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Mathematical modelling and simulation quarter car vehicle suspension

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Abstract—This paper is mainly discussing about the mathematical modelling and simulation study of two degree of freedom quarter car model. The state space mathematical model is derived using Newton's second law of motion and free body diagram concept and the vehicle body along with the wheel system is modelled as a two degree of freedom quarter car model. The performance of the system will be determined by computer simulation using MATLAB/SIMULINK. Passive, semi-active and active suspension systems connected in a single loop and tested under step and single bump input.

Keywords— quarter car, state space equation, two degree of freedom, passive suspension, active suspension

I. INTRODUCTION

The fundamental purpose of ground vehicle suspension system is to maintain continuous contact between the wheels and road surface, and to isolate passengers or cargo from the vibration induced by the road irregularities. These two purposes are responsible for the handling quality and ride comfort, respectively. However, these goals are generally contradictory. It is impossible for passive suspensions to achieve simultaneously a best performance of ride comfort and handling quality under all driving conditions [1]. The vehicle suspension system are categorised into following namely passive, semi active and fully active suspension system. Figure 1.a. shows Conventional suspension systems consist of spring and damper. These parameters spring constant and damping constant are fixed from the design stage itself, so it cannot control. Passive suspension systems with no controllable standard characteristics are the most Widespread on producing vehicles. Firstly, it is caused by an enough simple design, concerning high reliability, absence of necessity of power supply [2]. The problem of passive suspension is if it designs heavily damped or too hard suspension it will transfer a lot of road input or throwing the car on unevenness of the road [3].Many analytical and experimental studies on active and semi-active suspensions have been performed to improve ride quality and handling performance. The results of studies show that active and semi-active suspensions can provide substantial performance improvements over passive suspensions in general [4]. The Electronically controlled active suspension systems can potentially improve the ride comfort as well as the road handling of the vehicle simultaneously [5]. The ride comfort is improved by means of the reduction of the car body acceleration caused by the irregular road disturbances from smooth road [6]. Therefore the active suspension systems are superior in performance than passive and semi-active suspension.

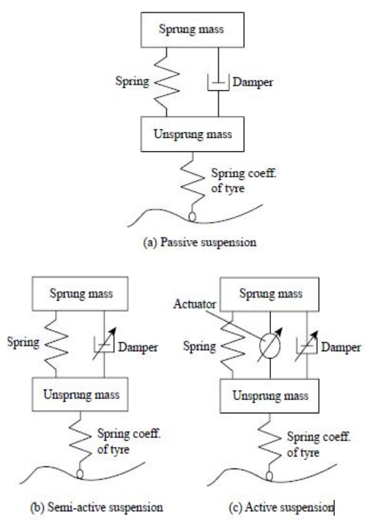

Figure 1: Quarter vehicle suspension models

II. MODELLING OF QUARTER CAR

The vehicle model considered in this study is quarter car model. The quarter car model suspension system consists of one-fourth of the body mass, suspension components and one wheel [7] as shown in Figure 1.The quarter car model for passive suspension system is shown in Figure 1(a).

The assumptions of a quarter car modelling are as follows: the tire is modelled as a linear spring without damping, there is no rotational motion in wheel and body, the behaviour of spring and damper are linear, the tire is always in contact with the road surface and effect of friction is neglected so that the residual structural damping is not considered into vehicle modelling [7]. The equations of motion for the sprung and unsprung masses of the passive quarter car model are given by.

$$M_s\ddot{Z}_s + C_s(\dot{Z}_s - \dot{Z}_{us}) + K_s(Z_s - Z_{us}) = 0 \quad (1)$$

$$M_{us}\ddot{Z}_{us} + C_s(\dot{Z}_{us} - \dot{Z}_s) + K_s(Z_{us} - Z_s) + K_{us}(Z_{us} - Z_r) = 0 \quad (2)$$

$$\ddot{Z}_s = \frac{1}{M_s}[C_s(\dot{Z}_{us} - \dot{Z}_s) + K_s(Z_{us} - Z_s)] \quad (3)$$

$$\ddot{Z}_{us} = \frac{1}{M_{us}}[C_s(\dot{Z}_s - \dot{Z}_{us}) + K_s(Z_s - Z_{us}) + K_{us}(Z_r - Z_{us})] \quad (4)$$

Let as assume the state variables are

$$Z_1 = Z_s - Z_w$$
$$Z_2 = \dot{Z}_s$$
$$Z_3 = Z_w - Z_r$$
$$Z_4 = \dot{Z}_w$$
$$\dot{Z}_1 = \dot{Z}_s - \dot{Z}_w \approx Z_s - Z_w$$
$$\dot{Z}_2 = \ddot{Z}_s$$
$$\dot{Z}_3 = \dot{Z}_w - \dot{Z}_r \approx Z_4 - \dot{Z}_r$$
$$\dot{Z}_4 = \ddot{Z}_w$$

General form of state space equation

$$\dot{Z} = AZ + BF_a + \dot{Z}_r \quad (5)$$

$$\begin{bmatrix}\dot{Z}_1\\ \dot{Z}_2\\ \dot{Z}_3\\ \dot{Z}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & -1\\ -K_s/M_{us} & -C_s/M_{us} & 0 & C_s/M_{us}\\ 0 & 0 & 0 & 1\\ -K_s/M_s & C_s/M_s & -K_{us}/M_s & -C_s/M_s\end{bmatrix}\begin{bmatrix}Z_1\\ Z_2\\ Z_3\\ Z_4\end{bmatrix} + \begin{bmatrix}0\\ 0\\ -1\\ 0\end{bmatrix}\dot{Z}_r \quad (6)$$

The equations of motion for the sprung and unsprung masses of the semi active and active suspension quarter car models are identical and given by equation (7) and (8). In case of semi active variable damper will provide a desire force. Other hand the Active suspension system requires an actuatorforce to provide a better ride and handling thanthe passive suspension system. The actuator force, F_a is an additional input to the suspensionsystem model. The model in Simulink was built based on the equations and the actuator and variable damper force iscontrolled by the PID which involves a feedback loop [8].

$$M_s\ddot{Z}_s + C_s(\dot{Z}_s - \dot{Z}_{us}) + K_s(Z_s - Z_{us}) + F_a = 0 \quad (7)$$

$$M_{us}\ddot{Z}_{us} + C_s(\dot{Z}_{us} - \dot{Z}_s) + K_s(Z_{us} - Z_s) + K_{us}(Z_{us} - Z_r) - F_a = 0 \quad (8)$$

$$\ddot{Z}_s = \frac{1}{M_s}[C_s(\dot{Z}_{us} - \dot{Z}_s) + K_s(Z_{us} - Z_s) - F_a] = 0 \quad (9)$$

$$\ddot{Z}_{us} = \frac{1}{M_s}[C_s(\dot{Z}_s - \dot{Z}_{us}) + K_s(Z_s - Z_{us}) + K_{us}(Z_r - Z_{us}) + F_a] = 0 \quad (10)$$

$$\begin{bmatrix}\dot{Z}_1\\ \dot{Z}_2\\ \dot{Z}_3\\ \dot{Z}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & -1\\ -K_s/M_{us} & -C_s/M_{us} & 0 & C_s/M_{us}\\ 0 & 0 & 0 & 1\\ -K_s/M_s & C_s/M_s & -K_{us}/M_s & -C_s/M_s\end{bmatrix}\begin{bmatrix}Z_1\\ Z_2\\ Z_3\\ Z_4\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{M_{us}}\\ 0\\ \frac{1}{M_s}\end{bmatrix}F_a + \begin{bmatrix}0\\ 0\\ -1\\ 0\end{bmatrix}\dot{Z}_r \quad (11)$$

This equation (11) is the state space equation for active and semi active suspension.

III. SIMULATION

The following parameters are used in the simulation. Body mass (M_s) = 290 kg, Suspension mass (M_{us}) = 60 kg, spring constant of Suspension system (K_s) = 16200 N/m, spring constant of wheel and tire (K_{us}) = 191000 N/m, damping constant of suspension system (C_s) = 1000 Ns/m, control force = F_a, Z_s, Z_{us} is sprung mass and unsprung mass displacement, $\dot{Z}s, \dot{Z}us, \ddot{Z}s, \ddot{Z}us$ is sprung mass and unsprung mass velocity and acceleration. The figure-2 shows that passive suspension system simulation block diagram [9].

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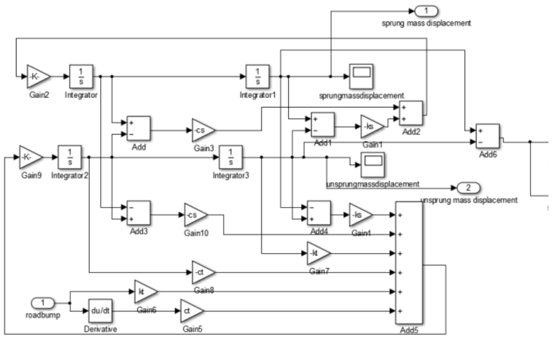

Figure-2 passive suspension simulation block diagram

IV. ROAD BUMP

A double bump road input, Zr as described by [10], which is used to simulate the road to verify the developed control system. The road input described by the following Equation (12).

$$Z_r = \begin{cases} a(1-\cos\omega t) & 1.25 \le t \le 1.5 \\ -a(1-\cos\omega t) & 4.25 \le t \le 4.5 \\ 0 & otherwise \end{cases} \qquad (12)$$

V. RESULT AND DISCUSSION

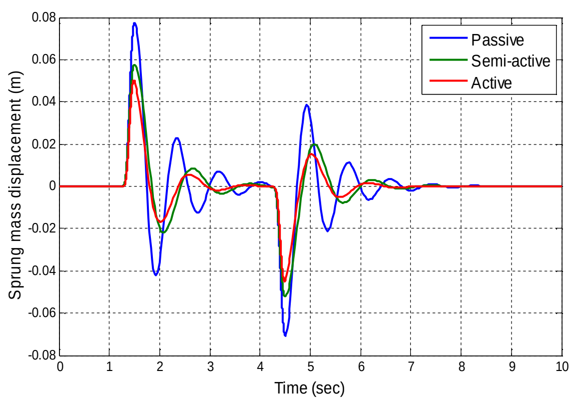

Figure-3 Sprung mass displacement

The figure-3 shows that sprung mass displacement with respect to the time trend of passive, semi-active and active suspension system. The peak point of passive is 0.08m and active suspension is 0.05m. and the root mean square value of active is 34% is lower than passive suspension system. The figure-4 shows that unsprung mass displacement with respect to time.

The figure-5 shows that suspension deflection or travel graph. This subtraction sprung and unsprung mass displacement. Also known as rattle space. Active suspension deflection 30% is lower than passive suspension

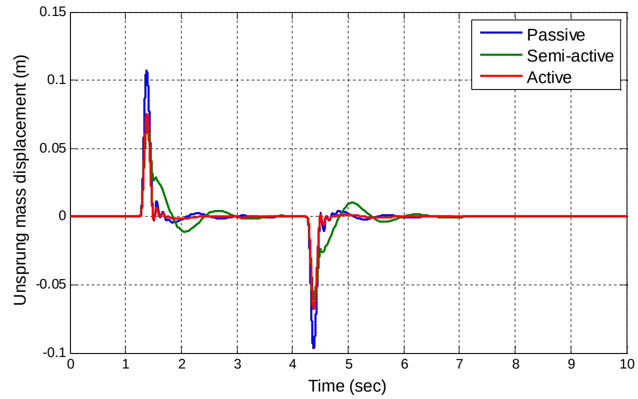

Figure-4 Unsprung mass displacement

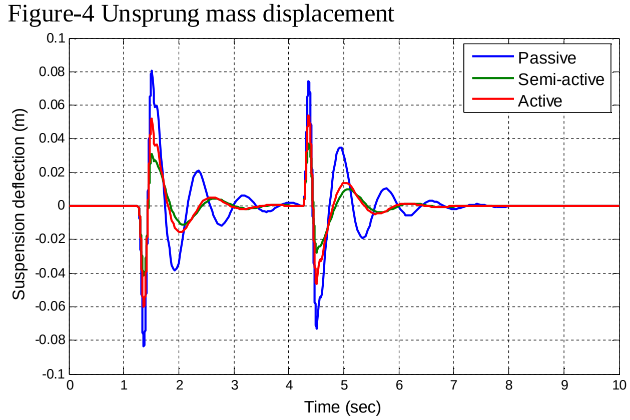

Figure-5 suspension deflection

TABLE I
RMS VALUES AND COMPARISON

parameters	**Passive**	**Semi-active**	**Active**	**% of reduction**
Sprung mass displacement (m)	0.0430	0.0335	0.0283	34.0
Sprung mass displacement (m)	0.0573	0.0415	0.0407	28.94
Suspension deflection (m)	0.0463	0.0227	0.0324	30.13

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VI. CONCLUSIONS

The passive, semi-active and active suspension system is simulated under Matlab/Simulink environment. From the result of simulation the active suspension is reducing peak amplitude and settling time than semi-active and passive suspension. Also the performance of active suspension is superior to semi-active and which superior to passive suspension.

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