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# Supply Chain Inventory Model for Deteriorating Items with Multiple Buyers Single Vendor under Time Dependent Demand and Variable Holding Cost 

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#### Abstract

An optimal policy for vendor and buyer is developed for items having deterioration and demand is linear function of time. A single vendor and multiple buyers system model is developed as profit maximization to determine the system's optimal cycle time (strategy). We also determine the profit of buyer-vendor jointly. Numerical illustrations show that both buyers and vendor earn significant profit in supply chain inventory system. Post-optimality analysis is also carried out for parameters.


KEYWORDS: Different Deterioration, Multiple Buyers, Optimal strategy, Single Vendor, Supply chain, Time dependent demand, Time varying holding cost

## I. Introduction

In supply chain inventory models many times a single buyer and single vendor manage collaborative inventory strategy to maximize joint profit or minimize total cost. But this strategy has limited global markets scenario. In recent times importance of supply chain inventory model develop memberships between the associated entities of supply chain. Buyer and vendor apply a strategy of business, generally, in classical methods of economic order quantity (EOQ) or Economic production quantity (EPQ). But according to today's business requirements, this concept is replaced by concept of integrated economic lot size (IELS) or supply chain strategy for improvement of market or demand of items. Supply chain inventory system is the collaborative process between individual business parties for achieving maximum joint profit or minimum joint cost.

An inventory model for purchaser and vendor under supply chain was derived by Banerjee [1]. He derived optimum quantity when vendor produce equal lot size under deterministic conditions. Hill [2] also derived supply chain model for global most favorable batching and delivery plan under manufacturing inventory strategy when single vendor and single buyer are considered in integrated system. Goyal [3] proposed and designed integrated inventory model that the consignment bulk would be estimated by first delivery amount. Wee and Yang [4] derived coordinated inventory model for buyer and vendor under deterministic demand when buyer and vendor worked together to minimize joint costs. Wee and Jong [5] formulated one buyer one producer collaborative inventory model where the influences of unit price, deterioration factor, producer's and buyer's unit cost was taken into consideration. Jauhari et al. [6] derived a supply chain inventory model where system leads to minimize joint cost of supply chain when single retailer and single purchaser is considered and adopted probabilistic demand. Seifert et al. [7] investigated two models such as without collaboration between various systems and with collaboration between two members only from entire supply chain. A time dependent coordinated inventory model for single seller and single purchaser derived by Gaobo [8] and computed expressions for optimal total cost for optimal order quantity and price discount. A collaborative inventory model was developed by Vijayashree and Uthayakumar [9] for single manufactured goods when the system consists of single retailer and single consumer. They assumed that manufacture rate is fixed/constant and lead time follows normal distribution. Ghiami and Williams [10] delivered two levels production inventory models with multiple buyers and one manufacturer when deteriorating items has fixed production rate and the order quantities are dispatched by the manufacturer to the consumers for definite period and the surplus inventory supplies for successive deliveries. Srinivas and Reddy [11] determined the integrated inventory model with consignment strategy when multiple buyers and single
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vendor included in supply chain inventory. The integrated inventory model optimized the order quantity and determined minimum joint cost. Saha [12] developed an inventory model for deteriorating items in a supply chain management system under fuzzy environment. Gani and Dharik [13] developed multiple buyers and single vendor supply chain inventory model based on a shipment stock and vendor followed inventory policy suggested by management where demand and manufacture rate of the vendor are considered trapezoidal fuzzy numbers. EOQ and EPQ inventory model derived by Jonrinaldi et al. [14] under supply chain strategy considered multiple buyers and single vendor with multiple items and determine the vendor's optimal lot size and buyers' number of orders for shipment policy by considering discrete and continuous demand. For multiple products when single supplier and multiple buyers consist in inventory model and derived integrated model by Powar and Nandurkar [15] under supply chain policy determined the optimal joint reorder point, shipments and order quantity for each buyer subject to decrease the co-ordinated cost of buyers and vendor. Three-tier supply chains inventory management models including supplier sector, a manufacturer and retailer was formulated by Ghourchiany and Bafrouei [16]. Expressions for obtaining optimal amount of raw material order, optimal amount of the production and optimal order of retailers was also derived. An algorithm was provided for obtaining these units.

A one vendor multiple buyers combined inventory models for varying deterioration for buyers and time varying holding cost for vendor buyer both under time dependent demand is considered in this chapter. We assume that vendor has better preservation technology, so preservation technology cost is included for vendor and therefore there is no deterioration cost for vendor.

## II. ASSUMPTIONS AND NOTATIONS

## NOTATIONS:

The first objective of this section is to list all the used notations in the subsequent sections for easy reference.
$D(t): a_{i}+b_{i} t$, where $a_{i}>0,0<b_{i}<1$
$\mathrm{I}_{\mathrm{b} i}(\mathrm{t}): \mathrm{i}^{\text {th }}$ buyer's inventory size at time t
$\mathrm{I}_{\mathrm{v}}(\mathrm{t})$ : Vendor's inventory size at time t
$A_{b i}$ : Ordering cost of $i^{\text {th }}$ buyer's per order
$A_{v}$ : Ordering cost of vendor per order
$\mathrm{c}_{\mathrm{b}} \quad$ : Unit purchasing cost of buyer
$\theta_{\mathrm{i}} \quad: \mathrm{i}^{\text {th }}$ buyer's deterioration rate during $\mathrm{t}_{1} \leq \mathrm{t} \leq \mathrm{t}_{2}, 0<\theta_{\mathrm{i}}<1$
$\theta_{\mathrm{i}} \mathrm{t} \quad: \mathrm{i}^{\text {th }}$ buyer's deterioration rate during, $\mathrm{t}_{2} \leq \mathrm{t} \leq\left(\mathrm{T} / \mathrm{n}_{\mathrm{i}}\right), 0<\theta_{\mathrm{i}}<1$
$\mathrm{x}_{\mathrm{bi}}: \mathrm{i}^{\text {th }}$ Buyer's fixed holding cost
$y_{b i}: i^{\text {th }}$ Buyer's varying holding cost
$\mathrm{X}_{\mathrm{v}} \quad$ : Fixed holding cost of vendor
$y_{v} \quad$ : Varying holding cost of vendor
$\mathrm{p}_{\mathrm{i}} \quad$ : Selling price of $\mathrm{i}^{\text {th }}$ buyer's per unit
m : Preservation technology cost for vendor (fixed)
$n_{i}$ : Number of time orders placed by $i^{\text {th }}$ buyer during cycle time.
$\mathrm{TP}_{\mathrm{bi}}$ : Total profit of $\mathrm{i}^{\text {th }}$ buyer
$\mathrm{TP}_{\mathrm{v}}$ : Vendor's total profit
TP : Integrated total profit for both vendor and buyers per time unit. $\mathrm{t}_{1}=\mathrm{v}_{1} *\left(\mathrm{~T} / \mathrm{n}_{\mathrm{i}}\right), \mathrm{t}_{2}=\mathrm{v}_{2} *\left(\mathrm{~T} / \mathrm{n}_{\mathrm{i}}\right)$, where $\mathrm{T}_{\mathrm{b}}=\mathrm{T} / \mathrm{n}_{\mathrm{i}}$
T : Cycle time of vendor (a decision variable).

## ASSUMPTIONS:

Further we present the assumptions related to the work.

- The demand is declining function of time.
- One vendor multiple buyers are considered.
- Stock out is not permitted.
- Lead time is zero.
- During the cycle time, no repairing or replacement of deteriorated units and deterioration is dependent on time for buyer's inventory.
- For buyer and vendor both, time varying holding cost is considered.
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## III. MATHEMATICAL MODEL AND ANALYSIS

Level of inventory of $i^{\text {th }}$ buyer's at time $t$ be given by $\mathrm{I}_{\mathrm{bi}}(\mathrm{t})\left(0 \leq \mathrm{t} \leq \mathrm{T} / \mathrm{n}_{\mathrm{i}}\right)$ is shown below:

## Buyer's Inventory



Fig. 1. Inventory size of buyer
Two situations are discussed. In the first situation, there is no collaboration between vendor and buyers, while in the second situation there is collaboration of buyers and vendor. Considering time and price dependent demand, inventory size is given for buyers and vendor.
Change in inventory sizes are given by following differential equations for buyers and vendor:
$\begin{array}{lr}\frac{d I_{b i}(t)}{d t}=-\left(a_{i}+b_{i} t\right), & 0 \leq t \leq t_{1} \\ \frac{d I_{b i}(t)}{d t}+\theta_{i} I_{b i}(t)=-\left(a_{i}+b_{i} t\right), & t_{1} \leq t \leq t_{2} \\ \frac{d I_{b i}(t)}{d t}+\theta_{i} I_{b i}(t)=-\left(a_{i}+b_{i} t\right), & t_{2} \leq t \leq \frac{T}{n_{i}} \\ \frac{d I_{v}(t)}{d t}=-\sum_{i=1}^{N}\left(a_{i}+b_{i} t\right), & 0 \leq t \leq T\end{array}$
initially taken conditions are: $\mathrm{I}_{\mathrm{bi}}(0)=\mathrm{Q}_{\mathrm{i}}, \mathrm{I}_{\mathrm{bi}}\left(\mathrm{t}_{1}\right)=\mathrm{S}_{\mathrm{li}}, \mathrm{I}_{\mathrm{bi}}\left(\frac{\mathrm{T}}{\mathrm{n}_{\mathrm{i}}}\right)=0$ and $\mathrm{I}_{\mathrm{v}}(\mathrm{T})=0$.
Solving equations (1) to (4) we have,
$I_{b i}(t)=Q_{i}-\left(a_{i} t+\frac{1}{2} b_{i} t^{2}\right)$,
$I_{b i}(t)=\left[\begin{array}{l}a_{i}\left(t_{1}-t\right)+\frac{1}{2} a_{i} \theta_{i}\left(t_{1}^{2}-t^{2}\right)+\frac{1}{2} b_{i}\left(t_{1}^{2}-t^{2}\right) \\ +\frac{1}{3} b_{i} \theta_{i}\left(t_{1}^{3}-t^{3}\right)-a_{i} \theta_{i} t\left(t_{1}-t\right)-\frac{1}{2} b_{i} \theta_{i} t\left(t_{1}^{2}-t^{2}\right)\end{array}\right]+S_{1 i}\left[1+\theta_{i}\left(t_{1}-t\right)\right]$
$I_{b i}(t)=\left[a_{i}\left(\frac{T}{n_{i}}-t\right)+\frac{1}{2} b_{i}\left(\frac{T^{2}}{n_{i}^{2}}-t^{2}\right)+\frac{1}{6} a_{i} \theta_{i}\left(\frac{T^{3}}{n_{i}^{3}}-t^{3}\right)+\frac{1}{8} b_{i} \theta_{i}\left(\frac{T^{4}}{n_{i}^{4}}-t^{4}\right)-\frac{1}{2} a_{i} \theta_{i} t^{2}\left(\frac{T}{n_{i}}-t\right)-\frac{1}{4} b_{i} \theta_{i} t^{2}\left(\frac{T^{2}}{n_{i}^{2}}-t^{2}\right)\right]$.
$I_{v}(t)=\sum_{i=1}^{N}\left[a_{i}(T-t)+\frac{1}{2} b_{i}\left(T^{2}-t^{2}\right)\right]$.
(by not considering higher powers of $\theta$ )
Substituting $t=t_{1}$, in (5) gives
$Q_{i}=S_{1 i}+\left(a_{i} t_{1}+\frac{1}{2} b_{i} t_{1}^{2}\right)$.
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From equations (6) and (7), putting $t=t_{2}$, we have

$$
\begin{align*}
& I_{b i}\left(t_{2}\right)=\left[\begin{array}{l}
a_{i}\left(t_{1}-t_{2}\right)+\frac{1}{2} a_{i} \theta_{i}\left(t_{1}^{2}-t_{2}^{2}\right)+\frac{1}{2} b_{i}\left(t_{1}^{2}-t_{2}^{2}\right) \\
+\frac{1}{3} b_{i} \theta_{i}\left(t_{1}^{3}-t_{2}^{3}\right)-a_{i} \theta_{i} t_{2}\left(t_{1}-t_{2}\right)-\frac{1}{2} b_{i} \theta_{i} t_{2}\left(t_{1}^{2}-t_{2}^{2}\right)
\end{array}\right]+S_{1 i}\left[1+\theta_{i}\left(t_{1}-t_{2}\right)\right]  \tag{10}\\
& I_{b i}\left(t_{2}\right)=\left[\begin{array}{l}
a_{i}\left(\frac{T}{n_{i}}-t_{2}\right)+\frac{1}{2} b_{i}\left(\frac{T^{2}}{n_{i}^{2}}-t_{2}^{2}\right)+\frac{1}{6} a_{i} \theta_{i}\left(\frac{T^{3}}{n_{i}^{3}}-t_{2}^{3}\right) \\
+\frac{1}{8} b_{i} \theta_{i}\left(\frac{T^{4}}{n_{i}^{4}}-t_{2}^{4}\right)-\frac{1}{2} a_{i} \theta_{i} t_{2}^{2}\left(\frac{T}{n_{i}}-t_{2}\right)-\frac{1}{4} b_{i} \theta_{i} t_{2}^{2}\left(\frac{T^{2}}{n_{i}^{2}}-t_{2}^{2}\right)
\end{array}\right] . \tag{11}
\end{align*}
$$

So from equations (10) and (11), we have

$$
S_{1 i}=\frac{1}{\left[1+\theta_{i}\left(t_{1}-t_{2}\right)\right]}\left[\begin{array}{l}
a_{i}\left(\frac{T}{n_{i}}-t_{2}\right)+\frac{1}{2} b_{i}\left(\frac{T^{2}}{n_{i}^{2}}-t_{2}^{2}\right)+\frac{1}{6} a_{i} \theta_{i}\left(\frac{T^{3}}{n_{i}^{3}}-t_{2}^{3}\right)+\frac{1}{8} b_{i} \theta_{i}\left(\frac{T^{4}}{n_{i}^{4}}-t_{2}^{4}\right)-\frac{1}{2} a_{i} \theta_{i} \mu_{2}^{2}\left(\frac{T}{n_{i}}-t_{2}\right)  \tag{12}\\
-\frac{1}{4} b_{i} \theta_{i} t_{2}^{2}\left(\frac{T^{2}}{n_{i}^{2}}-t_{2}^{2}\right)-a_{i}\left(t_{1}-t_{2}\right)-\frac{1}{2} a_{i} \theta_{i}\left(t_{1}^{2}-t_{2}^{2}\right)-\frac{1}{2} b_{i}\left(t_{1}^{2}-t_{2}^{2}\right)-\frac{1}{3} b_{i} \theta_{i}\left(t_{1}^{3}-t_{2}^{3}\right) \\
+a_{i} \theta_{i} t_{2}\left(t_{1}-t_{2}\right)-\rho_{i} p_{i} \theta_{i} t_{2}\left(t_{1}-t_{2}\right)+\frac{1}{2} b_{i} \theta_{i} t_{2}\left(t_{1}^{2}-t_{2}^{2}\right)
\end{array}\right] .
$$

Substituting from (12), value of $S_{1}$ in (6) gives

$$
\begin{align*}
& I_{b i}(t)= {\left[1+\theta_{i}\left(t_{1}-t\right)\right] } \\
& {\left[1+\theta_{i}\left(t_{1}-t_{2}\right)\right] }
\end{align*}\left[\begin{array}{l}
a_{i}\left(\frac{T}{n_{i}}-t_{2}\right)+\frac{1}{2} b_{i}\left(\frac{T^{2}}{n_{i}^{2}}-t_{2}^{2}\right)+\frac{1}{6} a_{i} \theta_{i}\left(\frac{T^{3}}{n_{i}^{3}}-t_{2}^{3}\right)+\frac{1}{8} b_{i} \theta_{i}\left(\frac{T^{4}}{n_{i}^{4}}-t_{2}^{4}\right) \\
-\frac{1}{2} b_{i}\left(\frac{T}{2}\left(t_{1}^{2}-t_{2}^{2}\right)-\frac{1}{n_{i}}-t_{2}\right)-\frac{1}{4} b_{i} \theta_{i} \theta_{i}\left(t_{2}^{2}\left(\frac{T_{1}^{3}}{n_{i}^{2}}-t_{2}^{2}\right)-t_{2}^{3}\right)+a_{i} \theta_{i} t_{1}-t_{2}\left(t_{1}-t_{2}\right)-\frac{1}{2} a_{i} \theta_{i}\left(t_{1}^{2}-t_{2}^{2}\right) \\
2  \tag{13}\\
b_{i} \theta_{i} t_{2}\left(t_{1}^{2}-t_{2}^{2}\right)
\end{array}\right] .
$$

Using (12) in (9), we have

$$
Q_{i}=\frac{1}{\left[1+\theta_{i}\left(t_{1}-t_{2}\right)\right]}\left[\begin{array}{l}
a_{i}\left(\frac{T}{n_{i}}-t_{2}\right)+\frac{1}{2} b_{i}\left(\frac{T^{2}}{n_{i}^{2}}-t_{2}^{2}\right)+\frac{1}{6} a_{i} \theta_{i}\left(\frac{T^{3}}{n_{i}^{3}}-t_{2}^{3}\right)+\frac{1}{8} b_{i} \theta_{i}\left(\frac{T^{4}}{n_{i}^{4}}-t_{2}^{4}\right)  \tag{1}\\
-\frac{1}{2} a_{i} \theta_{i} t^{2}\left(T_{b}-t_{2}\right)-\frac{1}{4} b_{i} \theta_{i} t_{2}^{2}\left(\frac{T^{2}}{n_{i}^{2}}-t_{2}^{2}\right)-a_{i}\left(t_{1}-t_{2}\right)-\frac{1}{2} a_{i} \theta_{i}\left(t_{1}^{2}-t_{2}^{2}\right) \\
-\frac{1}{2} b_{i}\left(t_{1}^{2}-t_{2}^{2}\right)-\frac{1}{3} b_{i} \theta_{i}\left(t_{1}^{3}-t_{2}^{3}\right)+a_{i} \theta_{i} t_{2}\left(t_{1}-t_{2}\right)+\frac{1}{2} b_{i} \theta_{i} t_{2}\left(t_{1}^{2}-t_{2}^{2}\right)
\end{array}\right]+\left(a_{i} t_{1}+\frac{1}{2} b_{i} t_{1}^{2}\right)
$$

Using value of Q in (5), we have

$$
I_{b i}(t)=\frac{1}{\left[1+\theta_{i}\left(t_{1}-t_{2}\right)\right]}\left[\begin{array}{l}
a_{i}\left(\frac{T}{n_{i}}-t_{2}\right)+\frac{1}{2} b_{i}\left(\frac{T^{2}}{n_{i}^{2}}-t_{2}^{2}\right)+\frac{1}{6} a_{i} \theta_{i}\left(\frac{T^{3}}{n_{i}^{3}}-t_{2}^{3}\right)+\frac{1}{8} b_{i} \theta_{i}\left(\frac{T^{4}}{n_{i}^{4}}-t_{2}^{4}\right)  \tag{15}\\
-\frac{1}{2} a_{i} \theta_{i} t_{2}^{2}\left(\frac{T}{n_{i}}-t_{2}\right)-\frac{1}{4} b_{i} \theta_{i} t_{2}^{2}\left(\frac{T^{2}}{n_{i}^{2}}-t_{2}^{2}\right)-a_{i}\left(t_{1}-t_{2}\right)-\frac{1}{2} a_{i} \theta_{i}\left(t_{1}^{2}-t_{2}^{2}\right)-\frac{1}{2} b_{i}\left(t_{1}^{2}-t_{2}^{2}\right) \\
-\frac{1}{3} b_{i} \theta_{i}\left(t_{1}^{3}-t_{2}^{3}\right)+a_{i} \theta_{i} t_{2}\left(t_{1}-t_{2}\right)-\rho_{i} p_{i} \theta_{i} \mu_{2}\left(\mu_{1}-\mu_{2}\right)+\frac{1}{2} b_{i} \theta_{i} \mu_{2}\left(\mu_{1}^{2}-\mu_{2}^{2}\right)
\end{array}\right]
$$

$$
+\left(\mathrm{a}_{\mathrm{i}} \mathrm{t}_{1}+\frac{1}{2} \mathrm{~b}_{\mathrm{i}} \mathrm{t}_{1}^{2}\right) .
$$

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Total relevant profit consisted of following elements:
Buyers' relevant costs:
(i) Ordering cost $\left(\mathrm{OC}_{\mathrm{b}}\right)=\sum_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{n}_{\mathrm{i}} \mathrm{A}_{\mathrm{b}}$
(ii) Holding Cost:

$$
\begin{align*}
& \left(\mathrm{HC}_{\mathrm{b}}\right)=\sum_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{n}_{\mathrm{i}}\left[\mathrm{x}_{\mathrm{bi}}\left\{\int_{0}^{\frac{\mathrm{T}}{n_{\mathrm{i}}}} \mathrm{I}_{\mathrm{bi}}(\mathrm{t}) \mathrm{dt}\right\}+\mathrm{y}_{\mathrm{bi}}\left\{\int_{0}^{\frac{\mathrm{T}}{n_{i}}} \mathrm{t} \mathrm{I}_{\mathrm{bi}}(\mathrm{t}) \mathrm{dt}\right\}\right] \\
& =\sum_{i=1}^{N} n_{i}\left[x_{b i}\left\{\int_{0}^{t_{1}} I_{b i}(t) d t+\int_{t_{1}}^{t_{2}} I_{b i}(t) d t+\int_{t_{2}}^{\frac{T}{n_{i}}} I_{b i}(t) d t\right\}+y_{b i}\left\{\int_{t_{1}}^{\frac{T}{n_{i}}} \mathrm{t}_{\mathrm{bi}^{2}}(t) d t+\int_{t_{1}}^{t_{2}} t t_{b i}(t) d t+\int_{t_{2}}^{\frac{T}{n_{i}}} \mathrm{t}_{\mathrm{bi}^{2}}(t) d t\right\}\right] \tag{17}
\end{align*}
$$

(iii) Deterioration Cost:

$$
\begin{equation*}
\left(\mathrm{DC}_{\mathrm{b}}\right)=\sum_{\mathrm{i}=1}^{N} \mathrm{n}_{\mathrm{i}} \mathrm{c}_{\mathrm{b}}\left[\int_{\mathrm{t}_{1}}^{\mathrm{t}_{2}} \theta_{\mathrm{i}} \mathrm{I}_{\mathrm{bi}}(\mathrm{t}) \mathrm{dt}+\int_{\mathrm{t}_{2}}^{\frac{\mathrm{T}}{n_{i}}} \theta_{\mathrm{i}} \mathrm{I}_{\mathrm{bi}}(\mathrm{t}) \mathrm{dt}\right] . \tag{18}
\end{equation*}
$$

(iv) Sales Revenue:

$$
\begin{equation*}
\left(\mathrm{SR}_{\mathrm{b}}\right)=\sum_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{n}_{\mathrm{i}} \mathrm{p}_{\mathrm{i}}\left[\int_{0}^{\frac{\mathrm{T}}{n_{\mathrm{i}}}}\left(\mathrm{a}_{\mathrm{i}}+\mathrm{b}_{\mathrm{i}} \mathrm{t}\right) \mathrm{dt}\right] \tag{19}
\end{equation*}
$$

(v) Total Profit (buyers):

$$
\begin{equation*}
\mathrm{TP}_{\mathrm{bi}}=\frac{1}{\mathrm{~T}}\left[\mathrm{SR}_{\mathrm{b}}-\mathrm{OC}_{\mathrm{b}}-\mathrm{HC}_{\mathrm{b}}-\mathrm{DC}_{\mathrm{b}}\right] \tag{20}
\end{equation*}
$$

## Vendor's relevant costs:

(i) Ordering cost $\left(\mathrm{OC}_{\mathrm{v}}\right)=\mathrm{Av}$
(ii) Holding Cost:

$$
\left.\left.\begin{array}{rl}
\left(H C_{v}\right)= & x_{v} \tag{21}
\end{array}\right] \int_{0}^{T} I_{v}(t) d t-\sum_{i=1}^{N} n_{i}\left[\left\{\int_{0}^{t_{1}} I_{b i}(t) d t+\int_{t_{1}}^{t_{2}} I_{b i}(t) d t+\int_{t_{2}}^{\frac{T}{n_{i}}} I_{b i}(t) d t\right\}\right]\right] .
$$

(iii) Preservation Technology Cost $\left(\mathrm{PTC}_{\mathrm{v}}\right)=\mathrm{m}$
(iv) Sales Revenue:

$$
\begin{equation*}
\left(\operatorname{SR}_{v}\right)=c_{b}\left(\sum_{\mathrm{i}=1}^{\mathrm{N}}\left[\int_{0}^{\mathrm{T}}\left(\mathrm{a}_{\mathrm{i}}+\mathrm{b}_{\mathrm{i}} \mathrm{t}\right) \mathrm{dt}\right]\right)=\mathrm{c}_{\mathrm{b}}\left(\sum_{\mathrm{i}=1}^{\mathrm{N}}\left(\mathrm{a}_{\mathrm{i}} \mathrm{~T}+\frac{1}{2} \mathrm{~b}_{\mathrm{i}} \mathrm{~T}^{2}\right)\right) \tag{23}
\end{equation*}
$$

(not considering higher powers of $\theta$ )
(v) Total Profit (vendor):

$$
\begin{equation*}
\mathrm{TP}_{\mathrm{v}}=\frac{1}{\mathrm{~T}}\left[\mathrm{SR}_{\mathrm{v}}-\mathrm{OC}_{\mathrm{v}}-\mathrm{HC}_{\mathrm{v}}-\mathrm{PTC}_{\mathrm{v}}\right] \tag{25}
\end{equation*}
$$

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## Situation I: Buyer and vendor take independent decision:

Here the buyers and vendor make decision independently. For given value of $\mathrm{n}, \mathrm{TP}_{\mathrm{b}}$ can be maximized by solving

$$
\begin{equation*}
\frac{\partial \mathrm{TP}_{\mathrm{b}}\left(\mathrm{~T}_{\mathrm{b}}\right)}{\partial \mathrm{T}_{\mathrm{b}}}=0, \text { where } \mathrm{T}_{\mathrm{b}}=\frac{\mathrm{T}}{\mathrm{n}_{\mathrm{i}}} \tag{26}
\end{equation*}
$$

provided it satisfies the second order condition

$$
\begin{equation*}
\frac{\partial^{2} \mathrm{TP}_{\mathrm{b}}\left(\mathrm{~T}_{\mathrm{b}}\right)}{\partial \mathrm{T}_{\mathrm{b}}^{2}}<0 \tag{27}
\end{equation*}
$$

This solution ( $\mathrm{n}, \mathrm{T}$ ) maximizes $\mathrm{TP}_{\mathrm{v}}$.
Then the total profit without collaboration is given by:
$\mathrm{TP}=\max \left(\mathrm{TP}_{\mathrm{b}}+\mathrm{TP}_{\mathrm{v}}\right)$.

## Situation-II: Joint decision of buyer and vendor:

Here joint decision is taken by buyers and vendor:
The optimum values of T must satisfy the following conditions which maximize total profit (TP) when buyers and vendor take joint decision. that $T$ is expressed in terms of $W$ and $t_{r}$, hence $T_{b}$ is not a decision variable.

$$
\begin{equation*}
\frac{\partial \mathrm{TP}_{\mathrm{v}}(\mathrm{~T})}{\partial \mathrm{T}}=0, \text { where } \mathrm{T}_{\mathrm{b}}=\frac{\mathrm{T}}{\mathrm{n}_{\mathrm{i}}} . \tag{28}
\end{equation*}
$$

provided it satisfies the second order condition

$$
\begin{equation*}
\frac{\partial^{2} \mathrm{TP}_{\mathrm{v}}(\mathrm{~T})}{\partial \mathrm{T}^{2}}<0 \tag{29}
\end{equation*}
$$

where total profit (TP) with collaboration is given by:
$\mathrm{TP}=\mathrm{TP}_{\mathrm{b}}+\mathrm{TP}_{\mathrm{v}}$

## IV. Numerical example

Various parameter values in appropriate units are taken for numerical illustration, $\mathrm{A}_{\mathrm{b} 1}=85, \mathrm{~A}_{\mathrm{b} 2}=65, \mathrm{a}_{1}=650, \mathrm{a}_{2}=$ $550, \mathrm{~b}_{1}=0.05, \mathrm{~b}_{2}=0.05, \theta_{1}=0.06, \theta_{2}=0.04, \mathrm{c}_{\mathrm{b}}=40, \mathrm{p}_{1}=77, \mathrm{p}_{2}=73, \mathrm{x}_{\mathrm{b} 1}=4.5, \mathrm{x}_{\mathrm{b} 2}=5.5, \mathrm{y}_{\mathrm{b} 1}=0.04, \mathrm{y}_{\mathrm{b} 2}=0.06, \mathrm{~m}=5, \mathrm{~A}_{\mathrm{v}}=2000$, $\mathrm{x}_{\mathrm{v}}=3, \mathrm{y}_{\mathrm{v}}=0.03, \mathrm{v}_{1}=0.3, \mathrm{v}_{2}=0.50, \mathrm{~N}=2$ in appropriate units.
Table provides optimum independent and joint values of T and profits for buyers and vendor. The second order conditions given in equation (27) and equation (29) are also satisfied.

Table 1
Without collaboration and with collaboration optimum solution

|  | Independent Decision | Joint Decision |
| :--- | :--- | :--- |
| $\mathrm{n}_{1}, \mathrm{n}_{2}$ | $\mathrm{n}_{1}=5, \mathrm{n}_{2}=5$ | $\mathrm{n}_{1}=3, \mathrm{n}_{2}=3$ |
| T | 1.0453 | 1.0225 |
| Buyers Profit | 88777.1911 | 88620.0355 |
| Vendor's Profit | 44572.8256 | 44809.9226 |
| Total Profit | 133350.0167 | 133429.9581 |

## V. Post-optimality computations

Study of one parameter at a time, table below gives post-optimality computations.
Table 2
Sensitivity Analysis
Independent Decision

| Parameter | $\%$ | $\mathrm{n}_{1}$ | $\mathrm{n}_{2}$ | Profit(b) | Profit(v) | Profit(bv) |
| :---: | :--- | ---: | :---: | :--- | :--- | :--- |
| $\mathrm{a}_{1}, \mathrm{a}_{2}$ | $+20 \%$ | 5 | 5 | 106682.4052 | 53846.5450 | 160528.9502 |
|  | $+10 \%$ | 5 | 5 | 97728.2627 | 49205.9595 | 146934.2222 |
|  | $-10 \%$ | 5 | 5 | 79829.6649 | 39948.2784 | 119777.9433 |
|  | $-20 \%$ | 5 | 5 | 70886.2947 | 35333.7917 | 106220.0864 |
|  | $+20 \%$ | 5 | 5 | 88640.1241 | 44595.1639 | 133235.2880 |

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|  | +10\% | 5 | 5 | 88707.1252 | 44587.9335 | 133295.0587 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | -10\% | 5 | 5 | 88850.7947 | 44547.2893 | 133398.0840 |
|  | -20\% | 6 | 6 | 88928.5449 | 44525.3209 | 133453.8658 |
| $\mathrm{X}_{\mathrm{b} 1}, \mathrm{X}_{\mathrm{b} 2}$ | +20\% | 5 | 5 | 88657.5352 | 44530.1876 | 133187.7228 |
|  | +10\% | 5 | 5 | 88716.1797 | 44553.3888 | 133269.5685 |
|  | -10\% | 5 | 5 | 88840.8765 | 44587.4166 | 133428.2931 |
|  | -20\% | 5 | 5 | 88907.6145 | 44595.5201 | 133503.1346 |
| $\theta_{1}, \theta_{1}$ | +20\% | 5 | 5 | 88760.5288 | 44567.4290 | 133327.9578 |
|  | +10\% | 5 | 5 | 88768.8304 | 44570.1675 | 133338.9979 |
|  | -10\% | 5 | 5 | 88785.6124 | 44575.3917 | 133361.0041 |
|  | -20\% | 5 | 5 | 88794.0957 | 44577.8210 | 133371.9167 |
| $\mathrm{A}_{\mathrm{v}}$ | +20\% | 5 | 5 | 88777.1911 | 44190.1603 | 132967.3514 |
|  | +10\% | 5 | 5 | 88777.1911 | 44381.4929 | 133158.6840 |
|  | -10\% | 4 | 4 | 88777.1911 | 44858.6907 | 133635.8818 |
|  | -20\% | 4 | 4 | 88777.1911 | 45050.0233 | 133827.2144 |
| $\mathrm{X}_{\mathrm{v}}$ | +20\% | 4 | 4 | 88777.1911 | 44385.2627 | 133162.4538 |
|  | +10\% | 4 | 4 | 88777.1911 | 44526.3103 | 133303.5015 |
|  | -10\% | 5 | 5 | 88777.1911 | 44723.3117 | 133500.5028 |
|  | -20\% | 5 | 5 | 88777.1911 | 44873.7979 | 133650.9890 |

Table 3
Sensitivity Analysis
Joint Decision

| Parameter | \% | $\mathrm{n}_{1}$ | $\mathrm{n}_{2}$ | Profit(b) | Profit(v) | Profit(bv) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{a}_{1}, \mathrm{a}_{2}$ | +20\% | 3 | 3 | 106510.8479 | 54106.2987 | 160617.1466 |
|  | +10\% | 3 | 3 | 97563.7945 | 49454.5984 | 147018.3929 |
|  | -10\% | 3 | 3 | 79680.2547 | 40173.1818 | 119853.4365 |
|  | -20\% | 3 | 3 | 70745.1025 | 35545.7782 | 106290.8807 |
| $\mathrm{A}_{\mathrm{b} 1}, \mathrm{~A}_{\mathrm{b} 2}$ | +20\% | 3 | 3 | 88520.3876 | 44822.3312 | 133342.7188 |
|  | +10\% | 3 | 3 | 88569.8309 | 44816.3139 | 133386.1448 |
|  | -10\% | 4 | 4 | 88803.9296 | 44679.6283 | 133483.5579 |
|  | -20\% | 4 | 4 | 88865.1698 | 44674.5836 | 133539.7534 |
| $\mathrm{x}_{\mathrm{b} 1}, \mathrm{x}_{\mathrm{b} 2}$ | +20\% | 4 | 4 | 88598.3719 | 44670.3827 | 133268.7546 |
|  | +10\% | 4 | 4 | 88670.1673 | 44677.5702 | 133347.7375 |
|  | -10\% | 3 | 3 | 88707.9214 | 44824.7561 | 133532.6775 |
|  | -20\% | 3 | 3 | 88798.9759 | 44838.6977 | 133637.6736 |
| $\theta_{1}, \theta_{1}$ | +20\% | 4 | 4 | 88726.4291 | 44682.5544 | 133408.9835 |
|  | +10\% | 4 | 4 | 88735.1385 | 44683.3493 | 133418.4878 |
|  | -10\% | 3 | 3 | 88630.4966 | 44811.9672 | 133442.4638 |
|  | -20\% | 3 | 3 | 88640.9679 | 44814.0427 | 133455.0106 |
| $\mathrm{A}_{\mathrm{v}}$ | +20\% | 4 | 4 | 88714.0147 | 44358.8485 | 133072.8632 |
|  | +10\% | 4 | 4 | 88729.3590 | 44517.9434 | 133247.3024 |
|  | -10\% | 3 | 3 | 88649.1132 | 44980.5467 | 133629.6599 |
|  | -20\% | 3 | 3 | 88677.6573 | 45160.6565 | 133838.3138 |
| $\mathrm{X}_{\mathrm{v}}$ | +20\% | 3 | 3 | 88653.0600 | 44537.6375 | 133190.6975 |
|  | +10\% | 3 | 3 | 88637.4089 | 44671.4866 | 133308.8955 |
|  | -10\% | 4 | 4 | 88731.6427 | 44845.2559 | 133576.8986 |
|  | -20\% | 4 | 4 | 88716.9816 | 45013.6688 | 133730.6504 |

Based on the results of Table 2 and Table 3, we can observe about the optimal length of order cycle T* and maximum total profits for independent as well as joint decisions.
There will be increase or decrease in value of profits when parameters ' $a_{1}$, $a_{2}$ ' increase/ decrease independently as well as jointly, however, when $\mathrm{A}_{\mathrm{b} 1}, \mathrm{~A}_{\mathrm{b} 2}, \mathrm{X}_{\mathrm{b} 1}, \mathrm{x}_{\mathrm{b} 2}, \mathrm{x}_{\mathrm{v}}, \mathrm{A}_{\mathrm{v}}$, and $\theta_{1}, \theta_{2}$ increase/decrease then total profit decrease/increase in independent and joint decision case.
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## VI. CONCLUSION

The result shows that the optimal cycle time is significantly decreased and total profit significantly increased when buyers and vendor take joint decision as compared to independent decision taken by buyers and vendor.

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