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[ijirset@gmail.com](mailto:ijirset@gmail.com)

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# Complex Some of the Results of the Analysis in Higher Mathematics

Sulaymonov Ilyosxo'ja Abdirasulo'g'li

2nd year student of the National University of Uzbekistan named after MirzoUlugbek

**ANNOTATION:** This article is about the application complex analysis in real analysis, which gives us the opportunity to calculate several important integrals in a simple and convenient way and get some additional results. It is also stated that we can get some results even from another important integral Eyler-Puaason integral.

**KEYWORDS:** application complex analysis, another important integral

$$F(t) = \int_a^t f(x)dx, \text{ integral, Eyler formulation,}$$

$$e^{i\varphi}, \int_0^{\infty} e^{-x^2} dx, \text{ Eyler-Puasson integral}$$

**1. Specific integrals depending on the parameter  $f(x)$  function  $[a, +\infty)$  in the range  $(a \in R)$  granted as voluntary  $[a, t]$  in  $(a \leq t < +\infty)$  let it be integrative. We enter the following**

$$F(t) = \int_a^t f(x)dx$$

It is known to us that  $t \rightarrow +\infty$  da  $F(t)$  this is the limit if the function has a limit  $f(x)$  function  $[a, +\infty)$  on the infinite interval, the singular is called an integral and

$$\int_a^{+\infty} f(x)dx$$

as defined.

## 2. Eyler-Poassan integral

$$I = \int_0^{\infty} e^{-x^2} dx$$

Let's look at the function below  $g(t) = (1 + t)e^{-t}$

$t = \pm x^2$  we perform the replacement and get the following

$$t = \pm x^2 \rightarrow \begin{cases} 0 < (1 - x^2)e^{x^2} < 1 \\ 0 < (1 + x^2)e^{-x^2} < 1 \end{cases} \rightarrow \begin{cases} 0 < (1 - x^2) < e^{-x^2} \\ 0 < e^{-x^2} < \frac{1}{1 + x^2} \end{cases}$$



from this we can have the following

$$1) (1 - x^2)^n < e^{-nx^2}, \quad 2) e^{-nx^2} < \frac{1}{(1 + x^2)^n}$$

We integrate 1-inequality (0,1) in the range, 2-inequality (0,∞) in the range

$$\int_0^1 (1 - x^2)^n dx < \int_0^1 e^{-nx^2} dx < \int_0^{+\infty} e^{-nx^2} dx < \int_0^{+\infty} \frac{dx}{(1 + x^2)^n} \Rightarrow \int_0^1 (1 - x^2)^n dx < \int_0^{+\infty} e^{-nx^2} dx < \int_0^{+\infty} \frac{dx}{(1 + x^2)^n}$$

$$\int_0^{+\infty} e^{-nx^2} dx = \left[ \begin{array}{l} p = \sqrt{nx} \\ \frac{dp}{\sqrt{n}} = dx \end{array} \right] = \frac{1}{\sqrt{n}} \int_0^{+\infty} e^{-p^2} dp = \frac{1}{\sqrt{n}} I \Rightarrow \int_0^1 (1 - x^2)^n dx < \frac{1}{\sqrt{n}} I < \int_0^{+\infty} \frac{dx}{(1 + x^2)^n}$$

Now we calculate these two integers

$$\int_0^1 (1 - x^2)^n dx = \left[ \begin{array}{l} x = \cos t \\ dx = -\sin t dt \\ x = 0 \rightarrow t = \frac{\pi}{2} \\ x = 1 \rightarrow t = 0 \end{array} \right] = -\int_{\frac{\pi}{2}}^0 \sin^{2n+1} t dt = \int_0^{\frac{\pi}{2}} \sin^{2n+1} t dt = \frac{(2n)!!}{(2n+1)!!}$$

$$\int_0^{+\infty} \frac{dx}{(1 + x^2)^n} = \left[ \begin{array}{l} x = ctgt \\ dx = -\frac{1}{\sin^2 t} dt \\ x = 0 \rightarrow t = \frac{\pi}{2} \\ x = +\infty \rightarrow t = 0 \end{array} \right] = -\int_{\frac{\pi}{2}}^0 \sin^{2n-2} t dt = \int_0^{\frac{\pi}{2}} \sin^{2n-2} t dt = \frac{(2n-3)!! \pi}{(2n-2)!! 2}$$

$$\frac{(2n)!!}{(2n-1)!!} < \frac{I}{\sqrt{n}} < \frac{(2n-3)!! \pi}{(2n-2)!! 2} \rightarrow \sqrt{n} \frac{(2n)!!}{(2n+1)!!} < I < \frac{(2n-3)!! \pi}{(2n-2)!! 2} \sqrt{n}$$

$$\frac{n}{2n+1} \left[ \frac{(2n)!!}{(2n-1)!!} \right]^2 \frac{1}{2n+1} < I^2 < \frac{n}{2n-1} \left[ \frac{(2n-3)!!}{(2n-2)!!} \right]^2 (2n-1) \frac{\pi^2}{4}$$

From this, according to the vallis formula, we get the following

$$\lim_{n \rightarrow \infty} \frac{n}{2n+1} \left[ \frac{(2n)!!}{(2n-1)!!} \right]^2 \frac{1}{2n+1} = \frac{\pi}{4}$$

$$\lim_{n \rightarrow \infty} \frac{n}{2n-1} \left[ \frac{(2n-3)!!}{(2n-2)!!} \right]^2 (2n-1) \frac{\pi^2}{4} = \frac{\pi}{4}$$



All in all

$$I^2 = \frac{\pi}{4} \Rightarrow I = \frac{\sqrt{\pi}}{2}$$

A little  $z \in \mathbb{C}$  Complex number this  $z = r(\cos \varphi + i \sin \varphi)$  let it be given in appearance. Complex is one of the following important formulas known from the course of analysis  $e^{i\varphi} = \cos \varphi + i \sin \varphi$

Euler is the formula. Now we calculate these integrals with the help of the properties Complex numbers and the Euler-Puasson integral the following multiple integrals without the presence of the following integrals[2].

**1-example.** Calculate the following specific integrals

$$I_1 = \int_0^{\infty} e^{-ax} \cos bxdx, \quad I_2 = \int_0^{\infty} e^{-ax} \sin bxdx, \quad (a > 0)$$

Solution We write the above two integrals as follows

$$I_1 + iI_2 = \int_0^A e^{-ax} \cos bxdx + i \int_0^A e^{-ax} \sin bxdx \quad (1)$$

We look at the integral below.

$$\int_0^A e^{-ax} e^{ibx} dx = \int_0^A e^{(-a+ib)x} dx = \frac{e^{(-a+ib)x}}{-a+ib} \Big|_0^A = -\frac{a+ib}{a^2+b^2} e^{(-a+ib)x} \Big|_0^A = -\frac{a+ib}{a^2+b^2} e^{-ax} e^{ibx} \Big|_0^A$$

Now according to Euler's formulation  $e^{ibx} = \cos(bx) + i \sin(bx)$  we can write the above example as follows

$$\begin{aligned} -\frac{(a+ib)(\cos(bx) + i \sin(bx))}{a^2+b^2} e^{-ax} &= \frac{b \sin(bx) - a \cos(bx)}{a^2+b^2} e^{-ax} + \\ + i \frac{-a \sin(bx) - b \cos(bx)}{a^2+b^2} e^{-ax} \end{aligned} \quad (2)$$

As a result of the equation of (1) and (2) expressions according to the above theorem, we get the following

$$I_1 = \int_0^A e^{-ax} \cos bxdx = \frac{b \sin(bx) - a \cos(bx)}{a^2+b^2} e^{-x^2} \Big|_0^A$$

$$I_2 = \int_0^A e^{-ax} \sin bxdx = \frac{-a \sin(bx) - b \cos(bx)}{a^2+b^2} e^{-x^2} \Big|_0^A$$

$I_1$  and  $I_2$  value of  $A \rightarrow \infty$  although will be as follows:

$$I_1 = \frac{a}{a^2+b^2}, \quad I_2 = \frac{b}{a^2+b^2}$$

**2-example.** Calculate the following specific integrals:

$$I_1 = \int_0^{\infty} \cos(x^2) dx, \quad I_2 = \int_0^{\infty} \sin(x^2) dx$$

Taking into account the presence of these integrals, we are dealing with their calculation



Solution

We write the given integers as follows

$$I_1 - iI_2 = \int_0^{\infty} \cos(x^2) dx - i \int_0^{\infty} \sin(x^2) dx = \int_0^{\infty} (\cos(-x^2) + i \sin(-x^2)) dx = \int_0^{\infty} e^{-ix^2} dx$$

Now this  $\int_0^{\infty} e^{-ix^2} dx$  we calculate the integral

$$\int_0^{\infty} e^{-ix^2} dx = \left[ \begin{array}{l} \sqrt{i}x = t \\ ix^2 = t^2 \\ dx = \frac{1}{\sqrt{i}} dt \end{array} \right] = \frac{1}{\sqrt{i}} \int_0^{\infty} e^{-t^2} dt = \frac{1}{\sqrt{i}} \frac{\sqrt{\pi}}{2}$$

$\frac{1}{\sqrt{i}}$  let's calculate.  $\frac{1}{\sqrt{i}} = -i^{\frac{3}{2}} = -\left(\cos\left(\frac{\pi}{2}\right) + i \sin\left(\frac{\pi}{2}\right)\right)^{\frac{3}{2}}$  According to the Muavr formulation

$$-\left(\cos\left(\frac{\pi}{2}\right) + i \sin\left(\frac{\pi}{2}\right)\right)^{\frac{3}{2}} = -\cos\left(\frac{3\pi}{2}\right) - i \sin\left(\frac{3\pi}{2}\right) = \frac{\sqrt{2}}{2} - i \frac{\sqrt{2}}{2}$$

$$\text{So } I_1 - iI_2 = \frac{\sqrt{2}}{2} \frac{\sqrt{\pi}}{2} - i \frac{\sqrt{2}}{2} \frac{\sqrt{\pi}}{2} = \frac{1}{2} \sqrt{\frac{\pi}{2}} - i \frac{1}{2} \sqrt{\frac{\pi}{2}}$$

Complex according to the theorem on the equality of numbers

$$I_1 = \int_0^{\infty} \cos(x^2) dx = \frac{1}{2} \sqrt{\frac{\pi}{2}}$$

$$I_2 = \int_0^{\infty} \sin(x^2) dx = \frac{1}{2} \sqrt{\frac{\pi}{2}}$$

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