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Fractional Derivatives of the Product of Multivariable H – Function of and General Class of Polynomials

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ABSTRACT: In this paper we first solve a convolution integral equation involving product of the general class of polynomials and the H-function of several variables. Due to general nature of the general class of polynomials and the H-function of several variables which occur as kernels in our main convolution integral equation, we can obtain from it solutions of a large number of convolution integral equations involving products of several useful polynomials and special functions as its special cases. We record here only one such special case which involves the product of general class of polynomials .

We solve two double convolution integral equations. The kernel of the first integral equation involves the product of a polynomial from a general and a generalized polynomial set and that of the other involves the product of two generalized polynomial sets. We also obtain the solutions of two double convolution integral equations involving two types of Gould and Hopper polynomials: generalized Laguerre polynomials and the Bessel polynomials. The single-variable analogue of the second main integral equation, which is quite general in nature and of interest in itself, is also given. Due to the general nature of the kernels involved in integral equations, solutions of a large number of other double and single convolution integral equations involving products of several useful polynomials can also be obtained. A large number of fractional integral formulas involving certain special functions and polynomials have been presented. Here, in this paper, we aim at establishing two fractional integral formulas involving the products of the multivariable H-function and a general class of polynomials by using generalized fractional integration operators given by Saigo and Maeda [M. Saigo, N. Maeda, Varna, Bulgaria, (1996), 386–400]. All the results derived here being of general character, they are seen to yield a number of results (known and new) regarding fractional integrals.

I. INTRODUCTION AND DEFINITIONS

Fractional calculus which are derivatives and integrals of arbitrary (real and complex) orders have found many applications in a variety of fields ranging from natural science to social science. In recent years, it has turned out that many phenomena in engineering, physics, chemistry and other sciences can be described very successfully by means of models using mathematical tools deduced from fractional calculus. For example, the *Corresponding author Email addresses: dinesh_dino03@yahoo.com (Dinesh Kumar), sunil_a_purohit@yahoo.com (S. D. Purohn junesang@mail.dongguk.ac.kr (Junesang Choi) Received 2015-04-15 P. Kumar, S. D. Purohit, J. Choi, J. Nonlinear Sci. Appl. 9 (2016), 8–21 9 nonlinear oscillation of earthquakes can be modeled with fractional derivatives and the fluid-dynamic traffic model with fractional derivatives can eliminate the deficiency arising from the assumption of continuum traffic flow. Fractional derivatives are also used in modeling many chemical processes, mathematical biology and many other problems in physics and engineering (see, e.g., [3]–[1], [4], [5], [6]). Under various fractional calculus operators, the computations of image formulas for special functions of one or more variables are important from the point of view of the usefulness of these results in the evaluation of generalized integrals and the solution of differential and integral equations (see, e.g., [7], [8],[9], [10], [11], [12], [13] and [14] and so on). Motivated essentially by diverse applications of fractional calculus we establish three image formulas for the product of multivariable H-function and general class of polynomials involving left and right sided fractional integral operators of Saigo-Maeda [15]. By virtue of the unified nature of our results, a large number of new and known results involving Saigo, Riemann-Liouville and Erd'elyi-Kober fractional integral operators and several special functions are shown to follow as special cases of our main results. The generalized fractional integral operators of arbitrary order involving Appell function F_3 in the kernel were defined and investigated by Saigo and Maeda [15, p. 393, Eqs. (4.12) and (4.13)] The fractional derivative of special function of one and more variables is important such as in the evaluation of series,[10,15] the derivation of generating function [12,chap.5] and the solution of differential equations [4,14;chap-3]



motivated by these and many other avenues of applications, the fractional differential operators $D_{k,\alpha,x}^n$ and ${}_x D_\alpha^\mu$ are much used in the theory of special function of one and more variables .

We use the fractional derivative operator defined in the following manner [16]

$$D_{k,\alpha,x}^n (x^\mu) = \prod_{r=0}^{n-1} \left[\frac{\sqrt{\mu + rk + 1}}{\sqrt{\mu + rk - \alpha + 1}} \right] x^{\mu+nk} \quad \dots(1.1)$$

Where $\alpha \neq \mu + 1$ and α and k are not necessarily integers

The present work is an attempt in the direction of obtaining fractional calculus formula by utilizing series expression method, introduced by srivastava [9]. The name general class of polynomials, itself indicates the importance of the results, because we can derive a number of fractional calculus formulae for various classical orthogonal polynomials.

In this paper we drive several fractional derivative formulas involving multivariable H-function which as defined by srivastav and panda [8, p.271 (4.1) et. Seq.] and studied systematically by then [20,21,22 also 23] for this multivariable H-function we adopt the contracted notations (due essentially to Srivastava and panda [13] thus following the various conventions and notations explained fairly and fully in their earlier works [6,7,8see also 23,24]

$$H [z_1, \dots, z_r] = H_{p,q; p_1, q_1, \dots, p_r, q_r}^{0,n; m_1, n_1, \dots, m_r, n_r} \left[\begin{matrix} z_1 \left(a_j, \alpha_j^{(1)} \dots \alpha_j^{(r)} \right)_{1,p} : \left(c_j^{(1)}, \gamma_j^{(1)} \right)_{1,p_1} \dots \left(c_j^{(r)}, \gamma_j^{(r)} \right)_{1,p_r} \\ \vdots \\ z_r \left(b_j, \beta_j^{(1)} \dots \beta_j^{(r)} \right)_{1,q} : \left(d_j^{(1)}, \delta_j^{(1)} \right)_{1,q_1} \dots \left(d_j^{(r)}, \delta_j^{(r)} \right)_{1,q_r} \end{matrix} \right] \quad \dots(1.2)$$

Denote the H-function of r-variables z_1, z_2, \dots, z_r here for convenience

$\left(a_j, \alpha_j^1, \dots, \alpha_j^{(r)} \right)_{1,p}$ Abbreviates the p- member array

$$\left(a_1, \alpha_1^1, \dots, \alpha_1^{(r)} \right), \dots, \left(a_p, \alpha_p^1, \dots, \alpha_p^{(r)} \right) \quad \dots(1.3)$$

While $\left(c_j^{(i)}, \gamma_j^{(i)} \right)_{1,p_2}$ Abbreviates the array of p_i pairs of parameters

$$\left(\alpha_j^{(i)}, \gamma_j^{(i)} \right), \dots, \left(\alpha_{p_i}^{(i)}, \gamma_{p_i}^{(i)} \right) \quad ; \quad (i=1, \dots, r) \quad \dots (1.4)$$

and so on ,suppose , asusual that the parameters

$$a_j, \quad j = 1, \dots, p; \quad c_j^{(i)}, \quad j = 1, \dots, p_i;$$

$$b_j, \quad j = 1, \dots, q; \quad d_j^{(i)}, \quad j = 1, \dots, q_i; \quad \forall i \in (i = 1, \dots, r) \quad \dots(1.5) \text{ complex number}$$

and the associated coefficients



$$\alpha_j, \quad j = 1, \dots, p; \quad \gamma_j^{(i)}, \quad j = 1 \dots p_i; \\ \beta_j, \quad j = 1, \dots, q; \quad \delta_j^{(i)}, \quad j = 1 \dots q_i; \quad \forall i \in (1, \dots, r) \quad \dots\dots(1.6)$$

Arepositive real numbers such that

$$\Lambda_i = \sum_{j=1}^r \alpha_j^{(i)} - \sum_{j=1}^q \beta_j^{(i)} + \sum_{j=1}^{p_i} \gamma_j^{(i)} - \sum_{j=1}^{q_i} \delta_j^{(i)} \leq 0 \quad \dots\dots(1.7)$$

and

$$\Omega_i = - \sum_{j=n+1}^p \alpha_j^{(i)} - \sum_{j=1}^q \beta_j^{(i)} + \sum_{j=1}^{n_i} \gamma_j^{(i)} - \sum_{j=n_i+1}^{p_i} \gamma_j^{(i)} + \sum_{j=1}^{m_i} \delta_j^{(i)} - \sum_{j=m_i+1}^{q_i} \delta_j^{(i)} > 0; \quad \forall i \in [1, \dots, r] \dots (1.8)$$

Where the integers $n, p, q, m_i, n_i, p_i, q_i$ are constrained by the inequalities $0 \leq n \leq p, q \geq 0, 1 \leq m_i \leq q_i, 0 \leq n_i \leq p_i$ [$i = 1, \dots, r$] and the equality (1.7) holds true for suitably restricted values of the complex variables z_1, \dots, z_r

Then it is known that the multiple Mellin-Barnes counter integral [11, p.251 (c.1)] representing the multivariable H-function (1.2) converges absolutely under the condition (1.8) when

$$|\arg(z_i)| < \frac{1}{2} \Lambda \Omega_i, \quad \forall i \in [1, \dots, r] \quad \dots\dots(1.9)$$

$$H[z_1, \dots, z_r] = \left| \begin{array}{l} 0(|z_1|^{\xi_1} \dots |z_r|^{\xi_r}), \quad (\max |z_1|, \dots, |z_r| \rightarrow 0) \\ 0(|z_1|^{\eta_1} \dots |z_r|^{\eta_r}), \quad (\eta = 0; \min |z_1|, \dots, |z_r| \rightarrow \infty) \end{array} \right| \quad \dots\dots (1.10)$$

Where with $i=1, \dots, r$

$$\xi_i = \min \left\{ \operatorname{Re} \left(\frac{d_j^{(i)}}{\delta_j^{(i)}} \right) \right\}, \quad (j = 1, \dots, m_i) \\ \eta_i = \max \left\{ \operatorname{Re} \left(\frac{c_j^{(i)} - 1}{\gamma_j^{(i)}} \right) \right\}, \quad (j = 1, \dots, n_i) \quad \dots\dots(1.11)$$

Provided that each of the inequalities(1.7),(1.8) and(1.9) holds true. Throughout the present paper .we assume that the convergence and existence condition corresponding appropriately to the ones detained above are satisfied by each of the various H-function involved in our results which are presented in the following sections

Keywords & pharse : Generalized fractional integral operators, Multivariable H-function, General class of Polynomials, Special-function

Subject Classification AMS 2020 MSC: 26A33, 33A20, 33C45, 33C60, 33C70

MAIN RESULT

RESULT-I

$$\Rightarrow D_{k,ax}^n \{x^u(ax^y + b)^\lambda S_{n_1}^{m_1} \dots S_{n_s}^{m_s} [y_1 x^{u_1}(ax^y + b)^{n_1} \dots y_s x^{u_s}(ax^y + b)^{n_s}]\}$$

$$\begin{aligned}
 H [z_1 x^{\lambda_1} \dots z_r x^{\lambda_r}] = & \\
 a^\lambda b^{-\delta} \sum_{J_1=0}^{N_1/m_1} \dots \sum_{J_s=0}^{N_s/N_s} \sum_{l,m=0}^{\infty} \frac{(-N_1)M_1 J_1}{l J_1} \dots \frac{(-N_s)M_s J_s}{l J_s} A(N_1 J_1 \dots N_s J_s) & \\
 \frac{(x^{v_1}/a)^l (x^{v_2}/b)^m}{l! m!} x^{u+u_1 J_1 + \dots + u_s J_s + nk} H_{P^1+n+2, q^1+n+2, P_1^1, q_1^1, \dots, P_r^1, q_r^1}^{0, n^1+n+2, m_1^1, n_1^1, \dots, m_r^1, n_r^1} & \\
 \left[\begin{aligned} & (-\lambda, \sigma_1, \dots, \sigma_r) (1-\delta-m, \delta_1, \dots, \delta_r) (-u-u_1 J_1 - \dots - u_s J_s - v_1 l - v_2 m - gk, p_1, p_2, \dots, p_r)_{g=0, n-1} \\ & (-\lambda+l, \sigma_1, \dots, \sigma_r) (1-\delta, \delta_1, \dots, \delta_r) (-u-u_1 J_1 - \dots - u_s J_s - v_1 l - v_2 m - gk + \alpha, p_1, p_2, \dots, p_r)_{g=0, n-1} \\ & (a_j; \alpha_j^1, \dots, \alpha_j^{(r)}): (c_j^1 r_j^1)_{1, p_1}, \dots, (c_j^r r_j^r)_{1, p_r} \\ & (b_j; \beta_j^1, \dots, \beta_j^{(r)}): (d_j^1 \delta_j^1)_{1, q_1}, \dots, (d_j^r \delta_j^r)_{1, q_r} \end{aligned} \right] \dots (2.1)
 \end{aligned}$$

Provided (in addition to the appropriate convergence and existence conditions that $\min(u_1, u_2, u_3, u_4, \sigma_i, \delta_i, h_i, g_i) > 0 (i = 1, \dots, r)$

$$\max \left\{ \left| \arg \left(x^{v_1}/a \right) \right|, \left| \arg \left(x^{v_2}/b \right) \right|, \left| \arg \left(y^{v_3}/c \right) \right|, \left| \arg \left(y^{v_4}/d \right) \right| \right\} < \pi$$

$$\operatorname{Re}(k) + \sum_{i=1}^r k_i \xi_i > -1 \operatorname{Re}(\lambda) + \sum_{i=1}^r \lambda_i \xi_i > -1 \text{ And where } \xi_1, \dots, \xi_r \text{ are given in (1.11)}$$

MAIN RESULT -II

$$\begin{aligned}
 D_n^u \{ x^k S_{N_1 N_2}^{M_1 M_2} \{ y_1 x^{u_1} y_2 x^{u_2} \} I_{0,x}^{\alpha, \beta, \eta} \{ t^\lambda H [z_1 x^{\lambda_1} \dots z_r x^{\lambda_r}] H[w_1 x^{\rho_1} \dots w_s x^{\rho_s}] \} & \\
 = x^{k+\lambda-u} \sum_{J_1=0}^{N_1/M_1} \sum_{J_2=0}^{N_2/M_2} \sum_{m=0}^{\infty} \frac{(-N_1)m_1 k_1 (-N_2)m_2 k_2}{l_1 l_2} \binom{u}{m} A[N_1 J_1, N_2 J_2] y_1^{J_1} y_2^{J_2} & \\
 \frac{[\beta - \lambda][\eta - \lambda]}{[-\lambda][\alpha + \beta + \eta - \lambda]} H_{P+1, q+1, p_1, q_1, \dots, p_r, q_r}^{0, n+1, m_1, n_1, \dots, m_r, n_r} & \\
 \left[\begin{aligned} & z_1 x^{\lambda_1} \left(\beta - \lambda, \lambda_1 \dots \lambda_r \right) (a_j \alpha_j^1, \dots, \alpha_j^{(r)})_{1,p} (c_j^1 r_j^1)_{p,1} \dots (c_j^r r_j^r)_{1,p_r} \\ & \vdots \\ & z_r x^{\lambda_r} \left(\beta + u - m - \lambda, \lambda_1, \dots, \lambda_r \right) (b_j \beta_j^1 \dots \beta_j^{(r)})_{1,q} (d_j^1 \delta_j^1)_{1,q_1} \dots (d_j^r \delta_j^r)_{1,q_r} \end{aligned} \right] & \\
 \times H_{P+1, q+1, p_1, q_1, \dots, p_s, q_s}^{0, n+1, m_1, n_1, \dots, m_s, n_s} & \\
 \left[\begin{aligned} & w_1 x^{\rho_1} \left(-k, p_1 \dots p_s \right) (-g_j, G_j^1 \dots G_j^s)_{1,p} (u_j^1, U_j^1)_{1,p} \dots (u_j^s, U_j^s)_{1,p_s} \\ & \vdots \\ & w_s x^{\rho_s} \left(m - k, P_1 \dots P_s \right) (h_j, H_j^1 \dots H_j^s)_{1,q} (v_j^1 V_j^1)_{1,q_1} \dots (v_j^s V_j^s)_{1,q_s} \end{aligned} \right] \dots (2.2)
 \end{aligned}$$

Provided (in addition to the appropriate convergence and existence conditions that $\min(u_1, u_2, u_3, u_4, \sigma_i, \delta_i, h_i, g_i) > 0 (i = 1, \dots, r)$

$$\max \left\{ \left| \arg \left(x^{\nu_1} / a \right) \right|, \left| \arg \left(x^{\nu_2} / b \right) \right|, \left| \arg \left(y^{\nu_3} / c \right) \right|, \left| \arg \left(y^{\nu_4} / d \right) \right| \right\} < \pi$$

$$\operatorname{Re}(k) + \sum_{i=1}^r k_i \xi_i > -1 \quad \operatorname{Re}(\lambda) + \sum_{i=1}^r \lambda_i \xi_i > -1 \quad \text{And where } \xi_1, \dots, \xi_r \text{ are given in (1.11)}$$

MAIN RESULT -III

$$\begin{aligned} & D_x^u \{x^k S_N^M \{yx^u\}\} \int_a^b (t-a)^{\alpha-1} (b-t)^{\beta-1} I_{0,x}^{\alpha,\beta,\eta} \{t^{\nu_1} H[z_1(g(t))^{\nu_1} \dots z_r(g(t))^{\nu_r}]\} dt \\ &= (b-a)^{\alpha+\beta-1} \sum_{l,m=0}^{\infty} \sum_{j=0}^{N/M} \frac{\left(\frac{B-A}{B}\right)^m \left(\frac{b-a}{au+v}\right)^m \left(\frac{u(b-a)}{au+v}\right)^l}{l! m!} A[N, J] (yx^u)^j x^{k+\lambda-\beta-u} \\ & \frac{[1+\lambda][1+\lambda-\beta+\eta]}{[1+\lambda-\beta][1+\lambda+\alpha+\eta]} \frac{[1+k+\lambda-\beta+u]}{[1+k+\lambda-\beta+uj-u]} H_{P+4}^{0, n+4, m_1, n_1, \dots, m_r, n_r}{}_{q+3, p_1, \dots, p_r, q_r} \\ & \left[\begin{array}{l} z_1 B^{-\nu_1} (b-a)^{(\eta+\delta)\nu_1} (au+v)^{-(\delta+\eta)\nu_1} \\ \vdots \\ z_r B^{-\nu_r} (b-a)^{(\eta+\delta)\nu_r} (au+v)^{-(\delta+\eta)\nu_r} \end{array} \right]_{(1, \nu_1, \dots, \nu_r) (1-l, (\delta+\eta)\nu_1, \dots, (\delta+\eta)\nu_r) (1-\beta, n\nu_1, \dots, n\nu_r)} \\ & (1-\alpha-l-m, \delta\nu_1, \dots, \delta\nu_r) (a_j, \alpha_j^1, \dots, \alpha_j^{(r)})_{1,p} (c_j^1 + \gamma_j^1)_{1,p_1} \dots (c_j^r + \gamma_j^r)_{1,p_r} \\ & \vdots \\ & (1, (\delta+\eta)\nu_1, \dots, (\delta+\eta)\nu_r) (b_j, \beta_j^1, \dots, \beta_j^{(r)})_{1,q} (d_j^i \delta_j^i)_{1,q_1} \dots (d_j^r \delta_j^r)_{1,q_r} \end{array} \right] \dots (2.3) \end{aligned}$$

Provided (in addition to the appropriate convergence and existence conditions that

$$\min(\nu_1, \nu_2, \nu_3, \nu_4, \sigma_i, \delta_i, h_i, g_i) > 0 \quad (i=1, \dots, r)$$

$$\max \left\{ \left| \arg \left(x^{\nu_1} / a \right) \right|, \left| \arg \left(x^{\nu_2} / b \right) \right|, \left| \arg \left(y^{\nu_3} / c \right) \right|, \left| \arg \left(y^{\nu_4} / d \right) \right| \right\} < \pi$$

$$\operatorname{Re}(k) + \sum_{i=1}^r k_i \xi_i > -1 \quad \operatorname{Re}(\lambda) + \sum_{i=1}^r \lambda_i \xi_i > -1 \quad \text{And where } \xi_1, \dots, \xi_r \text{ are given in (1.11)}$$

Proof:-Result -I First using The Fox's H-Function of multivariable. then we shall utilize following definition introduced by srivastava [18] or general class of polynomials for multivariables Now using by mellin barnes type contour integral for H- function for multivariable

We use the fractional derivative operator $D_{k,\alpha,x}^n(x^\mu)$ and after simplification we get required result .

Proof:-Result -2 First using The Fox's H-Function of multivariable. then apply Fractional integral formula

$$I_{0,x}^{\alpha,\beta,\eta} (t^\lambda) = \frac{[(1+\lambda)(1+\lambda-\beta+\eta)]}{[(1+\lambda-\beta)(1+\lambda+\alpha+\eta)]} x^{\lambda-\beta} \quad \text{provided that } \operatorname{Re}(\alpha) > \max [0, \operatorname{Re}(\beta-\eta)] - 1,$$

we shall utilize following definition introduced by srivastava [18] or general class of polynomials for two variables

$$\{ S_{N_1 N_2}^{M_1 M_2} \{y_1 x^{u_1} y_2 x^{u_2}\} =$$

$$\left\{ \sum_{J_1=0}^{N_1/M_1} \sum_{J_2=0}^{N_2/M_2} \frac{(-N_1)_{m_1 k_1}}{U_1} \frac{(-N_2)_{m_2 k_2}}{U_2} A[N_1 J_1, N_2 J_2] y_1^{J_1} x^{u_1 J_1} y_2^{J_2} x^{u_2 J_2} \right.$$

Where m is an arbitrary positive integer and the coefficient ($A_{l,j} > 0$)

are arbitrary constant real or complex

Now using by mellin barnes type contour integral for H- function for multivariable

We use the fractional derivative operator $D_x^u(x^\mu)$ and after simplification we get required result .

Proof:-Result -3 First using The Fox's H-Function of multivariable. then apply Fractional integral formula

$$I_{0,x}^{\alpha,\beta,\eta}(t^\lambda) = \frac{\Gamma(1+\lambda)\Gamma(1+\lambda-\beta+\eta)}{\Gamma(1+\lambda-\beta)\Gamma(1+\lambda+\alpha+\eta)} x^{1-\beta} \quad \text{provided that } \text{Re}(\alpha) > \max [0, \text{Re}(\beta-\eta)] - 1,$$

we shall utilize Beta function then we shall the following definition introduced by srivastava [18] or general class of polynomials for one variables then using by mellin Barnes type contour integral for H- function for multivariable and then

We use the fractional derivative operator $D_x^u(x^\mu)$ and after simplification we get required result .

II. CONCLUSION

In this paper we get fractional differential operator formulae involving special function and general class of polynomials. Here we presented two very generalized and unified theorems associated with the generalized fractional integral operators given by Saigo-Maeda. The main fractional integrals whose integrands being the products of multivariable H-functions and a general class of polynomials. The main results may find potentially useful applications in a variety of areas. The main fractional integrals whose integrands being the products of multivariable H-functions and a general class of polynomials, result can be specialized to yield a large number of simpler results. The main results may find potentially useful applications in a variety of areas. In this paper we get fractional differential operator formulae involving special function and general class of polynomials.

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