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Laplace Transforms in Science and Technology

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ABSTRACT: Our daily lives depend greatly on mathematics. One of the key techniques used by academics to solve numerous real-world issues that have been modelled as differential equations, simultaneous differential equations, or integral equations is the Laplace transform. Researchers from all scientific disciplines employ a variety of mathematical processes, tools, or models to explain and provide proof for their study findings. One of the main methods scientists and researchers utilise to solve their difficulties is the Laplace transform.

KEYWORDS: Laplace Transform, Properties and Applications

I. INTRODUCTION

The basic goal of the Laplace Transformation is to simplify complex systems of equations, differential equations, or integral equations so that we may easily find the answer. There are several puzzles that are extremely challenging to solve or have unusual algebraic structures. The Laplace transformation is essential for solving complex integral functions. To determine nth order linear differential equations involving constant coefficients, the Laplace transformation method is typically required.

A useful technique for analysing circuits containing sinusoidal or non-sinusoidal data sources is the Laplace transformation. The process of Laplace transformation is quite useful for examining the characteristics of a wide range of problems due to its broad range of applications. Laplace transform is typically used to solve differential equations of higher orders. It has a plethora of uses in the fields of applied sciences, engineering, and mathematics. The Laplace transform is a commonly used approach for dealing with higher order differential equations. It is used to make necessary adjustments to circuit, mechanical, electrical, aeronautical, picture-handling, and other frameworks.

We frequently employ mathematical models and their applications in daily life. An essential role is played by the population growth model. Laplace transform used in population growth forecast mathematical model. Laplace transform has straightforward applications in the engineering domains of transfer function of mechanical systems and nuclear physics. How Laplace transforms and the use of time derivative property utilising Laplace transforms are related to calculating discounted value in finance. A fundamental necessity of all engineering systems, the control system, is broken by the Laplace transform theory. The concept of partial fractions also makes use of the Laplace transform.

II. PRELIMINARIES

Laplace Transform: The Laplace transform is named for Pierre-Simon, marquis de Laplace, a mathematician and astronomer who employed a related transform in his research on probability theory. In *Essai philosophique sur les probabilités* (1814), Laplace wrote extensively about the usage of generating functions; as a result, the integral form of the Laplace transform naturally developed. Laplace used generating functions in a way that is comparable to the z-transform that is used today, and he paid minimal attention to the continuous-valued case that Niels Henrik Abel addressed. By Mathias Lerch, Oliver Heaviside, and Thomas Bromwich in the late 19th and early 20th centuries, the hypothesis was further developed.



The mathematical procedures known as the transformation of Laplace are those that change a certain function from the time domain to the complex frequency domain. In order to solve differential equations having boundary values without determining the values of arbitrary variables and the general solution, Laplace transforms are crucial.

$$f(t) \rightarrow F(s)$$

where(t) represent time and (s) represent complex frequency

Laplace transform definition is given by

$$F(S) = \int_0^{\infty} f(t)e^{-st} dt$$

Various properties of Laplace transforms:

1. Linearity property
2. Laplace transform for integral
3. Laplace transform for derivative
4. Initial value theorem
5. Final value theorem

1.1 Linearity Property in Laplace transforms:

According to the Laplace transform's linearity property, the weighted total of the individual Laplace transforms is identical to the Laplace transform of the weighted sum of two signals. [10]

Mathematically, if x(t) is a time domain function, then its Laplace transform is defined as –

$$L[x(t)] = X(s) = \int_0^{\infty} x(t)e^{-st} dt$$

1.2 Laplace transform for integral:

When the time domain function is integrated, its Laplace Transform can be expressed as,

$$L \left[\int_0^t f(u) du \right] = \frac{1}{s} \phi(s)$$

By definition,

$$L \left[\int_0^t f(u) du \right] = \int_0^{\infty} e^{-st} \left[\int_0^t f(u) du \right] dt. \text{ Integrating by parts,}$$

$$= \left[\int_0^t f(u) du \right] \left\{ -\frac{e^{-st}}{s} \right\} \Big|_0^{\infty} - \int_0^{\infty} \left[-\frac{e^{-st}}{s} \right] \frac{d}{dt} \int_0^t f(u) du$$

$$\text{But } \frac{d}{dt} \int_0^{\infty} f(u) du = f(t)$$

$$\therefore L \left[\int_0^t f(u) du \right] = \int_0^{\infty} \frac{1}{s} e^{-st} f(t) dt$$

$$= \frac{1}{s} L[f(t)]$$

$$\therefore L \left[\int_0^t f(u) du \right] = \frac{1}{s} \phi(s)$$



1.3 Laplace transform for derivative:

Let $f(t)$ be the time domain function. The Laplace Transform of its derivative can be expressed as [6]

$$L[f'(t)] = sL[f(t)] - f(0)$$

Proof By definition, $L[f'(t)] = \int_0^\infty e^{-st} f'(t) dt$

Integrating by parts,

$$\begin{aligned} L[f'(t)] &= [e^{-st} f(t)]_0^\infty - \int_0^\infty (-s)e^{-st} f(t) dt \\ &= f(0) + s \int_0^\infty e^{-st} f(t) dt \end{aligned}$$

$$\therefore L[f'(t)] = sL[f(t)] - f(0)$$

By differentiating the above equation with respect to variable t ,

$$L[f''(t)] = s^2 L[f(t)] - s f(0) - f'(0)$$

1.4 Initial Value Theorem:

This theorem is applicable only when we have the power of numerator polynomial less than that of denominator

$$\lim_{t \rightarrow 0} (f(t)) = \lim_{s \rightarrow \infty} s L[f(t)]$$

Applications:

1. Transient state gain can be calculated using the Initial Value Theorem.
2. With the use of the initial value theorem, we are able to obtain a signal's starting value, or $x(0)$, straight from its Z-transform, or $X(z)$, without first needing to figure out X 's inverse Z-transform (z). [4]
3. The initial value theorem is a theorem that connects frequency domain claims to time domain behaviours as time approaches zero in mathematical analysis.

1.5 Final value theorem:

This theorem is useful for finding the final value because it is almost always easier to derive the Laplace transform [2] and evaluate the limit on the right-hand side, than to derive the equation for $f(t)$ and evaluate the limit on the left-hand side.

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} s L\{f(t)\}$$

Applications:

1. Establishing the long-term stability of a system. [7]
2. In probability and statistics to calculate the moments of a random variable.
3. In fact, both the impulse response and step response oscillate, and (in this special case) the final value theorem describes the average values around which the responses oscillate. [8]



Some standard frequency and time domain relations:

<i>Frequency Domain Function</i>	<i>Inverse Laplace Transform</i>
$\frac{1}{s}$	$L^{-1}\left[\frac{1}{s}\right] = 1$
$\frac{1}{s+a}$	$L^{-1}\left[\frac{1}{s+a}\right] = e^{-at}$
$\frac{1}{s-a}$	$L^{-1}\left[\frac{1}{s-a}\right] = e^{at}$
$\frac{1}{s^n}$	$L^{-1}\left[\frac{1}{s^n}\right] = \frac{t^{n-1}}{\Gamma n}$
$\frac{1}{s^2+a^2}$	$L^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a} \sin at$
$\frac{s}{s^2+a^2}$	$L^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$
$\frac{1}{s^2-a^2}$	$L^{-1}\left[\frac{1}{s^2-a^2}\right] = \frac{1}{a} \sinh at$
$\frac{s}{s^2-a^2}$	$L^{-1}\left[\frac{s}{s^2-a^2}\right] = \cosh at$

III. APPLICATIONS

1. It is also utilised for a variety of engineering activities, including system modelling, digital signal processing, and electrical circuit analysis.[3]
2. It is used to translate derivatives into several domain variables and then, using the Inverse Laplace transform, to transform the polynomials back into the differential equation.
3. To transfer signals to both sides of the medium, it is utilised in the field of telecommunication. For instance, when signals are transmitted over a phone line, they are first transformed into time-varying wave before being superimposed on the medium.
4. It is used to simplify complicated differential equations by adding polynomials.[1]
5. Electronic engineers frequently employ the Laplace Transform to swiftly solve differential equations that arise during the analysis of electronic circuits.
6. A Laplace Transform is used to obtain the real form of radioactive decay. It makes it feasible to study the analytical portion of nuclear physics with ease.
7. For process controls, we use the Laplace transform. Analysing the variables that, when changed, cause desirable alterations in the outcome is helpful.
8. When many differential equations are utilised in system modelling, the Laplace Transform is used to streamline calculations.[9]
9. For finding the moment generating function in statistics.
10. By making a equation in simple form from hard one like equation of spring.
11. To determine the structure of astronomical object from spectrum.
12. Making the present value of consol or bond in economics.[8]
13. In semi conductor mobility and the call completion in wireless networks.

IV. CONCLUSION

The primary use of Laplace Transform of converting a time domain function into its frequency domain equivalent was also discussed.



It is examined and clarified how Laplace transforms are utilised to resolve issues and equations in fields other than mathematics, such as engineering, physics, and others. A few Laplace Transform applications are discussed and clearly explained. We have spoken about how to solve a variety of challenging and original issues.

The transformation of Laplace is therefore a key method in the study of circuits. Therefore, Laplace plays an important role in the field of engineering.

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