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# An Application of Neutrosophic Persistence in Decision-making Problem

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**ABSTRACT:** A relatively new field of data analysis, Topological Data Analysis (TDA) has endorsed very useful in a variety of applications. Topological Data Analysis is used for understanding the data better through visualization. Its main persistence tool is Persistence Homology, that examines data structures at different scales. Persistent Homology measures the topological features of the data that persists across multiple scales. Persistent Homology illustration include persistence barcodes and persistence graphs. Fuzzy and neutrosophic sets are two efficient tools for dealing with uncertainty and ambiguity in real-life problems. Neutrosophic set is more capable than fuzzy set to deal the uncertainties of a real-world problem. In this paper, we provide a conceptual overview of the major advancements in using Persistent Homology in neutrosophic graphs.

**KEYWORDS:** Topological Data Analysis, Persistent Homology, Neutrosophic Persistent Homology, truth membership, persistent truth value

## I. INTRODUCTION

Homology is the study of topological features. Persistent Homology is a tool used to find the most persistent homology group. A neutrosophic set is an extension of the classical set, fuzzy set, and intuitionistic fuzzy set. In the non-linear programming model introduced by Garg and Nancy [4] to solve a decision-making problem, the parameters are represented by interval neutrosophic numbers. Here, we have described the method to solve a decision-making problem by finding the highest truth membership with the help of Persistent Homology.

This paper is organized as follows. Section II introduces the background for topological data analysis. In Section III, an application is presented. Section IV concludes the paper.

## II. MATERIALS AND METHODS

In this section, we introduce the background notions at the basis of our work.

### 2.1 Simplicial complexes:

Simplicial complexes are a collection of well-glued bricks called simplices. A simplicial complex  $\Sigma$  is a finite set of simplices which satisfies the gluing conditions requiring that each face of a simplex in  $\Sigma$  belong to  $\Sigma$ , and each non-empty intersection of any two simplices in  $\Sigma$  is a face of both. [3]

### 2.2 Betti Numbers:

Consider a shape is given as a topological space  $S$ . Homology studies geometric patterns of  $S$  by examining its  $k$ -dimensional holes. For example, zero-dimensional holes are connected components, one-dimensional holes are loops, and two-dimensional holes are voids. These  $k$ -dimensional holes are often called the topological features of the shape. Let  $H_k(S)$  denotes the  $k^{\text{th}}$  homology group of  $S$ . The Betti number  $\beta_k$  [1] is the rank of  $H_k(S)$  and it counts the number of  $k$ -dimensional holes in  $S$ . Figure 1 presents three topological spaces and their Betti numbers. For example, the triangle in Figure 1b is one connected component with one hole ( $\beta_0 = 1$  and  $\beta_1 = 1$ ). Similarly, the Betti numbers of the circle in Figure 1b is  $\beta_0 = 1$  and  $\beta_1 = 1$ . Therefore, the Betti numbers of Figure 1b is two dimension-zero features and two dimension-one features ( $\beta_0 = 2$  and  $\beta_1 = 2$ ).

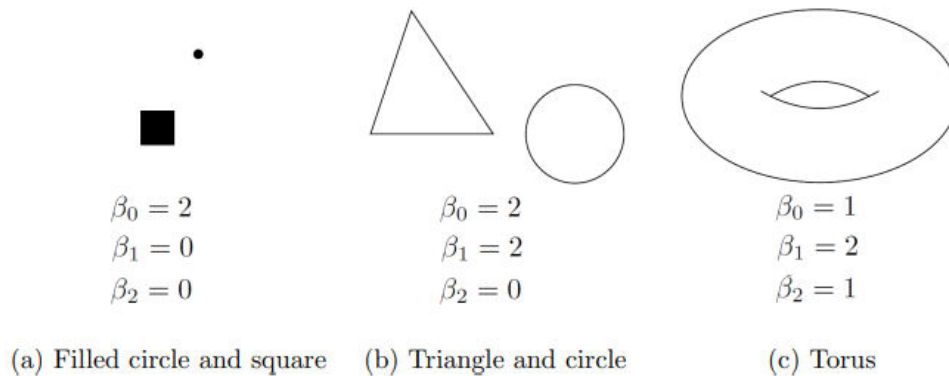


Fig.1: Topological space examples and their Betti numbers.  $\beta_0$ ,  $\beta_1$  and  $\beta_2$  represent the number of zero-, one-, and two-dimensional holes.

The persistent Betti numbers were introduced to count, for any intermediate complex, how many homological features contribute to the homology of the final object.

### 2.3 Persistence diagram:

A persistence diagram is a collection of points in  $\mathbb{R}^2$  where points in the diagram represent the birth and death values of features. Let  $P = \{(b, d) \in \mathbb{R}^2 : b < d\}$  be a persistence diagram such that  $(b, d)$  denotes the birth and death values and  $R$  be the space of persistence diagrams. The birth and death values can be plotted as points on the horizontal and vertical axis, respectively. Because the death of a topological feature comes after its birth, points of the persistence diagrams are placed above the diagonal line.[2]

### 2.4 Neutrosophic Sets:

A neutrosophic set has three membership grades: truth, indeterminacy and false of each and every element. These three membership grades are always independent and always lie in the interval  $[0,1]$ . [5]

## III. AN APPLICATION OF NEUTROSOPHIC PERSISTENCE

### 3.1 Algorithm:

Suppose that we have  $n$  alternatives and  $m$  criteria. The steps to compute neutrosophic persistence are presented as follows:

#### Step 1:

Construct the decision matrix  $X$  as follows:  $X = [x_{ij}]_{n \times m}$ , where  $x_{ij}$  denotes the performance value of the  $i^{\text{th}}$  alternative on  $j^{\text{th}}$  criterion and  $i \in \{1, 2, \dots, n\}, j \in \{1, 2, \dots, m\}$ .

#### Step 2:

Calculate the normalized decision matrix.

We use sum-based linear normalization of performance values as follows:

$$r_{ij} = \frac{x_{ij}}{\sum x_{ij}}$$

if  $j$  belongs to the benefit criteria and

$$r_{ij} = \frac{1}{\sum \frac{1}{x_{ij}}}$$



if  $j$  belongs to the cost criteria.

Step 3:

Calculate the weighted normalized decision matrix.

The weighted normalized performance values are calculated as follows:

Step 4:

Calculate the truth and falsity membership values by taking the sum of the benefit and cost criteria respectively.

Step 5:

Compute the persistence barcode and persistence diagram with falsity membership as the x-axis and truth membership as the y-axis.

### 3.2 Problem:

Consider a neutrosophic decision making problem for selecting the best mobile under the following criteria: Storage space, Camera quality and Price.

Let  $M = \{ M_1, M_2, M_3, M_4, M_5 \}$  be the set of mobile phones and  $C = \{ C_1, C_2, C_3 \}$  be the set of criteria taken into consideration and let  $C_1$  denote storage space,  $C_2$  denote the camera quality and  $C_3$  denote the price of the mobile.

TABLE I

	$C_1$	$C_2$	$C_3$
$M_1$	64 GB	12 MP	20,000
$M_2$	128 GB	16 MP	25,000
$M_3$	16 GB	8 MP	18,000
$M_4$	32 GB	8 MP	15,000
$M_5$	16 GB	12 MP	12,000
$\Sigma$	256	56	90,000
Weightage	40%	30%	30%

The storage space and camera quality are defined as benefit criteria and the price is defined as cost criterion.

TABLE II

	$C_1$	$C_2$	$C_3$	$\frac{1}{C_3}$
$M_1$	64 GB	12 MP	20,000	0.00005
$M_2$	128 GB	16 MP	25,000	0.00004
$M_3$	16 GB	8 MP	18,000	0.000056
$M_4$	32 GB	8 MP	15,000	0.000067
$M_5$	16 GB	12 MP	12,000	0.000083
	256	56	90,000	0.000296



We can calculate the normalized performance values using the formula mentioned in step 2. The normalized values are tabulated below.

TABLE III

	$C_1$	$C_2$	$C_3$
$M_1$	0.25	0.21	0.17
$M_2$	0.50	0.29	0.14
$M_3$	0.06	0.14	0.19
$M_4$	0.12	0.14	0.23
$M_5$	0.06	0.21	0.28
$w_j$	0.4	0.3	0.3

The weighted normalized performance values are calculated using the formula mentioned in step 3. The weighted normalized values are tabulated below:

TABLE IV

	$C_1$	$C_2$	$C_3$
$M_1$	0.25	0.063	0.051
$M_2$	0.20	0.087	0.042
$M_3$	0.025	0.042	0.057
$M_4$	0.050	0.042	0.069
$M_5$	0.025	0.063	0.084

The truth membership (T) is the sum of C1 and C2 and, C3 is the falsity membership. The design/looks is considered as the indeterminacy membership. Thus, the neutrosophic decision matrix is

TABLE V

	T	F
$M_1$	0.313	0.051
$M_2$	0.287	0.042
$M_3$	0.067	0.057
$M_4$	0.092	0.069
$M_5$	0.088	0.084

A neutrosophic persistence pair (F,T) represents the point that appears at F and vanishes at T. In this case,  $T - F$  corresponds to the lifespan of that point. That is, the homology class is born at the point where falsity ends and dies at the point upto which the truth value persists.

Barcodes are widely used because of their stability even in the presence of noise. A neutrosophic persistence barcode is a collection of horizontal bars in which the horizontal axis and vertical axis correspond to the falsity and truth values respectively. It is used to visualize the persistence of the truth membership. In the barcode, short bars correspond to the points closer to the diagonal and long bars correspond to points farther from the diagonal. In other words, short bars correspond to cities with lesser truth value persistence and long bars correspond to cities with higher truth value persistence.

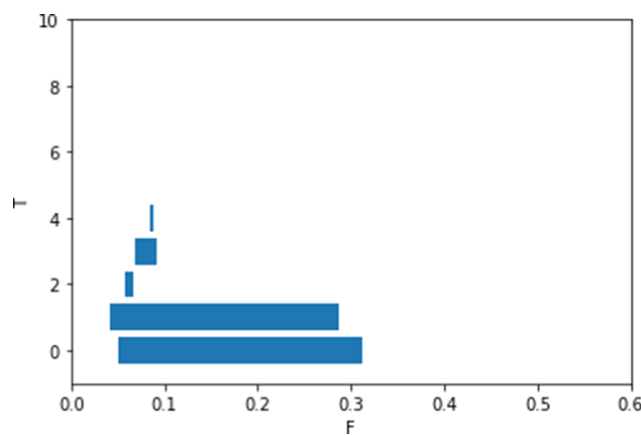


Fig.2: Neutrosophic Persistence Barcode

The neutrosophic persistence diagram is the representation of the persistence pairs (F,T) in the Cartesian plane. The lifespan of the point (F,T) can be found by measuring the distance of the point from the diagonal. The points that are closer to the diagonal are the ones with higher falsity value. The more a point is depicted away from the diagonal, the highest truth membership it has.

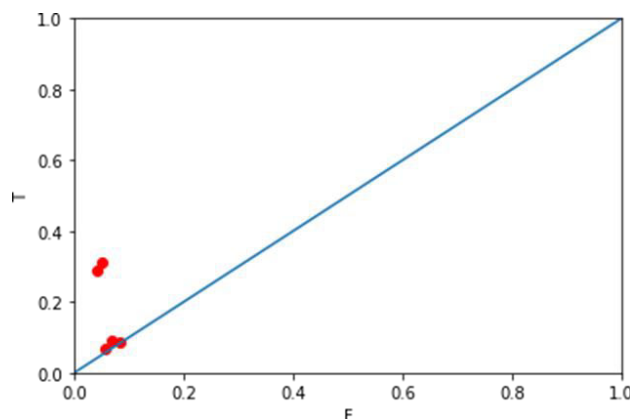


Fig.3: Neutrosophic Persistence Diagram

From the persistence diagram, we observe that the point (0.051,0.313) is comparatively more away from the diagonal. This point corresponds to Mobile 1. Thus, Mobile 1 has the highest truth membership. Hence, it is appropriate to choose Mobile 1.



#### IV. CONCLUSION

There are many ways to solve decision-making problems. Neutrosophic Persistence can be applied to real-world problems which aim at finding the most desirable value. The findings of this paper help to choose the best mobile phone to be purchased.

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