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Obtaining Stability Regions using Random Sampling in Time Delay Handling Method in Dominant Pole Placement based PID Controller Design

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ABSTRACT: This is a new method of Proportional-Integral-Derivative (PID) controller design using the dominant pole placement method for handling second order pulse time delay (SOPTD) systems. To handle the time-delay term this method transforms the transcendental exponential delay term of the plant into finite number of discrete-time poles by a suitable choice of sampling time. The PID controller has been represented by Tustin's discretization method and the PID controller gains are obtained using the dominant pole placement criterion where the plant is discretized using the pole-zero matching method. Random search and optimization technique is used to obtain the stability regions in the desired closed loop parameters space by minimizing the integral squared error (ISE) criterion by randomly sampling from the regions which can be stabilized and these closed loop parameters are mapped on to the PID controller gains.

I. INTRODUCTION

Several methods have been proposed in the last few decades to design PID controllers to handle the generic SOPTD systems that are slow and oscillatory. The dominant pole placement method is quite challenging among different PID design methods to avoid the problem of infinite order which arises due to the presence of the exponential delay term. This paper proposes a new dominant pole placement based PID controller design method to control SOPTD systems which is an extension of the previous works reported in (Das et al. 2018).

In order to eliminate the problem of infinite order due to finite term approximation of the exponential delay term, we propose a method of mapping the time delay term as a finite number of poles in discrete time domain using pole-zero matching, considering that the delay term is an integer multiple of sampling time. To overcome the infinite dimension problem of the closed loop time delay systems for the design of dominant pole placement controllers, considering the sampling time to be sufficiently smaller to that of open loop time constant and delay term.

We assume that the ratio to the delay and sampling time to be an integer in handling the finite number of poles in complex z-plane. We utilize the discretization method to transform the delay term into finite poles. Thus, this method works for systems with larger time delays. We use Tustin's method to discretize the PID controller as it is not possible to discretize continuous time PID controller using pole-zero matching method since at $s=0$ the DC gain of the controller will be infinite. Using coefficient matching method PID controller gains are obtained in comparison to the polynomial equation with desired closed loop specifications. The obtained controller gains are then mapped back into the continuous time to handle the continuous time SOPTD systems.



2. Theoretical Formulation of Dominant Pole Placement based PID controller design methodology:

The design methodology of dominant pole placement based PID controller to handle SOPTD systems is given under this section of the paper.

2. 1. Discretization of SOPTD system using pole-zero matching method:

We discretize the system using pole-zero matching method as guided step by step in the following way

Step – 1: Transforming the poles and zeros from continuous time domain i.e. S to discrete time domain i.e. Z using

$$z = e^{sT_s}$$

$$e^{-Ls} = z^{-n}; n = L/T_s$$

Step – 2: Limits of S-plane are mapped onto the Z-Plane as

Poles/Zeros at $S = -\infty$ are mapped onto $Z = 0$ and

Poles/Zeros at $S = 0$ are mapped onto $Z = 1$

Step – 3: The discrete open loop system transfer function

Consider the open loop SOPTD system as

$$G(S) = \frac{Ke^{Ls}}{\tau^2 s^2 + 2\zeta_{ol}\tau s + 1}$$

with poles as

$$S_{1,2} = -\zeta_{ol}\omega_{ol} \pm j\omega_{ol}\sqrt{1 - \zeta_{ol}^2}; \omega_{ol} = \frac{1}{\tau}$$

converting from S to Z the system becomes

$$G(Z) = \frac{\tilde{K}}{z^n(z - z_1)(z - z_2)}$$

here

$$z_1 = e^{s_1 T_s}, z_2 = e^{s_2 T_s}$$

Step – 4: The DC gain of the discrete system can be obtained as

$$G(s)_{s=0} = G(z)_{z=1}$$

$$\left. \frac{Ke^{Ls}}{\tau^2 s^2 + 2\zeta_{ol}\tau s + 1} \right|_{s=0} = \left. \frac{\tilde{K}}{z^n(z - z_1)(z - z_2)} \right|_{z=1}$$

this results as

$$\tilde{K} = K(z - z_1)(z - z_2)$$

2. 2. Discretization of PID controller with Tustin’s approximation:

The continuous time PID controller is discretized using Tustin’s discretization method in the following steps the continuous time PID is



$$C(s) = K_p + \frac{K_i}{s} + K_d s$$

the Tustin's approximation is given as

$$S = \frac{2(1 - z^{-1})}{T_s(1 + z^{-1})}$$

then the discretized PID controller will be

$$C(z) = \frac{(4K_d + 2K_p T_s + K_i T_s^2)z^2 + (2K_i T_s^2 - 8K_d)z + (4K_d - 2K_p T_s + K_i T_s^2)}{2T_s(z^2 - 1)}$$

The discretized closed loop system characteristic equation will be

$$1 + C(z)G(z) = 0$$

on substituting C(Z) and G(Z), we get

$$2T_s z^n (z - z_1)(z - z_2)(z^2 - 1) + K[(4K_d + 2K_p T_s + K_i T_s^2)]z^2 + K[(2K_i T_s^2 - 8K_d)]z + K[(4K_d - 2K_p T_s + K_i T_s^2)] = 0$$

it is an $(n + 4^{\text{th}})$ order equation with same number of roots as the order of the system out of which two are dominant poles and the rest i.e. $(n+2)$ are non-dominant poles. We can modify the closed loop poles/zeros by controlling the PID controller gains i.e. $\{K_p, K_i, K_d\}$ and the sampling time.

2. 3. Deriving the expressions for PID controller gains:

We obtain the expressions for the PID controller gains by matching the coefficients of the discrete-time finite-term characteristic polynomial for the given set of open loop parameters $\{K, L, \tau, \zeta_{ol}, \omega_{ol}\}$ with the desired characteristic polynomial which contains the user defined closed loop specifications $\{m, \zeta_{cl}, \omega_{cl}\}$.

In the $(n + 4^{\text{th}})$ order equation the controller gains are associated with the order 0,1 and 2 powers of the discrete time operator 'z'. The desired characteristic polynomial can be designed in two different ways two ways based on the nature of the non-dominant poles i.e. all real and all complex.

2. 3. 1. All real non-dominant poles:

The two dominant complex conjugate poles $(S_{1,2})^d$ and the non-dominant all real poles (S_3^{nd}) of the continuous time desired closed loop can be considered as

$$S_{1,2}^d = -\zeta_{cl}\omega_{cl} \pm j\omega_{cl}\sqrt{1 - \zeta_{cl}^2}; S_3^{nd} = -m\zeta_{cl}\omega_{cl}$$

the corresponding poles in Z are

$$Z_1^d = e^{s_1^d T_s}, Z_2^d = e^{s_2^d T_s}, Z_3^{nd} = e^{s_3^{nd} T_s}$$

then the desired polynomial formed by the above roots will be

$$(z - z_1^d)(z - z_2^d)(z - z_3^{nd})^{(n+2)} = 0$$



this equation after expanding can be

$$A_{n+4}z^{n+4} + A_{n+3}z^{n+3} + \dots + A_2z^2 + A_1z^1 + A_0z^0 = 0$$

on matching the coefficients of this equation with the (n + 4th) order equation with PID controller gains gives

$$z^0: K(4K_d - 2K_pT_s + K_iT_s^2) = A_0$$

$$z^1: K(2K_iT_s^2 - 8K_d) = A_1$$

$$z^2: K(4K_d + 2K_pT_s + K_iT_s^2) = A_2$$

hence the desired gains will be

$$K_i = \frac{(A_0 + A_1 + A_2)}{4KT_s^2}; K_d = \frac{K_iT_s^2}{4} - \frac{A_1}{8K}; K_p = \frac{1}{2T_s} \left(4K_d + K_iT_s^2 - \frac{A_0}{K} \right)$$

2. 3. 2. All complex conjugate non-dominant poles:

The two dominant complex conjugate poles ($S_{1,2}^d$) and the non-dominant all real poles ($S_{3,4}^{nd}$) of the continuous time desired closed loop can be considered as

$$s_{1,2}^d = -\zeta_{cl}\omega_{cl} \pm j\omega_{cl}\sqrt{1 - \zeta_{cl}^2}; s_{3,4}^{nd} = m \left(-\zeta_{cl}\omega_{cl} \pm j\omega_{cl}\sqrt{1 - \zeta_{cl}^2} \right)$$

the corresponding poles in Z are

$$Z_1^d = e^{s_1^d T_s}, Z_2^d = e^{s_2^d T_s}, z_3^{nd} = e^{s_3^{nd} T_s}, z_4^{nd} = e^{s_4^{nd} T_s}$$

the desired polynomial will be

$$(z - z_1^d)(z - z_2^d)(z - z_3^{nd})^{\frac{(n+2)}{2}}(z - z_4^{nd})^{\frac{(n+2)}{2}} = 0$$

this equation after expanding can be

$$A_{n+4}z^{n+4} + A_{n+3}z^{n+3} + \dots + A_2z^2 + A_1z^1 + A_0z^0 = 0$$

on matching the coefficients of this equation with the (n + 4th) order equation with PID controller gains gives

$$z^0: K(4K_d - 2K_pT_s + K_i) = A_0$$

$$z^1: K(2K_iT_s^2 - 8K_d) = A_1$$

$$z^2: K(4K_d + 2K_pT_s + K_iT_s^2) = A_2$$

hence the desired gains will be

$$K_i = \frac{(A_0 + A_1 + A_2)}{4KT_s^2}; K_d = \frac{K_iT_s^2}{4} - \frac{A_1}{8K}; K_p = \frac{1}{2T_s} \left(4K_d + K_iT_s^2 - \frac{A_0}{K} \right)$$



III. SIMULATION AND RESULTS

To analyse the idea proposed we consider nine test bench system with different types are given below and are mentioned in Table – 1 namely lag dominant G1–G3(L<), balanced G4–G6(L=) and delay dominant G7–G9(L>) which are further of different type of damping i.e. under damped systems G1,G4,G7(ol<1), critically damped systems G2,G5,G8(ol=1) and over damped systems G3,G6,G9(ol>1) along with detailed open loop parameters of each plant.

Table – 1: Nine test bench systems with open loop parameter specifications

Plant	Type	K	L	τ	ζ	ω_n
P_1	Lag dominant under damped	1	1	3	0.4	0.33
P_2	Lag dominant critically damped	1	0.8	1	1	1
P_3	Lag dominant over damped	1	2	6.32	1.1	0.16
P_4	Balanced under damped	0.5	1	1	0.6	1
P_5	Balanced critically damped	1	1	1	1	1
P_6	Balanced over damped	1	3	3	4	0.33
P_7	Delay dominant under damped	0.33	1.28	1.03	0.5	0.97
P_8	Delay dominant critically damped	1	10	1	1	1
P_9	Delay dominant over damped	0.81	2	0.31	1.72	3.21

The open loop transfer functions of the nine test bench SOPTD systems are

G1 is Lag dominant and Under damped system

$$G1S = \frac{19S^2 + 2.4S + 1}{e^{-s}}$$

G2 is Lag dominant and Critically damped system

$$G2S = \frac{1S^2 + 2S + 1}{e^{-0.8s}}$$

G3 is Lag dominant and Over damped system

$$G3S = \frac{11 + 10S + 4S^2}{e^{-2s}}$$

G4 is Balanced and Under damped system

$$G4S = \frac{0.5S^2 + 1.2S + 1}{e^{-s}}$$

G5 is Balanced and Critically damped system

$$G5S = \frac{11 + S^2}{e^{-s}}$$



G6 is Balanced and Over damped system

$$G6S = 19S^2 + 24S + 1e - 3s$$

G7 is Delay dominant and Under damped system

$$G7S = 13.2158S^2 + 3.1614S + 3.0568e - 1.2755s$$

G8 is Delay dominant and Critically damped system

$$G8S = 1S + 12e - 10s$$

G9 is Delay dominant and Over damped system

$$G9S = 10.12S^2 + 1.33S + 1.24e - 2s$$

The two different PID controller expressions are used to determine the PID controller gains using specified sampling time for all the nine test bench systems. Based on the choice sampling time, the closed loop design parameters may vary hence using a random search and optimization help in determining the closed loop parameters. Here, Particle Swarm Optimization (PSO) algorithm is used to obtain the optimum values of the closed loop parameters where the parameter search range of the parameters is given as $m(0,20)$, $ol(0,5)$, $ol(0,20)$.

Table – 2: PSO based optimum closed loop design parameters and PID controller parameters

Plant	Snd	Ts	m	ol	ol	Kp	Ki	Kd
G1	All real	0.005	16.0623	0.2568	8.2407	0.00000002	0.12462740	0
		0.01	13.2818	0.3006	7.4728	0.00000013	0.12462763	0.00000321
		0.02	4.3607	2.5405	2.3045	0.00000102	0.12462838	0.00001305
	All Complex	0.005	5.5959	0.3924	15.3917	0.00000002	0.12462934	0.00000079
		0.01	1.5817	4.8462	3.8605	0.00000013	0.12448930	0.00000320
		0.02	14.4470	0.1666	10.6461	0.00000097	0.12462829	0.00001187
G2	All real	0.005	1.4359	3.9174	6.6913	0.00000004	0.34216861	0.00000218
		0.01	5.6485	0.4011	14.4150	0.00000035	0.34217031	0.00000886
		0.02	0.6954	3.2629	11.4256	0.00000281	0.34217858	0.00003612
	All Complex	0.005	13.9068	0.5150	5.2783	0.00000004	0.34216852	0.00000218
		0.01	8.8372	0.6373	5.7739	0.00000035	0.34217031	0.00000888
		0.02	11.9460	4.3698	0.5345	0.00000280	0.34217910	0.00003588



G3	All real	0.01	3.3276	0.3730	15.0295	0.00000001	0.07669311	0.00000048
		0.01	3.8042	3.5571	1.2243	0.00000008	0.07669315	0.00000195
		0.02	20.0000	1.4490	0.5013	0.00000062	0.07669330	0.00000790
	All Complex	0.005	0.8456	2.8471	7.7168	0.00000001	0.07669311	0.00000048
		0.01	3.3449	0.4663	10.5862	0.00000008	0.07669315	0.00000195
		0.02	8.8356	1.0149	1.6182	0.00000062	0.07669330	0.00000790
G4	All real	0.005	5.8575	3.4244	1.5387	0.00000008	0.61576541	0
		0.01	6.7181	0.3056	13.0071	0.00000063	0.61656335	0.00001588
		0.02	1.0387	4.2892	4.9060	0.00000505	0.61578624	0.00006465
	All Complex	0.005	6.3039	0.6956	7.0329	0.00000008	0.61576549	0.00000391
		0.01	9.8957	4.8140	0.5624	0.00000062	0.61576941	0.00001585
		0.02	7.0000	1.2469	2.5905	0.00000504	0.61578623	0.00006454
G5	All real	0.005	2.61	1.8913	6.2400	0.00000004	0.30789366	0.00000196
		0.01	12.8415	1.0794	1.9295	0.00000031	0.30789494	0.00000793
		0.02	8.2242	0.4209	6.5194	0.00000252	0.30790144	0.00003233
	All Complex	0.005	7.1645	0.9614	4.4820	0.00000004	0.30789353	0.00000196
		0.01	5.2752	1.9267	2.6304	0.00000031	0.30789495	0.00000792
		0.02	20.0000	0.1590	7.0610	0.00000240	0.30790064	0.00002915
G6	All real	0.005	20.0000	5.0000	0.1217	0.00000001	0.09063826	0.00000057
		0.01	1.3519	1.1976	6.6332	0.00000009	0.09063827	0.00000229
		0.02	0.6111	4.2722	3.5303	0.00000073	0.09063838	0.00000926
	All Complex	0.005	1.5180	0.7173	11.1547	0.00000001	0.09063826	0.00000057
		0.01	8.1539	1.0958	1.2069	0.00000009	0.09063827	0.00000229
		0.02	5.0686	0.8179	2.2637	0.00000073	0.09063838	0.00000926
G7	All real	0.005	16.2347	0.1869	8.1286	0.00000010	0.82971756	0.00000526
		0.01	12.6640	0.2608	6.4837	0.00000084	0.82972223	0.00002126
		0.02	0.7641	1.7574	13.0506	0.00000678	0.82974191	0.00008652
	All Complex	0.005	18.8368	0.2807	4.6378	0.00000010	0.82960126	0.00000526
		0.01	4.2810	0.9761	5.1117	0.00000084	0.82972229	0.00002126



		0.02	8.8453	0.4895	4.1174	0.00000681	0.82973400	0.00008737
G8	All real	0.005	20.0000	0.0631	2.9383	0.00000001	0.06241485	0.00000039
		0.01	2.5617	0.4622	2.7802	0.00000006	0.06249392	0.00000157
		0.02	0.5694	0.9216	5.4501	0.00000050	0.06249401	0.00000630
	All Complex	0.005	0.1111	4.5444	7.2841	0.00000001	0.06249379	0.00000039
		0.01	0.2314	1.7977	7.8528	0.00000006	0.06249392	0.00000157
		0.02	8.9848	0.6853	0.4675	0.00000050	0.06249401	0.00000630
G9	All real	0.005	6.5852	1.0844	2.1079	0.00000004	0.32875691	0
		0.01	0.2671	3.8142	12.3083	0.00000033	0.32875831	0.00000836
		0.02	2.6349	4.0728	1.0140	0.00000267	0.32876409	0.00003392
	All Complex	0.005	5.7626	0.9134	2.8594	0.00000004	0.32875764	0.00000207
		0.01	4.1118	0.7240	4.3556	0.00000033	0.32875828	0.00000836
		0.02	2.0871	2.4744	2.1067	0.00000267	0.32876388	0.00003390

Using these parameter values we can simulate the responses of the closed loop response of that system and PID setup using MATLAB software. Choose a specific sampling time and choose the specified values from the table to synthesize the responses.

Considering the sampling time to be 0.2 sec for testing the method, we simulated the results using MATLAB software.

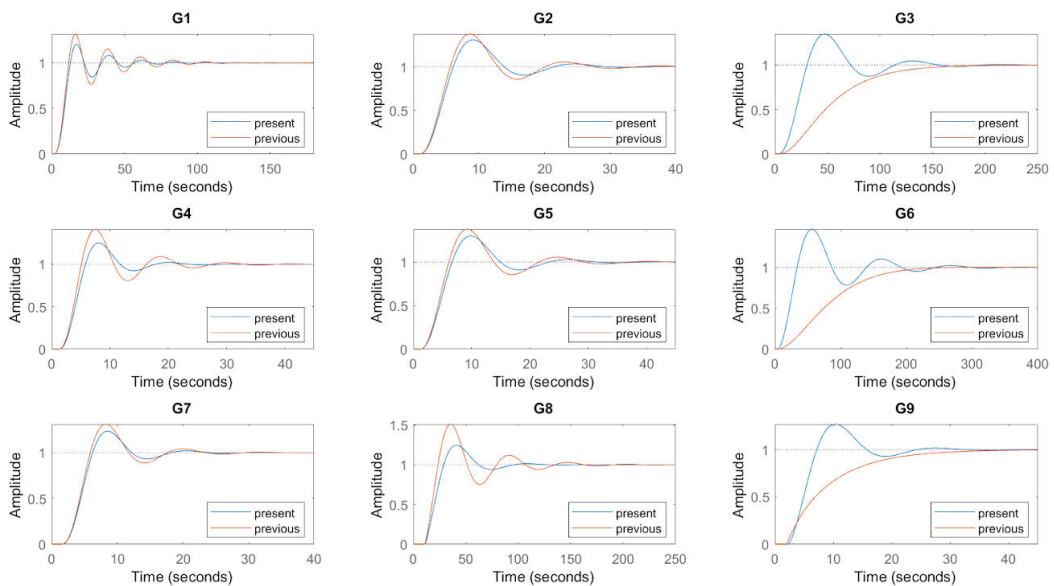


Figure – 1: Responses of the nine test bench systems for step input at set point



The figure has results of current method represented (as present) in blue colour comparing to those of the results from the base paper mentioned (as previous) in graphs with red colour notation.

IV. DISCUSSIONS

There are several optimization techniques to determine the PID controller gains for a controller design, these optimization methods for determining the PID controller parameters do not provide any option for choosing the dominant pole placement parameters which are directly related to the closed loop response. We in this paper use a random search and optimization technique using PSO algorithm to determine the three desired closed loop parameters $\{m, c_1, c_2\}$ in the first stem and derive the PID controller parameters using the proposed analytical expressions which ensures dominant pole placement for the SOPTD systems while choosing different types of non-dominant poles. The derivations of PID controller expressions for the dominant pole placement of SOPTD systems without approximating the delay term is the main contribution of this paper.

We use two different PID controller expressions from two different non dominant pole type to obtain stability regions in the closed loop parameter space $\{m, c_1, c_2\}$. However the delay term is mapped as finite number of poles in the z-domain rather being approximated by fixed Pade. This is due to the Pade approximation of the time delay term increases the number of poles of the open loop time delay system.

In this paper we do not specify the desired closed loop parameters $\{m, c_1, c_2\}$ manually rather we selected the optimum closed loop parameters using the PSO algorithm. In Das et al. 2020 Euler's discretization was used for PID controller but we use more accurate Tustin's discretization, as a result present time response results are mostly insensitive to the choice of non dominant pole type.

V. CONCLUSION

In this paper dominant pole placement based continuous time PID controller has been designed for controlling the SOPTD systems. Two different transformations of PID controller gains have been obtained with two different non dominant pole types. Using these different controller expressions, PSO based random search and optimization method has been used to draw samples from the stability region in the dominant pole placement and controller parameter space for different sampling times. The obtained controller gains are very close to each other from these two different expressions for a fixed sampling time. We found that the control performances remain almost unaltered for each plant when the PID controller gains are obtained from the two alternative expressions of non dominant pole types. The proposed discrete time transformation of continuous time PID controllers using Tustin's discretization results better in time responses compared to the use of Euler's discretization.

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