\ \therefore H(s) &= \frac{Q_o(s)}{C_s} \end{aligned}$$

Using this in equation (i), we get

$$\begin{aligned} \frac{Q_o(s)}{C_s Q_i(s)} &= \frac{1}{sA_s + C_s} \\ \therefore \frac{Q_o(s)}{Q_i(s)} &= \frac{1}{1 + \left(\frac{A_s}{C_s}\right)s} \end{aligned}$$

This equation represents the transfer function of system.

$$\text{timeconstant, } \tau = \frac{A_s}{C_s} \therefore$$



III. CONCLUSION

Through this paper, we present the applications of Laplace Transformation in different Science & Engineering fields, like Electronics, Mechanical and Physics etc. Besides these, Laplace Transform is a very effective tool to simplify very complex problems in the area of Process and control. It goes without saying that Laplace Transform has tremendous applications in Science & Engineering field.

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