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Common Coupled fixed point theorems for maps under (E.A) and occasionally w-compatible properties in Multiplicative and non-Multiplicative metric Spaces

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ABSTRACT: In this paper, we prove a unique common coupled fixed point theorems for four mappings using (E.A) property in multiplicative metric space and a unique common coupled fixed point theorem for maps satisfying occasionally w- compatible property in a non-multiplicative metric space.

KEYWORDS: Multiplicative metric space, E. A. property, occasionally w- compatible

I. INTRODUCTION AND PRELIMINARIES

The notion of multiplicative metric was introduced by Bashirov et al. [3] in 2008. After that Ozavsar and Cevikel [8] investigated its topological properties and proved some fixed point theorems. For more works on fixed, common fixed point theorems in Multiplicative metric spaces, we refer [5, 2, 6, 9].

The Coupled fixed point is introduced by Bhaskar and Lakshmikantham [4]. Later some of authors proved coupled fixed and coupled common fixed point theorems (See [1, 7, 10, 11, 12]).

The main aim of this paper is to prove a unique common coupled fixed point theorems for four mappings using (E.A) property in multiplicative metric space and a unique common coupled fixed point theorem for maps satisfying occasionally w- compatible property in a non-multiplicative metric space.

Now we recall some basic definitions and examples in multiplicative metric spaces.

Definition 1.1 ([3]): Multiplicative metric on a nonempty set X is a mapping $d: X \times X \rightarrow \mathbb{R}^+$

Satisfying the following conditions:

(d₁) $d(x, y) \geq 1$ for all $x, y \in X$,

(d₂) $d(x, y) = 1$ if and only if $x = y$,

(d₃) $d(x, y) = d(y, x)$,

(d₄) $d(x, y) \leq d(x, z).d(z, y)$ for all $x, y, z \in X$.

The pair (X, d) is called a multiplicative metric space.

Example 1.2 ([9]): Let R_+^n be the collection of all n-tuples of positive real numbers. And let $d^*: R_+^n \times R_+^n \rightarrow R$ be defined as

$$d^*(x, y) = \left| \frac{x_1}{y_1} \right|^*, \left| \frac{x_2}{y_2} \right|^*, \dots, \left| \frac{x_n}{y_n} \right|^*$$

Where $x = (x_1, x_2, \dots, x_n)$, $y = (y_1, y_2, \dots, y_n) \in R_+^n$ and $|\cdot|^*: R_+ \rightarrow R_+$ is defined as follows

$$|a|^* = \begin{cases} a & \text{if } a \geq 1 \\ \frac{1}{a} & \text{if } a < 1 \end{cases}$$

Then clearly $d^*(x, y)$ is a multiplicative metric.

Example 1.3 ([9]): Let $d: R \times R \rightarrow [1, \infty)$ be defined as $d(x, y) = a^{|x-y|}$ where $x, y \in R$ and $a > 1$. Then $d(x, y)$ is multiplicative metric.

Definition 1.4 ([8]): Let (X, d) be a multiplicative metric space, x_0 an arbitrary point in X , and $\epsilon > 1$. A multiplicative open ball $B(x_0, \epsilon)$ of radius ϵ centred at x_0 is the set $\{z \in X: d(z, x_0) < \epsilon\}$.

A sequence $\{x_n\}$ in a multiplicative metric space (X, d) is said to be multiplicative convergent to some point $x \in X$, if for any given $\epsilon > 1$, then there exists $n_0 \in N$ such that $x_n \in B(x, \epsilon)$ for all $n \geq n_0$. If $\{x_n\}$ converges to x , we write $x_n \rightarrow x$ as $n \rightarrow \infty$.

Definition 1.5 ([8]): A sequence $\{x_n\}$ in a multiplicative metric space (X, d) is said to be multiplicative convergent to x in X if and only if $d(x_n, x) \rightarrow 1$ as $n \rightarrow \infty$.

Note 1.5(a): Let $\{x_n\}$ be a sequence converging to x in a multiplicative metric space (X, d) and then $\lim_{n \rightarrow \infty} d(x_n, y) = d(x, y)$, for all $y \in X$.

Definition 1.6 ([8]): Let (X, d_x) and (Y, d_y) be two multiplicative metric space and x_0 an arbitrary but fixed element of X . A mapping $f: X \rightarrow Y$ is said to be multiplicative continuous at x_0 if and only if $x_n \rightarrow x_0$ in (X, d_x) implies that $f(x_n) \rightarrow f(x_0)$ in (Y, d_y) . That is, given arbitrary $\epsilon > 1$, then there exists $\delta > 1$ which depends on x_0 and ϵ such that $d_y(fx, fx_0) < \epsilon$ for all those x in X for which $d_x(x, x_0) < \delta$.

Definition 1.7 ([8]): A sequence $\{x_n\}$ in a multiplicative metric space (X, d) is said to be multiplicative Cauchy sequence if, for any $\epsilon > 1$, there exists $n_0 \in N$ such that $d(x_m, x_n) < \epsilon$ for all $m, n \geq n_0$.

Definition 1.8 ([8]): A multiplicative metric space (X, d) is said to be complete if every multiplicative Cauchy sequence $\{x_n\}$ in X is multiplicative convergent in X .

Definition 1.9 ([2]): Let Ψ denote the class of functions $\phi: [1, \infty) \rightarrow [1, \infty)$ which are continuous, nondecreasing and $\phi(t) = 1$ if and only if $t = 1$.

Note 1.9(a): If $\phi, \psi \in \Psi$ then $\psi(t) \leq \frac{\psi(t)}{\phi(t)}$ implies $t = 1$.

Definition 1.10 ([4]): An element $(x, y) \in X \times X$ is called a coupled fixed point of mapping $F: X \times X \rightarrow X$ if $x = F(x, y)$ and $y = F(y, x)$.

Definition 1.11 ([7]): An element $(x, y) \in X \times X$ is called (g_1) a coupled coincident point of mappings $F: X \times X \rightarrow X$ and $S: X \rightarrow X$ if $Sx = F(x, y)$ and $Sy = F(y, x)$. (g_2) a common coupled fixed point of mappings $F: X \times X \rightarrow X$ and $S: X \rightarrow X$ if $x = Sx = F(x, y)$ and $y = Sy = F(y, x)$.

Definition 1.12 ([1]): The mappings $F: X \times X \rightarrow X$ and $S: X \rightarrow X$ are called W-compatible if $S(F(x, y)) = F(Sx, Sy)$ and $S(F(y, x)) = F(Sy, Sx)$ whenever $Sx = F(x, y)$ and $Sy = F(y, x)$.

Now we define the following definitions.

Definition 1.13: Let (X, d) be a multiplicative metric space and $F: X \times X \rightarrow X$ and $S: X \rightarrow X$ be mappings. Then the pair (F, S) is said to satisfy (E.A) property if there exist sequences x_n and y_n in X such that $\lim_{n \rightarrow \infty} F(x_n, y_n) = \lim_{n \rightarrow \infty} Sx_n = t$ and $\lim_{n \rightarrow \infty} F(y_n, x_n) = \lim_{n \rightarrow \infty} Sy_n = t^1$ for so $t, t^1 \in X$

Definition 1.14: Let X be a non-empty set and $F: X \times X \rightarrow X$ and $S: X \rightarrow X$ be mappings. Then the pair (F, S) is said to be Occasionally w-compatible if there exist $x, y \in X$ such that $F(x, y) = Sx$, $F(y, x) = Sy$, $S(F(x, y)) = F(Sx, Sy)$ and $S(F(y, x)) = F(Sy, Sx)$.



MAIN RESULTS

Theorem 2.1

Let (X, d) be a multiplicative metric space and $F, G : X \times X \rightarrow X$ and $S, T : X \rightarrow X$ be Mappings satisfying the following

(2.1.1) $\Psi(d(F(x, y), G(u, v))) \leq \frac{\Psi(M(x, y, u, v))}{\varphi(M(x, y, u, v))}$ for all $x, y, u, v \in X$ where $\varphi, \Psi \in \Psi$ and

$$M(x, y, u, v) = \max \left\{ \begin{array}{l} d(Sx, Tu), d(Sy, Tv), d(Sx, F(x, y)), d(Sy, F(y, x)), \\ d(Tu, G(u, v)), d(Tv, G(v, u)), d(Sx, G(u, v)), \\ d(Sy, G(v, u)), d(Tu, F(x, y)), d(Tv, F(y, x)), \\ \frac{d(Sx, F(x, y))d(Tu, G(u, v))}{d(Sx, Tu)d(Sy, Tv)}, \frac{d(Sy, F(y, x))d(Tv, G(v, u))}{d(Sx, Tu)d(Sy, Tv)}, \\ \frac{d(Tu, F(x, y))d(Sx, G(u, v))}{d(Sx, Tu)d(Sy, Tv)}, \frac{d(Tv, F(y, x))d(Sy, G(v, u))}{d(Sx, Tu)d(Sy, Tv)} \end{array} \right.$$

(2.1.2) the pairs (F, S) and (G, T) are w - compatible.

(2.1.3) (a) (F, S) satisfies (E.A) property, $F(X \times X) \subseteq T(X)$ and $S(X)$ is a closed subspace of X (or)

(b) (G, T) satisfies (E.A) property, $G(X \times X) \subseteq S(X)$ and $T(X)$ is a closed subspace of X .

Then F, G, S and T have a unique common coupled fixed point.

Proof. Suppose (2.1.3) (a) holds. Then there exist sequence $\{x_n\}$ and $\{y_n\}$ in X such that

$$\begin{aligned} \lim_{n \rightarrow \infty} F(x_n, y_n) &= \lim_{n \rightarrow \infty} Sx_n = t \text{ and} \\ \lim_{n \rightarrow \infty} F(y_n, x_n) &= \lim_{n \rightarrow \infty} Sy_n = t^1 \end{aligned} \tag{1}$$

for some $t, t^1 \in X$.

Since $F(X \times X) \subseteq T(X)$, there exist sequences $\{z_n\}$ and $\{w_n\}$ in X such that $F(x_n, y_n) = Tz_n \rightarrow t$ and $F(y_n, x_n) = Tw_n \rightarrow t^1$ as $n \rightarrow \infty$. (2)

Consider

$$\Psi \lim_{n \rightarrow \infty} d(t, G(z_n, w_n)) \leq \Psi \lim_{n \rightarrow \infty} d(t, F(x_n, y_n))d(F(x_n, y_n), G(z_n, w_n))$$

from (d₄) and ψ is non – decreasing

$$= \psi \lim_{n \rightarrow \infty} d(F(x_n, y_n), G(z_n, w_n))$$

$$\leq \lim_{n \rightarrow \infty} \frac{\psi(M(x_n, y_n, z_n, w_n))}{\phi(M(x_n, y_n, z_n, w_n))}, \text{ where}$$

$$M(x_n, y_n, z_n, w_n) = \max \left\{ \begin{array}{l} d(Sx_n, Tz_n), d(Sy_n, Tw_n), d(Sx_n, F(x_n, y_n)), \\ d(Sy_n, F(y_n, x_n)), d(Tz_n, G(z_n, w_n)), d(Tw_n, G(w_n, z_n)) \\ d(Sx_n, G(z_n, w_n)), d(Sy_n, G(w_n, z_n)), d(Tz_n, F(x_n, y_n)) \\ d(Tw_n, F(y_n, x_n)), \frac{d(Sx_n, F(x_n, y_n))d(Tz_n, G(z_n, w_n))}{d(Sx_n, Tz_n)d(Sy_n, Tw_n)} \\ \frac{d(Sy_n, F(y_n, x_n))d(Tw_n, G(w_n, z_n))}{d(Sx_n, Tz_n)d(Sy_n, Tw_n)}, \frac{d(Tz_n, F(x_n, y_n))d(Sx_n, G(z_n, w_n))}{d(Sx_n, Tz_n)d(Sy_n, Tw_n)} \\ \frac{d(Tw_n, F(y_n, x_n))d(Sy_n, G(w_n, z_n))}{d(Sx_n, Tz_n)d(Sy_n, Tw_n)} \end{array} \right\}$$

$$\rightarrow \max \left\{ \begin{array}{l} 1, 1, 1, 1, \lim_{n \rightarrow \infty} d(t, G(z_n, w_n)), \\ \lim_{n \rightarrow \infty} d(t^1, G(w_n, z_n)), \lim_{n \rightarrow \infty} d(t, G(z_n, w_n)), \\ \lim_{n \rightarrow \infty} d(t^1, G(w_n, z_n)), 1, 1, \\ \lim_{n \rightarrow \infty} d(t, G(z_n, w_n)), \lim_{n \rightarrow \infty} d(t^1, G(w_n, z_n)) \\ \lim_{n \rightarrow \infty} d(t, G(z_n, w_n)), \lim_{n \rightarrow \infty} d(t^1, G(w_n, z_n)) \end{array} \right\}$$

$$\frac{d(Tw_n, F(y_n, x_n))d(Sy_n, G(w_n, z_n))}{d(Sx_n, Tz_n)d(Sy_n, Tw_n)}$$

from (1), (2), Note 1.5(a)

$$(\lim_{n \rightarrow \infty} d(t, G(z_n, w_n))) \leq \frac{\psi \max \lim_{n \rightarrow \infty} d(t, G(z_n, w_n)), \lim_{n \rightarrow \infty} d(t^1, G(w_n, z_n))}{\phi \max \lim_{n \rightarrow \infty} d(t, G(z_n, w_n)), \lim_{n \rightarrow \infty} d(t^1, G(w_n, z_n))}$$

Similarly we can show that

$$\psi \lim_{n \rightarrow \infty} d(t^1, G(w_n, z_n)) \leq \frac{\psi \max \lim_{n \rightarrow \infty} d(t, G(z_n, w_n)), \lim_{n \rightarrow \infty} d(t^1, G(w_n, z_n))}{\phi \max \lim_{n \rightarrow \infty} d(t, G(z_n, w_n)), \lim_{n \rightarrow \infty} d(t^1, G(w_n, z_n))}$$

Now

$$\psi \{ \max \{ \lim_{n \rightarrow \infty} d(t, G(z_n, w_n)), \lim_{n \rightarrow \infty} d(t^1, G(w_n, z_n)) \}, \max \{ \lim_{n \rightarrow \infty} d(t, G(z_n, w_n)), \psi \lim_{n \rightarrow \infty} d(t^1, G(w_n, z_n)) \} \}$$

$$\leq \frac{\psi \max \lim_{n \rightarrow \infty} d(t, G(z_n, w_n)), \lim_{n \rightarrow \infty} d(t^1, G(w_n, z_n))}{\phi \max \lim_{n \rightarrow \infty} d(t, G(z_n, w_n)), \lim_{n \rightarrow \infty} d(t^1, G(w_n, z_n))}$$

From Note 1.9(a), we have

$$\max \{ \lim_{n \rightarrow \infty} d(t, G(z_n, w_n)), \lim_{n \rightarrow \infty} d(t^1, G(w_n, z_n)) \} = 1$$



Which implies that

$$\lim_{n \rightarrow \infty} d(t, G(z_n, w_n)) = 1 = \lim_{n \rightarrow \infty} d(t^1, G(w_n, z_n))$$

Thus

$$\lim_{n \rightarrow \infty} G(z_n, w_n) = t \text{ and } \lim_{n \rightarrow \infty} G(w_n, z_n) = t^1 \tag{3}$$

Since $S(X)$ is a closed subspace of X , there exist $u, v \in X$ such that

$$\lim_{n \rightarrow \infty} Sx_n = t = Su \text{ and } \lim_{n \rightarrow \infty} Sy_n = t^1 = Sv. \tag{4}$$

From Note 1.5(a), (3),(4) and using continuity of ψ , we have

$$\psi(d(F(u, v), Su)) = \lim_{n \rightarrow \infty} \psi(d(F(u, v), G(z_n, w_n)))$$

$$\leq \lim_{n \rightarrow \infty} \frac{\psi(M(u, v, z_n, w_n))}{\varphi(M(u, v, z_n, w_n))} \tag{From (2.1.1)}$$

$$\leq \frac{\psi(\lim_{n \rightarrow \infty} M(u, v, z_n, w_n))}{\varphi(\lim_{n \rightarrow \infty} M(u, v, z_n, w_n))} \text{ Since } \psi, \varphi \text{ are continuous.}$$

$$M(u, v, z_n, w_n) = \max \left\{ \begin{array}{l} d(Su, Tz_n), d(Sv, Tw_n), d(Su, F(u, v)), \\ d(Sv, F(v, u)), d(Tz_n, G(z_n, w_n)), d(Tw_n, G(w_n, z_n)), \\ d(Su, G(z_n, w_n)), d(Sv, G(w_n, z_n)), d(Tz_n, F(u, v)), \\ d(Tw_n, F(v, u)), \frac{d(Su, F(u, v))d(Tz_n, G(z_n, w_n))}{d(Su, Tz_n)d(Sv, Tw_n)}, \\ \frac{d(Sv, F(v, u))d(Tw_n, G(w_n, z_n))}{d(Su, Tz_n)d(Sv, Tw_n)}, \frac{d(Su, G(z_n, w_n))d(Tz_n, F(u, v))}{d(Su, Tz_n)d(Sv, Tw_n)}, \\ \frac{d(Sv, G(w_n, z_n))d(Tw_n, F(v, u))}{d(Su, Tz_n)d(Sv, Tw_n)}, \end{array} \right.$$

$$\rightarrow \max \{d(Su, F(u, v)), d(Sv, F(v, u))\} \text{ as } n \rightarrow \infty$$

Thus

$$\psi(d(F(u, v), Su)) \leq \frac{\psi(\max\{d(Su, F(u, v)), d(Sv, F(v, u))\})}{\varphi(\max\{d(Su, F(u, v)), d(Sv, F(v, u))\})}$$

Similarly we can show that

$$\psi(d(F(u, v), Sv)) \leq \frac{\psi(\max\{d(Su, F(u, v)), d(Sv, F(v, u))\})}{\varphi(\max\{d(Su, F(u, v)), d(Sv, F(v, u))\})}$$

Thus

$$\psi(\max\{d(F(u, v), Su), d(F(v, u), Sv)\}) = \max\{\psi(d(F(u, v), Su)), \psi(d(F(v, u), Sv))\}$$

$$\leq \frac{\psi(\max\{d(Su, F(u, v)), d(Sv, F(v, u))\})}{\varphi(\max\{d(Su, F(u, v)), d(Sv, F(v, u))\})}$$

From Note 1.9 (a), we have

$$F(u, v) = Su = t \text{ and } F(v, u) = Sv = t^1. \tag{5}$$

Since the pair (F, S) is w -compatible, we have

$$F(t, t^1) = St \text{ and } F(t^1, t) = St^1. \tag{6}$$

Since $F(X \times X) \subseteq T(X)$ there exist $\alpha, \beta \in X$ such that $t = F(u, v) = T\alpha$

And $t^1 = F(v, u) = T\beta$.

Consider

$$\Psi(d(t, G(\alpha, \beta))) = \psi(d(F(u, v), G(\alpha, \beta)))$$

$$\leq \frac{\psi(M(u, v, \alpha, \beta))}{\varphi(M(u, v, \alpha, \beta))}$$



$$M(u, v, \alpha, \beta) = \max \left\{ \begin{array}{l} d(Su, T\alpha), d(Sv, T\beta), d(Sv, F(u, v)) \\ d(Sv, F(v, u)), d(T\alpha, G(\alpha, \beta)), d(\beta, G(\beta, \alpha)), \\ d(Su, G(\alpha, \beta)), d(Sv, G(\beta, \alpha)), d(T\alpha, F(u, v)), \\ d(T\beta, F(v, u)), \frac{d(Sx_n, F(u, v))d(T\alpha, G(\alpha, \beta))}{d(Su, T\alpha) + d(Sv, T\beta)}, \\ \frac{d(Sv, F(v, u))d(T\beta, G(\beta, \alpha))}{d(Su, T\alpha) + d(Sv, T\beta)}, \frac{d(T\alpha, F(u, v))d(Su, G(\alpha, \beta))}{d(Su, T\alpha) + d(Sv, T\beta)}, \\ \frac{d(T\beta, F(v, u))d(Sv, G(\beta, \alpha))}{d(Su, T\alpha) + d(Sv, T\beta)} \end{array} \right\}$$

→ max {d(t, G(α, β)), d(t¹, G(β, α))}

$$\psi(d(t, G(\alpha, \beta))) \leq \frac{\psi(\max\{d(t, G(\alpha, \beta)), d(t^1, G(\beta, \alpha))\})}{\varphi(\max\{d(t, G(\alpha, \beta)), d(t^1, G(\beta, \alpha))\})}$$

Similarly we can show that

$$\psi(d(t^1, G(\beta, \alpha))) \leq \frac{\psi(\max\{d(t, G(\alpha, \beta)), d(t^1, G(\beta, \alpha))\})}{\varphi(\max\{d(t, G(\alpha, \beta)), d(t^1, G(\beta, \alpha))\})}$$

Now

$$\psi(\max\{d(t, G(\alpha, \beta)), d(t^1, G(\beta, \alpha))\}) = \max\{\psi(d(t, G(\alpha, \beta))), \psi(d(t^1, G(\beta, \alpha)))\}$$

Hence

$$\begin{aligned} &\leq \frac{\psi(\max\{d(t, G(\alpha, \beta)), d(t^1, G(\beta, \alpha))\})}{\varphi(\max\{d(t, G(\alpha, \beta)), d(t^1, G(\beta, \alpha))\})} \\ &\varphi(\max\{d(t, G(\alpha, \beta)), d(t^1, G(\beta, \alpha))\}) \leq 1 \end{aligned}$$

But $\varphi(\max\{d(t, G(\alpha, \beta)), d(t^1, G(\beta, \alpha))\}) \geq 1$

Hence $\varphi(\max\{d(t, G(\alpha, \beta)), d(t^1, G(\beta, \alpha))\}) = 1$

Implies $(\max\{d(t, G(\alpha, \beta)), d(t^1, G(\beta, \alpha))\}) = 1$

Implies

$$d(t, G(\alpha, \beta)) = 1 = d(t^1, G(\beta, \alpha))$$

$$G(\alpha, \beta) = T\alpha = t \text{ and } G(\beta, \alpha) = T\beta = t^1. \tag{7}$$

Since (G, T) is w-compatible, we have

$$G(t, t^1) = Tt \text{ and } G(t^1, t) = Tt^1 \tag{8}$$

Now

$$\begin{aligned} \Psi(d(St, t)) &\leq \psi(d(F(t, t^1), t)) \\ &= \psi \lim_{n \rightarrow \infty} d(F(t, t^1), G(z_n, w_n)) \\ &= \lim_{n \rightarrow \infty} \psi(d(F(t, t^1), G(z_n, w_n))) \\ &\leq \lim_{n \rightarrow \infty} \frac{\psi(M(t, t^1, z_n, w_n))}{\varphi(M(t, t^1, z_n, w_n))} \end{aligned}$$



$$M(t, t^1, z_n, w_n) = \max \left\{ \begin{array}{l} d(St, Tz_n), d(s t^1, Tw_n), d(St, F(t, t^1)), \\ d(St^1, F(t^1, t)), d(T, G(z_n, w_n)), d(Tw_n, G(w_n, z_n)), \\ d(St, G(z_n, w_n)), d(S t^1, G(w_n, z_n)), d(Tz_n, F(t, t^1)), \\ d(Tw_n, F(t^1, t)), \frac{d(St, F(t, t^1))d(Tz_n, G(z_n, w_n))}{d(St, Tz_n)d(St^1, Tw_n)} \\ \frac{d(S t^1, F(t^1, t))d(Tw_n, G(w_n, z_n))}{d(St, Tz_n)d(S t^1, Tw_n)}, \frac{d(Tz_n, F(t, t^1)) d(S t^1, G(w_n, z_n))}{d(St, Tz_n)d(S t^1, Tw_n)} \\ \frac{d(Tw_n, F(t^1, t))d(St, G(w_n, z_n))}{d(St, Tz_n)d(St^1, Tw_n)} \end{array} \right\}$$

Thus

$$\begin{aligned} &\rightarrow \max \{d(St, t), d(St^1, t^1)\}, \text{ from (2), (3), (6)} \\ &\psi(d(St, t)) \leq \frac{\psi(\max\{d(St, t), d(St^1, t^1)\})}{\varphi(\max\{d(St, t), d(St^1, t^1)\})} \end{aligned}$$

Similarly we can show that

$$\psi(d(St^1, t^1)) \leq \frac{\psi(\max\{d(St, t), d(St^1, t^1)\})}{\varphi(\max\{d(St, t), d(St^1, t^1)\})}$$

Now

$$\begin{aligned} \Psi(\max \{d(St, t), d(St^1, t^1)\}) &= \max\{\psi(d(St, t)), \psi(d(St^1, t^1))\} \\ &\leq \frac{\psi(\max\{d(St, t), d(St^1, t^1)\})}{\varphi(\max\{d(St, t), d(St^1, t^1)\})} \end{aligned}$$

By Note 1.9 (a), we have $\max \{d(St, t), d(St^1, t^1)\} = 1$. Which in turn implies that $St = t$ and $St^1 = t^1$.

Similarly we can show that $Tt = t$ and $Tt^1 = t^1$.

Now from (6) and (8), it follows that (t, t^1) is a common coupled fixed point of F, G, S and T . Uniqueness of common coupled fixed point follows easily from (2.2.1). Similarly the theorem follows when 2.1.3(b) holds.

Now we give following example to illustrate Theorem 2.1.

Example 2.2

Suppose that $X = [0, 1]$. Define $d(x, y) = 2^{|x-y|}$ for all $x, y \in X$. Let $F, G: X \times X \rightarrow X$ and $S, T: X \times X \rightarrow X$ be defined as

$$F(x, y) = \frac{x^2 + y^2}{16}, G(x, y) = \frac{x + y}{16}, S_x = \frac{x^2}{2}, T_x = \frac{x}{2}, \psi(t) = t, \varphi(t) = t^{\frac{3}{4}}$$

Then (F, S) satisfies (E.A) property with $\{x_n\} = \{\frac{1}{n}\}$ and $\{y_n\} = \{\frac{1}{2n}\} \in X$.

$$\begin{aligned} d(F(x, y), G(u, v)) &= 2^{\left| \frac{x^2+y^2}{16} - \frac{u+v}{16} \right|} \\ &\leq 2^{\frac{1}{16}(|x^2-u| + |y^2-v|)} \\ &\leq 2^{\frac{1}{16} \max\{|x^2-u|, |y^2-v|\}} \\ &= \max \left\{ 2^{\frac{|x^2-u|}{8}}, 2^{\frac{|y^2-v|}{8}} \right\} \\ &= \max \left\{ 2^{\left| \frac{x^2}{2} - \frac{u}{2} \right|}, 2^{\left| \frac{y^2}{2} - \frac{v}{2} \right|} \right\}^{\frac{1}{4}} \end{aligned}$$



$$= \frac{\psi(\max\{d(Sx, Tu), d(Sy, Tv)\})^{\frac{1}{4}}}{\varphi(M(x, y, u, v))}$$

Thus, the inequality (2.1.1) is satisfied. One can easily verify the remaining conditions of Theorem 2.1. Clearly (0, 0) is unique common coupled fixed point of F, G, S and T.

Remark: There are some functions $d: X \times X \rightarrow [1, \infty)$ such that $d(x, y) = 1 \Leftrightarrow x = y$ and $d(x, y) \neq d(y, x)$ and $d(x, y) \not\leq d(x, z)d(z, y)$.

For example, Let $X = \{1, 2, 3\}$, $d(1, 1) = d(2, 2) = d(3, 3) = 1$, $d(1, 2) = 2$, $d(2, 1) = 1.2$, $d(1, 3) = d(3, 1) = 1.3$, $d(2, 3) = d(3, 2) = 1.5$.

Then $d(1, 2) \neq d(2, 1)$ and $d(1, 2) \not\leq d(1, 3)d(3, 2)$.

Theorem 2.3 Let X be a non-empty set and $d: X \times X \rightarrow [1, \infty)$ be a function such that $d(x, y) = 1$ iff $x = y$. Let $F, G: X \times X \rightarrow X$ and $S, T: X \rightarrow X$ be satisfying the following

(2.3.1)

$$\psi(d(F(x, y), G(u, v))) \leq \frac{\psi(M(x, y, u, v))}{\varphi(M(x, y, u, v))} \text{ for all } x, y, u, v \in X,$$

Where $\psi, \varphi: [1, \infty) \rightarrow [1, \infty)$ be such that $\varphi(t) = 1$ iff $t = 1$ and $\psi(t) = 1$ iff $t = 1$ and

$$M(x, y, u, v) = \max \left\{ \begin{array}{l} d(Sx, Tu), d(Sy, Tv), d(Sx, F(x, y)), d(Sy, F(y, x)), \\ d(Tu, G(u, v)), d(Tv, G(v, u)), d(F(x, y), Tu) \\ d(F(y, x), Tv), d(Sx, G(u, v)), d(Sy, G(v, u)), \\ \frac{d(Sx, F(x, y))d(Tu, G(u, v))}{d(Sx, Tu)d(Sy, Tv)}, \frac{d(Sy, F(y, x))d(Tv, G(v, u))}{d(Sy, Tv)d(Sx, Tu)} \\ \frac{d(Sx, G(u, v))d(F(x, y), Tu)}{d(Sx, Tu)d(Sy, Tv)}, \frac{d(Sy, G(v, u))d(F(y, x), Tv)}{d(Sy, Tv)d(Sx, Tu)} \end{array} \right\}$$

(2.2.2) the pairs (F, S) and (G, T) are occasionally w-compatible Then there exists a unique

Common coupled fixed point of F, G, S and T

Proof. Since the pairs (F, S) and (G, T) are occasionally w-compatible, there exist points $(x, y), (\tilde{x}, \tilde{y}) \in X \times X$ such that

$$\begin{aligned} Sx &= F(x, y), Sy = F(y, x) \text{ and} \\ S(F(x, y)) &= F(Sx, Sy), S(F(y, x)) = F(Sy, Sx) \end{aligned} \tag{9}$$

$$T\tilde{x} = G(\tilde{x}, y), Ty = G(y, \tilde{x}) \text{ and}$$

$$T(G(\tilde{x}, \tilde{y})) = G(T\tilde{x}, T\tilde{y}), T(G(\tilde{y}, \tilde{x})) = G(T\tilde{y}, T\tilde{x}) \tag{10}$$

Now

$$S^2x = F(Sx, Sy), S^2y = F(Sy, Sx) \tag{11}$$

$$T^2\tilde{x} = G(T\tilde{x}, T\tilde{y}), T^2\tilde{y} = G(T\tilde{y}, T\tilde{x}) \tag{12}$$

Now, we shall show that $Sx = T\tilde{x}$ and $Sy = T\tilde{y}$

From (2.3.1), consider y^*

$$\psi(d(F(x, y), G(\tilde{x}, y^*))) \leq \frac{\psi(M(x, y, \tilde{x}, y^*))}{\varphi(M(x, y, \tilde{x}, y^*))}$$

where

$$M(x, y, \tilde{x}, \tilde{y}) = \max \left\{ \begin{array}{l} d(Sx, T\tilde{x}), d(Sy, T\tilde{y}), d(Sx, F(x, y)), d(Sy, F(y, x)), \\ d(T\tilde{x}, G(\tilde{x}, \tilde{y})), d(T\tilde{y}, G(\tilde{y}, \tilde{x})), d(F(x, y), T\tilde{x}), \\ d(F(y, x), T\tilde{y}), d(Sx, G(\tilde{x}, \tilde{y})), d(Sy, G(\tilde{y}, \tilde{x})), \\ \frac{d(Sx, F(x, y))d(T\tilde{x}, G(\tilde{x}, \tilde{y}))}{d(Sx, T\tilde{x})d(Sy, T\tilde{y})}, \frac{d(Sy, F(y, x))d(T\tilde{y}, G(\tilde{y}, \tilde{x}))}{d(Sx, T\tilde{x})d(Sy, T\tilde{y})} \\ \frac{d(Sx, G(\tilde{x}, \tilde{y}))d(F(x, y), T\tilde{x})}{d(Sx, T\tilde{x})d(Sy, T\tilde{y})}, \frac{d(Sy, G(\tilde{y}, \tilde{x}))d(F(y, x), T\tilde{y})}{d(Sx, T\tilde{x})d(Sy, T\tilde{y})} \\ \frac{d(Sx, G(\tilde{x}, \tilde{y}))d(F(x, y), T\tilde{x})}{d(Sx, T\tilde{x})d(Sy, T\tilde{y})}, \frac{d(Sy, G(\tilde{y}, \tilde{x}))d(F(y, x), T\tilde{y})}{d(Sx, T\tilde{x})d(Sy, T\tilde{y})} \end{array} \right\}$$

$$= \max \{d(Sx, T\tilde{x}), d(Sy, T\tilde{y})\}$$

Thus

$$\psi(d(Sx, T\tilde{x})) \leq \frac{\psi(\max\{d(Sx, T\tilde{x}), d(Sy, T\tilde{y})\})}{\phi(\max\{d(Sx, T\tilde{x}), d(Sy, T\tilde{y})\})} \tag{13}$$

Similarly we can show that

$$\psi(d(Sy, T\tilde{y})) \leq \frac{\psi(\max\{d(Sx, T\tilde{x}), d(Sy, T\tilde{y})\})}{\phi(\max\{d(Sx, T\tilde{x}), d(Sy, T\tilde{y})\})} \tag{14}$$

Now we have

$$\begin{aligned} \psi(\max\{d(Sx, T\tilde{x}), d(Sy, T\tilde{y})\}) &= \max\{\psi(d(Sx, T\tilde{x})), \psi(d(Sy, T\tilde{y}))\} \\ &\leq \frac{\psi(\max\{d(Sx, T\tilde{x}), d(Sy, T\tilde{y})\})}{\phi(\max\{d(Sx, T\tilde{x}), d(Sy, T\tilde{y})\})} \end{aligned}$$

From (13), (14)

Hence $\phi(\max\{d(Sx, T\tilde{x}), d(Sy, T\tilde{y})\}) \leq 1$ so that

$$\max\{d(Sx, T\tilde{x}), d(Sy, T\tilde{y})\} = 1$$

Hence We have $Sx = T\tilde{x}$ and $Sy = T\tilde{y}$

Now write

$$u = Sx = T\tilde{x} \text{ and } v = Sy = T\tilde{y}. \tag{15}$$

Thus, from (11) and (12), we get

$$Su = S^2x = F(Sx, Sy) = F(u, v), Sv = S^2y = F(Sy, Sx) = F(v, u) \tag{16}$$

$$Tu = T^2\tilde{x} = G(S\tilde{x}, S\tilde{y}) = G(u, v), Tv = T^2\tilde{y} = G(S\tilde{y}, S\tilde{x}) = G(v, u). \tag{17}$$

Now we shall show that $u = Su = Tu$ and $v = Sv = Tv$.

From (2.1.2), consider

$$\psi(d(F(u, v), G(\tilde{x}, \tilde{y}))) \leq \frac{\psi(M(u, v, \tilde{x}, \tilde{y}))}{\phi(M(u, v, \tilde{x}, \tilde{y}))}$$

where



$$M(u, v, \tilde{x}, \tilde{y}) = \max \left\{ \begin{array}{l} d(Su, T\tilde{x}), d(Sv, T\tilde{y}), d(Su, F(u, v)), d(Sv, F(v, u)), \\ d(T\tilde{x}, G(\tilde{x}, \tilde{y})), d(T\tilde{y}, G(\tilde{y}, \tilde{x})), d(F(u, v), T\tilde{x}), \\ d(F(v, u), T\tilde{y}), d(Su, G(\tilde{x}, \tilde{y})), d(Sv, G(\tilde{y}, \tilde{x})), \\ \frac{d(Su, F(u, v))d(T\tilde{x}, G(\tilde{x}, \tilde{y}))}{d(Su, T\tilde{x})d(Sv, T\tilde{y})}, \frac{d(Sv, F(v, u))d(T\tilde{y}, G(\tilde{y}, \tilde{x}))}{d(Sv, T\tilde{y})d(Su, T\tilde{x})}, \\ \frac{d(Su, G(\tilde{x}, \tilde{y}))d(F(u, v), T\tilde{x})}{d(Su, T\tilde{x})d(Sv, T\tilde{y})}, \frac{d(Sv, G(\tilde{y}, \tilde{x}))d(F(v, u), T\tilde{y})}{d(Sv, T\tilde{y})d(Su, T\tilde{x})} \end{array} \right\}$$

$$= \max d(Su, u), d(Sv, v)$$

Thus

$$\psi(d(Su, u)) \leq \frac{\psi(\max\{d(Su, u), d(Sv, v)\})}{\varphi(\max\{d(Su, u), d(Sv, v)\})} \tag{18}$$

Similarly we can show that

$$\psi(d(Sv, v)) \leq \frac{\psi(\max\{d(Su, u), d(Sv, v)\})}{\varphi(\max\{d(Su, u), d(Sv, v)\})} \tag{19}$$

Now

$$\begin{aligned} \psi(\max\{d(Su, u), d(Sv, v)\}) &= \max\{\psi(d(Su, u)), \psi(d(Sv, v))\} \\ &\leq \frac{\psi(\max\{d(Su, u), d(Sv, v)\})}{\varphi(\max\{d(Su, u), d(Sv, v)\})} \end{aligned}$$

From (18), (19)

Which in turn yields that $\varphi(\max\{d(Su, u), d(Sv, v)\}) \leq 1$

So that $\varphi(\max\{d(Su, u), d(Sv, v)\}) = 1$ implies $\max\{d(Su, u), d(Sv, v)\} = 1$ Hence

$$Su = u \text{ and } Sv = v \tag{20}$$

In the similar lines we can show that

$$Tu = u \text{ and } Tv = v \tag{21}$$

Now from (16), (17), (20) and (21) it follows that (u, v) is a common coupledfixed point of F, G, S and T .

Uniqueness of common coupled fixed point follows easily from (2.3.1).

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