



e-ISSN: 2319-8753 | p-ISSN: 2347-6710

IJIRSET

International Journal of Innovative Research in
SCIENCE | ENGINEERING | TECHNOLOGY

INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH

IN SCIENCE | ENGINEERING | TECHNOLOGY

Volume 11, Issue 8, August 2022

ISSN INTERNATIONAL
STANDARD
SERIAL
NUMBER
INDIA

Impact Factor: 8.118

 9940 572 462

 6381 907 438

 ijirset@gmail.com

 www.ijirset.com

Some More Results on Generalized R-Norm Information Measure of Type Alpha and Degree Beta in Quantum Logic

Amandeep

Assistant Professor, Arya PG College, Panipat (Haryana), India

ABSTRACT: In the present communication, I deal with the mathematical modelling of R-Norm Information Measure of type α and degree β (written as $RIM_{(\alpha,\beta)}$) in quantum logic. Some results are extended concerning $RIM_{(\alpha,\beta)}$, Joint and Conditional $RIM_{(\alpha,\beta)}$ to the quantum logic.

KEYWORDS: R-Norm Information Measure, R-Norm Information Measure of type α and degree β , Quantum logic.

I. INTRODUCTION

Birkhoff and Neumann introduce the approach of quantum logic. The concept of information measure has been applied in many areas such as in physics, information theory, computer science, general systems theory, sociology, statistics, biology, chemistry and many other fields. There are several information measures presented in history such as Shannon's information measure [1] and the measures of order α and of type β , introduced by Renyi [2], Havrda-Charvat [3] and Daroczy [4] respectively.

We define Δ_n as the set of all n-ary probability distributions

$$\Delta_n = \left\{ P = (p_1, p_2, p_3, \dots, p_n); p_i \geq 0; i = 1, 2, \dots, n; \sum_{i=1}^n p_i = 1 \right\}$$

Information measure given by Shannon [1] is

$$H(P) = - \sum_{i=1}^n p_i \log p_i \quad (1)$$

The R-Norm Information Measure of the distribution P is given by Boekke and Lubbe [5] and is defined for $R \in \mathfrak{R}$ by

$$H_R(P) = \frac{R}{R-1} \left[1 - \left[\sum_{i=1}^n p_i^R \right]^{\frac{1}{R}} \right] \quad (2)$$

Where $\mathfrak{R} = \{R: R > 0, R \neq 1\}$ and the R-Norm Information measure is a real function from Δ_n to \mathfrak{R} defined on Δ_n ($n \geq 2$) R is real numbers set (see [4]). When $R \rightarrow 1$, R-Norm Information Measure approaches to Shannon's information measure and when $R \rightarrow \infty$, $H_R(P) \rightarrow (1 - \max p_i); i = 1, 2, \dots, n$.

Hooda and Sharma [6] studied R-Norm Information Measure of type α and degree β which is given by



$$H_R^{\alpha,\beta} = \frac{R}{R + \beta - 2\alpha} \left[1 - \left(\sum_{i=1}^n p_i^{\frac{R}{2\alpha-\beta}} \right)^{\frac{2\alpha-\beta}{R}} \right]; \alpha \geq 0, 0 \leq \beta \leq 1, R \in \mathfrak{R}, R + \beta \neq 2\alpha. \tag{3}$$

In (Ebrahimzadeh[7]–[9]), (Khare[10], [11]) and (Zhao [12]) the concepts of information measure, joint information measure and conditional information measure of partitions and dynamical systems have been studied in quantum logic. We can study the quantum information theory in an easy way by using the concepts of information measures in quantum logic (see [13]). In this paper, we study $RIM_{(\alpha,\beta)}$, Joint $RIM_{(\alpha,\beta)}$ and Conditional $RIM_{(\alpha,\beta)}$ in quantum logic. Some of the results of the information measures are generalized to the quantum logic.

II. SOME DEFINITIONS

Definition 1. [11] Consider a quantum logic QL. This is a σ -orthomodular lattice, i.e., a lattice $\Gamma(\Gamma, R, \vee, \wedge, 0, 1)$ where 0 is the minimal element and 1 is the maximal element. We can define an operation $'$: $\Gamma \rightarrow \Gamma$ such that the following properties hold $\forall u, v \in \Gamma$:

- (i) $(u')' = u$.
- (ii) $uRv \Rightarrow v'Ru'$.
- (iii) Given any finite sequence $\langle u_i \rangle, u_i R u'_j$, then $\bigvee_{i \in \mathbb{N}} u_i$ lies in Γ .
- (iv) Γ is orthomodular: $uRv \Rightarrow v = u \vee (v \wedge u')$.

In quantum logic we have $u \wedge u' = 0$, and $u \vee u' = 1$.

Definition 2.[11] Consider a quantum logic Γ . A state ξ is a mapping $\Gamma \rightarrow [0,1]$ such that:

- (i) $\xi(1) = 1$.
- (ii) $\xi(u \vee v) = \xi(u) + \xi(v)$ if $u \perp v$.

It may be observed that $\xi(0) = 0$, ξ is monotone and $\xi(u') = 1 - \xi(u), u \in \Gamma$.

Definition 3.[11] Consider u_1, \dots, u_n are the finite elements of a quantum logic. $P = \{u_1, \dots, u_n\}$ is \vee -orthogonal iff $\bigvee_{i=1}^k u_i \perp u_{k+1}, \forall k = 1, 2, \dots, n - 1$.

Definition 4.[14] Consider Γ is a quantum logic. $P = \{u_1, \dots, u_n\} \subset \Gamma$ is called a partition of Γ w.r.t. a state ξ on Γ if:

- (i) P is \vee -orthogonal.
- (ii) $\xi(\bigvee_{i=1}^n u_i) = 1$.

Remarks 1. From the above definitions, we can find that $\sum_{i=1}^n \xi(u_i) = 1$.

Definition 5.[11] Let (Γ, ξ) be any couple in quantum logic and $P = \{u_1, u_2, \dots, u_n\}$ and $Q = \{v_1, v_2, \dots, v_m\}$ be any two partitions of it. Q is said to be a ξ -refinement of P if \exists a partition $I(1), \dots, I(n)$ of the set $\{1, \dots, m\}$ s.t.

$$u_i = \bigvee_{j \in I(i)} v_j, \forall i = 1, \dots, n.$$

Definition 6.[11] Consider a partition $Q = \{v_1, \dots, v_m\}$ of a couple (Γ, ξ) and $u \in \Gamma$. The state ξ satisfy Bayes' property if

$$\xi \left(\bigvee_{j=1}^m (u \wedge v_j) \right) = \xi(u).$$



In this case, we get

$$\sum_{j=1}^m \xi(u \wedge v_j) = \xi(u).$$

Let us consider $P = \{u_1, u_2, \dots, u_n\}$ and $Q = \{v_1, \dots, v_m\}$ are the two partitions of (Γ, ξ) . Now, $P \vee Q$ is a partition of (Γ, ξ) if ξ has Bayes' property. Then the refinement of this partition is given by

$$P \vee Q = \{u_i \wedge v_j | u_i \in P, v_j \in Q\}.$$

Definition 7. Two partitions P and Q are said to be ξ -independent if $\xi(u \wedge v) = \xi(u)\xi(v)$ for $u \in P$ and $v \in Q$.

Remarks 2. The well-known Minkowski inequality [15] is given by

Form ≥ 1 ,

$$\left(\sum_{i=1}^k x_i^m\right)^{\frac{1}{m}} + \left(\sum_{i=1}^k y_i^m\right)^{\frac{1}{m}} \geq \left(\sum_{i=1}^k (x_i + y_i)^m\right)^{\frac{1}{m}}$$

where x_i, y_j are non negative numbers.

Remarks 3. Also, the well-known Jensen inequality [15] is given by

Case1. If ϕ is a real convex function,

$$\phi\left(\sum_{i=1}^n u_i x_i\right) \leq \sum_{i=1}^n u_i \phi(x_i)$$

Case2. If ϕ is a real concave function,

$$\phi\left(\sum_{i=1}^n u_i x_i\right) \geq \sum_{i=1}^n u_i \phi(x_i)$$

where x_i 's are real numbers and u_i 's are non negative real numbers with condition $\sum_{i=1}^n u_i = 1$.

III. $RIM_{(\alpha,\beta)}$ IN QUANTUM LOGIC

In this section, we define $RIM_{(\alpha,\beta)}$ for a finite partition on a quantum logic. Some of the properties of suggested measure are proved on Quantum logic.

Definition 8. Let us consider a partition $P = \{u_1, u_2, \dots, u_n\}$ of a couple (Γ, ξ) . We can define the $RIM_{(\alpha,\beta)}$ w.r.t. state ξ as:

$$H_{R(\alpha,\beta)}^\xi(P) = \frac{R}{R+\beta-2\alpha} \left(1 - \left(\sum_{i=1}^n (\xi(u_i))^{\frac{R}{2\alpha-\beta}} \right)^{\frac{2\alpha-\beta}{R}} \right); \alpha \geq 0, 0 \leq \beta \leq 1, R \in \mathfrak{R}, R + \beta \neq 2\alpha. \tag{4}$$

Theorem 1. For a partition $P = \{u_1, u_2, \dots, u_n\}$ of a couple (Γ, ξ) , $H_{R(\alpha,\beta)}^\xi(P) \geq 0$.

Proof. For $R \leq 2\alpha - \beta$, we have

$$(\xi(u_i))^{\frac{R}{2\alpha-\beta}} \geq \xi(u_i), i = 1, 2, \dots, n.$$



therefore,

$$\sum_{i=1}^n (\xi(u_i))^{2\alpha-\beta} \geq \sum_{i=1}^n \xi(u_i) = 1$$

it follows that

$$\left(\sum_{i=1}^n (\xi(u_i))^{2\alpha-\beta} \right)^{\frac{2\alpha-\beta}{R}} \geq 1$$

Hence, for $R \leq 2\alpha - \beta$, we have $\frac{R}{R+\beta-2\alpha} \leq 0$. So, from eq. (4) we have $H_{R(\alpha,\beta)}^\xi(P) \geq 0$.

Theorem 2. Let us consider a family of all the states denoted by K , on quantum logic. Let $\xi, \eta \in K$ and P is a partition of $(\Gamma, \xi), (\Gamma, \eta)$. Then for $\lambda \in [0,1]$, we have

$$\lambda H_{R(\alpha,\beta)}^\xi(P) + (1 - \lambda) H_{R(\alpha,\beta)}^\eta(P) \leq H_{R(\alpha,\beta)}^{\lambda\xi+(1-\lambda)\eta}(P).$$

Proof. Let $P = \{u_1, u_2, \dots, u_n\}$ be a partition. In the Minkowski inequality put $x_i = \lambda\xi(u_i)$ and $y_i = (1 - \lambda)\eta(u_i)$, then for $R \geq 2\alpha - \beta$,

$$\lambda \left(\sum_{i=1}^n (\xi(u_i))^{2\alpha-\beta} \right)^{\frac{2\alpha-\beta}{R}} + (1 - \lambda) \left(\sum_{i=1}^n (\eta(u_i))^{2\alpha-\beta} \right)^{\frac{2\alpha-\beta}{R}} \geq \left(\sum_{i=1}^n (\lambda\xi(u_i) + (1 - \lambda)\eta(u_i))^{2\alpha-\beta} \right)^{\frac{2\alpha-\beta}{R}}.$$

Since, $\frac{R}{R+\beta-2\alpha} \geq 0$ for $R \geq 2\alpha - \beta$, therefore we get

$$\lambda H_{R(\alpha,\beta)}^\xi(P) + (1 - \lambda) H_{R(\alpha,\beta)}^\eta(P) \leq H_{R(\alpha,\beta)}^{\lambda\xi+(1-\lambda)\eta}(P).$$

Theorem 3. Let us consider a couple (Γ, ξ) . Let $P = \{u_1, u_2, \dots, u_n\}$ and $Q = \{v_1, v_2, \dots, v_m\}$ are two partitions of it and Q is a ξ -refinement of P . Then,

$$H_{R(\alpha,\beta)}^\xi(P) \leq H_{R(\alpha,\beta)}^\xi(Q).$$

Proof. Since, Q is a ξ -refinement of P , therefore \exists a partition I_1, I_2, \dots, I_n of the set $\{1, 2, \dots, m\}$ s.t. $u_i = \bigvee_{j \in I_i} v_j, \forall i \in \{1, 2, \dots, n\}$. From Definition 2, we have $\xi(u_i) = \sum_{j \in I_i} \xi(v_j)$.

Now, for $R \geq 2\alpha - \beta$,

$$(\xi(u_i))^{2\alpha-\beta} = \left(\sum_{j \in I_i} \xi(v_j) \right)^{\frac{R}{2\alpha-\beta}} \geq \sum_{j \in I_i} (\xi(v_j))^{2\alpha-\beta} \tag{5}$$

Since, the partition of the set $\{1, 2, \dots, m\}$ is I_1, I_2, \dots, I_n , therefore, we have

$$\bigcup_{i=1}^n I_i = \bigcup_{j=1}^m \{j\}$$



and

$$I_p \cap I_q = \phi; \forall p, q \in \{1, 2, \dots, n\}.$$

So,

$$\sum_{i=1}^n \sum_{j \in I_i} (\xi(v_j))^{\frac{R}{2\alpha-\beta}} = \sum_{j=1}^m (\xi(v_j))^{\frac{R}{2\alpha-\beta}} \tag{6}$$

Taking summation on both sides of Eq(5) w.r.t. ivaries from 1 to n and using Eq. (6), we get

$$\sum_{i=1}^n (\xi(u_i))^{\frac{R}{2\alpha-\beta}} \geq \sum_{i=1}^n \sum_{j \in I_i} (\xi(v_j))^{\frac{R}{2\alpha-\beta}} = \sum_{j=1}^m (\xi(v_j))^{\frac{R}{2\alpha-\beta}} \tag{7}$$

Thus, we have

$$\sum_{i=1}^n (\xi(u_i))^{\frac{R}{2\alpha-\beta}} \geq \sum_{j=1}^m (\xi(v_j))^{\frac{R}{2\alpha-\beta}} \tag{8}$$

Raising power to $\frac{2\alpha-\beta}{R}$ on both sides, we have

$$\left(\sum_{i=1}^n (\xi(u_i))^{\frac{R}{2\alpha-\beta}} \right)^{\frac{2\alpha-\beta}{R}} \geq \left(\sum_{j=1}^m (\xi(v_j))^{\frac{R}{2\alpha-\beta}} \right)^{\frac{2\alpha-\beta}{R}} \tag{9}$$

Now, $\frac{R}{R+\beta-2\alpha} \geq 0$ (for $R \geq 2\alpha - \beta$), so we have

$$\frac{R}{R + \beta - 2\alpha} \left(1 - \left(\sum_{i=1}^n (\xi(u_i))^{\frac{R}{2\alpha-\beta}} \right)^{\frac{2\alpha-\beta}{R}} \right) \leq \frac{R}{R + \beta - 2\alpha} \left(1 - \left(\sum_{j=1}^m (\xi(v_j))^{\frac{R}{2\alpha-\beta}} \right)^{\frac{2\alpha-\beta}{R}} \right)$$

i.e.

$$H_{R(\alpha,\beta)}^\xi(P) \leq H_{R(\alpha,\beta)}^\xi(Q). \tag{10}$$

Theorem 4. Let $P = \{u_1, u_2, \dots, u_n\}$ and $Q = \{v_1, v_2, \dots, v_m\}$ are two partitions of a couple (Γ, ξ) satisfying Bayes' property, then

$$\max \{H_{R(\alpha,\beta)}^\xi(P), H_{R(\alpha,\beta)}^\xi(Q)\} \leq H_{R(\alpha,\beta)}^\xi(P \vee Q). \tag{11}$$



Proof. As we know that $(P \vee Q)$ is ξ -refinement of both P and Q Therefore, by using result of Theorem.3, we have

$$H_{R(\alpha,\beta)}^\xi(P) \leq H_{R(\alpha,\beta)}^\xi(P \vee Q)$$

and

$$H_{R(\alpha,\beta)}^\xi(Q) \leq H_{R(\alpha,\beta)}^\xi(P \vee Q).$$

Hence, the result.

Theorem 5. Let $P = \{u_1, u_2, \dots, u_n\}$ and $Q = \{v_1, v_2, \dots, v_m\}$ be partitions of a couple (Γ, ξ) having Bayes' property, then the $RIM_{(\alpha,\beta)}$ with respect to state ξ is sub-additive. i.e

$$H_{R(\alpha,\beta)}^\xi(P \vee Q) = H_{R(\alpha,\beta)}^\xi(P) + H_{R(\alpha,\beta)}^\xi(Q) - \frac{R + \beta - 2\alpha}{R} H_{R(\alpha,\beta)}^\xi(P) H_{R(\alpha,\beta)}^\xi(Q). \tag{12}$$

where P and Q are ξ -independent partitions.

Proof. Consider

$$\begin{aligned} & H_{R(\alpha,\beta)}^\xi(P) + H_{R(\alpha,\beta)}^\xi(Q) - \frac{R + \beta - 2\alpha}{R} H_{R(\alpha,\beta)}^\xi(P) H_{R(\alpha,\beta)}^\xi(Q) \\ &= \frac{R}{R + \beta - 2\alpha} \left(1 - \left(\sum_{i=1}^n (\xi(u_i))^{\frac{R}{2\alpha-\beta}} \right)^{\frac{2\alpha-\beta}{R}} \right) + \frac{R}{R + \beta - 2\alpha} \left(1 - \left(\sum_{j=1}^m (\xi(v_j))^{\frac{R}{2\alpha-\beta}} \right)^{\frac{2\alpha-\beta}{R}} \right) \\ &\quad - \frac{R + \beta - 2\alpha}{R} \left(\frac{R}{R + \beta - 2\alpha} \right)^2 \left(1 - \left(\sum_{i=1}^n (\xi(u_i))^{\frac{R}{2\alpha-\beta}} \right)^{\frac{2\alpha-\beta}{R}} \right) \left(1 - \left(\sum_{j=1}^m (\xi(v_j))^{\frac{R}{2\alpha-\beta}} \right)^{\frac{2\alpha-\beta}{R}} \right) \\ &= \frac{R}{R + \beta - 2\alpha} \left(1 - \left(\sum_{i=1}^n (\xi(u_i))^{\frac{R}{2\alpha-\beta}} \right)^{\frac{2\alpha-\beta}{R}} \left(\sum_{j=1}^m (\xi(v_j))^{\frac{R}{2\alpha-\beta}} \right)^{\frac{2\alpha-\beta}{R}} \right) \\ &= \frac{R}{R + \beta - 2\alpha} \left(1 - \left(\sum_{i=1}^n \sum_{j=1}^m (\xi(u_i) \xi(v_j))^{\frac{R}{2\alpha-\beta}} \right)^{\frac{2\alpha-\beta}{R}} \right) \\ &= \frac{R}{R + \beta - 2\alpha} \left(1 - \left(\sum_{i=1}^n \sum_{j=1}^m (\xi(u_i \wedge v_j))^{\frac{R}{2\alpha-\beta}} \right)^{\frac{2\alpha-\beta}{R}} \right) \\ &= H_{R(\alpha,\beta)}^\xi(P \vee Q) \end{aligned}$$

Hence, the result.

IV. JOINT AND CONDITIONAL $RIM_{(\alpha,\beta)}$ IN QUANTUM LOGIC

Let us assume that Γ is a quantum logic. Let $P = \{u_1, u_2, \dots, u_n\}$ and $Q = \{v_1, v_2, \dots, v_m\}$ are two partitions of Γ corresponding to a state ξ on Γ . Let us consider a set π given by



$$\pi = \{\pi_{11}, \pi_{12}, \dots, \pi_{nm}\}.$$

where $\pi = x_{ij}y_j = y_{ji}x_i; \forall i = 1, 2, \dots, n; j = 1, 2, \dots, m.$

Also,

$$x_{ij} = \frac{\xi(u_i \wedge v_j)}{\xi(v_j)}, \quad y_{ji} = \frac{\xi(v_j \wedge u_i)}{\xi(u_i)}, \quad \forall i = 1, 2, \dots, n; j = 1, 2, \dots, m. \tag{13}$$

and

$$x_i = \sum_{j=1}^m \pi_{ij}, \quad y_i = \sum_{j=1}^n \pi_{ij}, \quad \forall i = 1, 2, \dots, n; j = 1, 2, \dots, m. \tag{14}$$

Definition 9. The Joint $RIM_{(\alpha, \beta)}$ of partition P and Q w.r.t. state ξ is defined by:

$$H_{R(\alpha, \beta)}^\xi(P, Q) = \frac{R}{R + \beta - 2\alpha} \left(1 - \left(\sum_{i=1}^n \sum_{j=1}^m (\pi_{ij})^{\frac{R}{2\alpha - \beta}} \right)^{\frac{2\alpha - \beta}{R}} \right) \tag{15}$$

Definition 10. The Conditional $RIM_{(\alpha, \beta)}$ of P given Q w.r.t. state ξ is defined by:

$$'H_{R(\alpha, \beta)}^\xi(Q|P) = \frac{R}{R + \beta - 2\alpha} \left[1 - \sum_{i=1}^n \xi(u_i) \left(\sum_{j=1}^m \left(\frac{\xi(v_j \wedge u_i)}{\xi(u_i)} \right)^{\frac{R}{2\alpha - \beta}} \right)^{\frac{2\alpha - \beta}{R}} \right] \tag{16}$$

and

$$''H_{R(\alpha, \beta)}^\xi(Q|P) = \frac{R}{R + \beta - 2\alpha} \left[1 - \left(\sum_{i=1}^n \xi(u_i) \sum_{j=1}^m \left(\frac{\xi(v_j \wedge u_i)}{\xi(u_i)} \right)^{\frac{R}{2\alpha - \beta}} \right)^{\frac{2\alpha - \beta}{R}} \right] \tag{17}$$

for $R > 0 (R \neq 2\alpha - \beta).$

Theorem 6. Let Γ is a quantum logic w.r.t. a state ξ . If P and Q are two partitions of it satisfying Bayes' property. Then the following results hold:

- (i). $'H_{R(\alpha, \beta)}^\xi(Q|P) \leq H_{R(\alpha, \beta)}^\xi(Q),$
- (ii). $''H_{R(\alpha, \beta)}^\xi(Q|P) \leq H_{R(\alpha, \beta)}^\xi(Q),$
- (iii). $''H_{R(\alpha, \beta)}^\xi(Q|P) \leq 'H_{R(\alpha, \beta)}^\xi(Q|P).$



Proof. (i) For $R \geq 2\alpha - \beta$, (i.e. $\frac{R}{2\alpha - \beta} \geq 1$) using Jensen's inequality [15], we get

$$\left[\sum_{j=1}^m \left(\sum_{i=1}^n \pi_{ij} \right)^{\frac{R}{2\alpha - \beta}} \right]^{\frac{2\alpha - \beta}{R}} \geq \left[\sum_{i=1}^n \left(\sum_{j=1}^m (\pi_{ij})^{\frac{R}{2\alpha - \beta}} \right)^{\frac{2\alpha - \beta}{R}} \right]$$

Using Eq. (14) on LHS and $\pi_{ij} = y_{ji}x_i$ on RHS, we have

$$\left[\sum_{j=1}^m (y_j)^{\frac{R}{2\alpha - \beta}} \right]^{\frac{2\alpha - \beta}{R}} \geq \left[\sum_{i=1}^n \left(\sum_{j=1}^m (y_{ji}x_i)^{\frac{R}{2\alpha - \beta}} \right)^{\frac{2\alpha - \beta}{R}} \right]$$

i.e.

$$1 - \left[\sum_{i=1}^n \left(\sum_{j=1}^m (y_{ji}x_i)^{\frac{R}{2\alpha - \beta}} \right)^{\frac{2\alpha - \beta}{R}} \right] \leq 1 - \left[\sum_{j=1}^m (y_j)^{\frac{R}{2\alpha - \beta}} \right]^{\frac{2\alpha - \beta}{R}}$$

i.e.

$$1 - \left[\sum_{i=1}^n x_i \left(\sum_{j=1}^m (y_{ji})^{\frac{R}{2\alpha - \beta}} \right)^{\frac{2\alpha - \beta}{R}} \right] \leq 1 - \left[\sum_{j=1}^m (y_j)^{\frac{R}{2\alpha - \beta}} \right]^{\frac{2\alpha - \beta}{R}}$$

Putting the values from Eq.(13), we get

$$1 - \sum_{i=1}^n \xi(u_i) \left(\sum_{j=1}^m \left(\frac{\xi(v_j \wedge u_i)}{\xi(u_i)} \right)^{\frac{R}{2\alpha - \beta}} \right)^{\frac{2\alpha - \beta}{R}} \leq 1 - \left(\sum_{j=1}^m (\xi(v_j))^{\frac{R}{2\alpha - \beta}} \right)^{\frac{2\alpha - \beta}{R}}$$

Using $\frac{R}{R + \beta - 2\alpha} \geq 0$ (for $R \geq 2\alpha - \beta$), we have

$${}^I H_{R(\alpha, \beta)}^{\xi}(Q|P) \leq H_{R(\alpha, \beta)}^{\xi}(Q)$$

Also, the equality sign holds iff $\pi_{ij} = x_i y_j$.

(ii) For $R \geq 2\alpha - \beta$, (i.e. $\frac{R}{2\alpha - \beta} \geq 1$) using Jensen's inequality [15], we get

$$\sum_{i=1}^n x_i y_{ji}^{\frac{R}{2\alpha - \beta}} \geq \left[\sum_{i=1}^n x_i y_{ji} \right]^{\frac{R}{2\alpha - \beta}}$$

use Eq. (13) and (14), we get

$$\sum_{i=1}^n \xi(u_i) \left(\frac{\xi(v_j \wedge u_i)}{\xi(u_i)} \right)^{\frac{R}{2\alpha - \beta}} \geq \left(\sum_{i=1}^n \xi(u_i) \left(\frac{\xi(v_j \wedge u_i)}{\xi(u_i)} \right) \right)^{\frac{R}{2\alpha - \beta}} = \xi(v_j)^{\frac{R}{2\alpha - \beta}}$$



i.e.

$$\sum_{i=1}^n \xi(u_i) \left(\frac{\xi(v_j \wedge u_i)}{\xi(u_i)} \right)^{\frac{R}{2\alpha-\beta}} \geq \xi(v_j)^{\frac{R}{2\alpha-\beta}}$$

Take summation w.r.t j varying from 1 to m and then raising power to $\frac{2\alpha-\beta}{R}$ on both sides, we have

$$\left(\sum_{i=1}^n \xi(u_i) \sum_{j=1}^m \left(\frac{\xi(v_j \wedge u_i)}{\xi(u_i)} \right)^{\frac{R}{2\alpha-\beta}} \right)^{\frac{2\alpha-\beta}{R}} \geq \left(\sum_{j=1}^m \xi(v_j)^{\frac{R}{2\alpha-\beta}} \right)^{\frac{2\alpha-\beta}{R}}$$

i.e.

$$1 - \left(\sum_{i=1}^n \xi(u_i) \sum_{j=1}^m \left(\frac{\xi(v_j \wedge u_i)}{\xi(u_i)} \right)^{\frac{R}{2\alpha-\beta}} \right)^{\frac{2\alpha-\beta}{R}} \leq 1 - \left(\sum_{j=1}^m \xi(v_j)^{\frac{R}{2\alpha-\beta}} \right)^{\frac{2\alpha-\beta}{R}}$$

Using $\frac{R}{R+\beta-2\alpha} \geq 0$ (for $R \geq 2\alpha - \beta$), we have

$${}^{\xi}H_{R(\alpha,\beta)}^{\xi}(Q|P) \leq H_{R(\alpha,\beta)}^{\xi}(Q)$$

(iii) For $R \geq 2\alpha - \beta$, (i.e. $\frac{R}{2\alpha-\beta} \geq 1$) using Jensen's inequality [15], we get

$$\left[\sum_{i=1}^n x_i \left(\sum_{j=1}^m y_{ji}^{\frac{R}{2\alpha-\beta}} \right)^{\frac{2\alpha-\beta}{R}} \right] \leq \left[\sum_{i=1}^n x_i \sum_{j=1}^m y_{ji}^{\frac{R}{2\alpha-\beta}} \right]^{\frac{2\alpha-\beta}{R}}$$

use Eq.(13) and(14) in above eq., we get

$$\left[\sum_{i=1}^n \xi(u_i) \left(\sum_{j=1}^m \left\{ \frac{\xi(v_j \wedge u_i)}{\xi(u_i)} \right\}^{\frac{R}{2\alpha-\beta}} \right)^{\frac{2\alpha-\beta}{R}} \right] \leq \left[\sum_{i=1}^n \xi(u_i) \sum_{j=1}^m \left\{ \frac{\xi(v_j \wedge u_i)}{\xi(u_i)} \right\}^{\frac{R}{2\alpha-\beta}} \right]^{\frac{2\alpha-\beta}{R}}$$

i.e.

$$1 - \left[\sum_{i=1}^n \xi(u_i) \left(\sum_{j=1}^m \left\{ \frac{\xi(v_j \wedge u_i)}{\xi(u_i)} \right\}^{\frac{R}{2\alpha-\beta}} \right)^{\frac{2\alpha-\beta}{R}} \right] \geq 1 - \left[\sum_{i=1}^n \xi(u_i) \sum_{j=1}^m \left\{ \frac{\xi(v_j \wedge u_i)}{\xi(u_i)} \right\}^{\frac{R}{2\alpha-\beta}} \right]^{\frac{2\alpha-\beta}{R}}$$

Using $\frac{R}{R+\beta-2\alpha} \geq 0$ (for $R \geq 2\alpha - \beta$), we have

$${}^{\xi}H_{R(\alpha,\beta)}^{\xi}(Q|P) \leq {}^{\xi}H_{R(\alpha,\beta)}^{\xi}(Q|P)$$



V. CONCLUSION

In this paper, I have defined the $RIM_{(\alpha,\beta)}$, joint and conditional $RIM_{(\alpha,\beta)}$ in quantum logic. I generalize R-norm information measure introduced by Boekee and Van der Lubbe [5] in quantum logic.

REFERENCES

- [1] C. E. Shannon, "A mathematical Theory of Communication," *Bell Syst. Tech. J.*, vol. 27, no. 3, pp. 212–214, 1948.
- [2] Alfred Renyi, "On Measures of Entropy and Information," *Fourth Berkeley Symp. Math. Stat. Probab.*, vol. 1, pp. 547–561, 1961.
- [3] J. Havrda and F. Charvát, "Quantification Method of Classification Processes," *Kybernetika*, vol. 3, no. 1, pp. 30–35, 1967.
- [4] Z. Daróczy, "Generalized information functions," *Inf. Control*, vol. 16, no. 1, pp. 36–51, 1970, doi: 10.1016/S0019-9958(70)80040-7.
- [5] D. E. Boekee and J. C. A. Van der Lubbe, "The R-norm information measure," *Inf. Control*, vol. 45, no. 2, pp. 136–155, May 1980, doi: 10.1016/S0019-9958(80)90292-2.
- [6] D. S. Hooda and D. K. Sharma, "Generalized R-norm Information Measures," vol. 4, no. 2, pp. 153–168, 2008.
- [7] "Conditional entropy of infinite partitions on quantum logic | Semantic Scholar." <https://www.semanticscholar.org/paper/Conditional-entropy-of-infinite-partitions-on-logic-Ebrahimzadeh/1f0eed144a1a2e61b54c0e75e00d5e695afd8d32> (accessed Jul. 28, 2022).
- [8] D. Markechová, B. Mosapour, and A. Ebrahimzadeh, "R-norm entropy and R-norm divergence in fuzzy probability spaces," *Entropy*, vol. 20, no. 4, pp. 1–18, 2018, doi: 10.3390/e20040272.
- [9] A. Ebrahimzadeh, "Logical entropy of quantum dynamical systems," *Open Phys.*, vol. 14, no. 1, pp. 1–5, Jan. 2016, doi: 10.1515/PHYS-2015-0058.
- [10] M. Khare and S. Roy, "Conditional Entropy and the Rokhlin Metric on an Orthomodular Lattice with Bayesian State," *Int. J. Theor. Phys. 2007 475*, vol. 47, no. 5, pp. 1386–1396, Oct. 2007, doi: 10.1007/S10773-007-9581-1.
- [11] M. Khare and S. Roy, "Conditional Entropy and the Rokhlin Metric on an Orthomodular Lattice with Bayesian State," *Int. J. Theor. Phys. 2007 475*, vol. 47, no. 5, pp. 1386–1396, Oct. 2007, doi: 10.1007/S10773-007-9581-1.
- [12] Y. X. Zhao and Z. H. Ma, "Conditional entropy of partitions on quantum logic," *Commun. Theor. Phys.*, vol. 48, no. 1, pp. 11–13, Jul. 2007, doi: 10.1088/0253-6102/48/1/003.
- [13] "2005- the Zhang Lab." <https://www.zhanglab.tch.harvard.edu/2005> (accessed Jul. 29, 2022).
- [14] A. Ebrahimzadeh and Z. E. Giski, "ENTROPY OF QUANTUM DYNAMICAL SYSTEMS WITH INFINITE PARTITIONS."
- [15] E. Beckenbach and R. Bellman, "an Introduction To Inequalities," *An Introd. to Inequalities*, pp. 1–2, 2012, doi: 10.5948/upo9780883859216.002.



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