

An Unsteady Free Convection Slip Flow along a Vertical Plate Embedded in a Porous Medium with Effects of Hall Current and Thermal Radiation

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ABSTRACT: An unsteady free convection slip flow with Heat and Mass transfer effect of viscous incompressible fluid through a porous medium in a rotating system with Hall Effect, thermal radiation and chemical reaction have been studied. The plate moves with constant velocity u_0 in its own plane. A perturbation technique is used to obtain the expressions for velocity, temperature and concentration. Expressions for Skin friction, Nusselt number and Sherwood number are also derived.

KEYWORDS: Hall current, Thermal radiation, porous medium, chemical reaction

I. INTRODUCTION

Magneto hydrodynamic (MHD) free convection flow through porous media are very important particularly in the fields of petroleum technology for the flow of oil through porous rocks, in chemical engineering for the purification and filtration processes and in the cases like drug permeation through human skin. Free convection arises in the fluid when temperature and concentration changes cause density variation leading to buoyancy forces acting on the fluid elements. The mass transfer differences affect the rate of heat transfer. Recently, it is of great interest to study the effect of magnetic field on the temperature distribution and heat transfer when the fluid is not only an electrical conductor but also when it is capable of emitting and absorbing thermal radiation. The radiative free convective flows are encountered in countless industrial and environment processes e.g. heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle reentry. The radiative heat transfer plays an important role in manufacturing industries for the design of reliable equipment. This chapter deals with the effects of hall current, thermal radiation, chemical reaction, soret number etc., on an unsteady, rotating, free convection slip flow past an infinite vertical porous plate, embedded in a porous medium, with constant heat and mass flux at the plate. The analytical expressions for the velocity, temperature and concentration are obtained using perturbation technique. The effects on velocity, temperature and concentration are discussed through graphs.

II. RELATED WORK

Sivaiah et al. [1] and Das and Jana [2] have studied heat and mass transfer effects on MHD free convection flow past a vertical plate with different boundary conditions. Chaudhary and Jain [3] have also studied combined heat and mass transfer effects on MHD free convection flow past an oscillating plate. Aboeldahab et al. [4] and Shit [5] studied the effects of Hall currents on MHD free convection flow with mass transfer. Acharya et al. [6] studied the magnetic effects on the free convection and mass transfer flow through porous medium. The no slip boundary condition is valid only when particles close to the surface do not move with the flow; however this is only true macroscopically. Effect of Hall currents, with slip conditions at the boundary have been studied by Abbas and Asghar [7] and Jain and Gupta [8]. Jain and Singh [9], have observed Hall and thermal radiative effects on an unsteady rotating free convection slip flow. Anjalidevi and Kandasamy [11] investigated the effect of chemical reaction on the flow in the presence of heat transfer

and magnetic field. Chemical reaction effects on heat and mass transfer in laminar boundary layer flow have been studied by several scholars e.g. Kandasamy et al. [12], Afify [10], Takhar et al. [14] and Mansour et al. [13] etc.

III. MATHEMATICAL FORMULATION

We consider an unsteady free convective flow of an incompressible viscous fluid through a porous medium past an infinite vertical porous plate, with a constant heat and mass flux at $z^* = 0$. Both the fluid and the plate are assumed to be in a state of rigid rotation with uniform angular velocity Ω along z^* axis taken normal to the plate. A uniform magnetic field H_0 is applied along the axis of rotation. The plate is considered to be electrically non-conducting and the permeability of the porous medium is assumed to be constant. The equation of continuity $\nabla \cdot q = 0$ gives on integration, $W^* = -W_0(1 + \epsilon e^{-nt})$, the negative sign indicates the suction velocity is towards the plate where $q = (u, v, w)$. Thus for the present configuration, the governing equations are:

Continuity Equation:

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{2.1}$$

Momentum Equations:

$$\begin{aligned} \frac{\partial u^*}{\partial t^*} - W_0(1 + \epsilon e^{-nt}) \frac{\partial u^*}{\partial z^*} = & -\frac{1}{\rho} \frac{\partial p^*}{\partial x^*} + \nu \frac{\partial^2 u^*}{\partial z^{*2}} + 2\Omega v^* \\ & + \frac{\sigma H_0^2 (mv^* - u^*)}{\rho(1+m^2)} + g\beta(T^* - T_\infty^*) + g\beta^*(C^* - C_\infty^*) - \frac{\nu u^*}{k} \end{aligned} \tag{2.2}$$

$$\frac{\partial v^*}{\partial t^*} - W_0(1 + \epsilon e^{-nt}) \frac{\partial v^*}{\partial z^*} = \nu \frac{\partial^2 v^*}{\partial z^{*2}} - 2\Omega u^* - \frac{\nu v^*}{k} - \frac{\mu_e H_0}{\rho} J_x \tag{2.3}$$

Energy Equation:

$$\frac{\partial T^*}{\partial t^*} - W_0(1 + \epsilon e^{-nt}) \frac{\partial T^*}{\partial z^*} = \frac{k}{\rho C_p} \frac{\partial^2 T^*}{\partial z^{*2}} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial z^*} + \frac{Q_0}{\rho C_p} \frac{\partial T^*}{\partial z^*} \tag{2.4}$$

Concentration Equation:

$$\frac{\partial C^*}{\partial t^*} - W_0(1 + \epsilon e^{-nt}) \frac{\partial C^*}{\partial z^*} = D \frac{\partial^2 C^*}{\partial z^{*2}} - kr^*(C^* - C_\infty^*) + Dl \frac{\partial^2 T^*}{\partial z^{*2}} \tag{2.5}$$

Where $J_x = \frac{\sigma \mu_e H_0}{1+m^2} (v + mu)$, $J_y = \frac{\sigma \mu_e H_0}{1+m^2} (-u + mv)$

The local radiant is expressed by:

$$\frac{\partial q_r}{\partial z^*} = -4a^* \sigma^* (T_\infty^4 - T^4) \tag{2.6}$$

It is assumed that the temperature differences within the flow are sufficiently small such that

$$T^4 \text{ in a Taylor series about } T_\infty \text{ and neglecting higher order, thus } T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{2.7}$$

By using (2.6) and (2.7), we obtain

$$\frac{\partial q_r}{\partial z^*} = -16a^* \sigma^* T_\infty^3 (T_\infty - T) \tag{2.8}$$

Where σ^* is the Stephen-Boltzmann constant and a^* is absorption coefficient, England and Emery[15].

The corresponding boundary conditions of the problem for velocity, temperature and concentration fields are

$$\text{At } z^* = 0, \quad u^* = U_0 + L \frac{\partial u^*}{\partial z^*}, v^* = L \frac{\partial v^*}{\partial z^*}, \frac{\partial T^*}{\partial z^*} = -\frac{q}{k}, \frac{\partial C^*}{\partial z^*} = -\frac{m_0}{D}.$$

$$\text{As } z^* \rightarrow \infty, \quad u^* \rightarrow \infty, v^* \rightarrow 0, T^* \rightarrow T_\infty^*, C^* \rightarrow C_\infty^*$$

Where q and m_0 are uniform heat and concentration flux at the plate respectively, $m = W_e \tau_e$ is the hall parameter, W_e is the cyclotron frequency and τ_e is the collision time of electrons, σ is the fluid conductivity, μ_e is the magnetic permeability, u and v are the velocities in x and y directions respectively. T is the temperature of the fluid, T_∞ is the temperature of the fluid far away from the plate, β is the coefficient of thermal expansion, k is the thermal conductivity, C_p is the specific heat at constant pressure, K is the permeability of the medium, ρ is the density, ν is the kinematic viscosity, g is the acceleration due to gravity, β^* is the coefficient of expansion due to concentration, μ is the coefficient of viscosity, D is molecular diffusivity, D_l is thermal diffusivity.

On introducing the following non-dimensional quantities

$$u = \frac{u^*}{U_0}, v = \frac{v^*}{U_0}, z = \frac{w_0 z^*}{\nu}, n = U_\infty = \frac{U_\infty^*}{U_0}, t = \frac{t^* w_0^2}{\nu}, n = \frac{n^* \nu}{w_0^2}, k = \frac{k^* \nu^2}{\nu^2}$$

$$kr = \frac{kr^* \nu}{w_0^2}, k = \frac{k^* w_0^2}{\nu^2}, \theta = \frac{(T^* - T_\infty^*) k w_0}{\nu q}, C = \frac{(C^* - C_\infty^*) D w_0}{\nu m_0}, \quad (2.8)$$

in equations(2.2) - (2.5) yields

$$\frac{\partial u}{\partial t} - (1 + \varepsilon e^{-nt}) \frac{\partial u}{\partial z} = \frac{dU_\infty}{dt} + \frac{\partial^2 u}{\partial z^2} + 2\Omega v + Gr\theta + GcC$$

$$+ \frac{M^2 (mv - u - U)}{1 + m^2} - N(u - U_\infty) + 2Ev \quad (2.9)$$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon e^{-nt}) \frac{\partial \theta}{\partial z} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial z^2} - \frac{R}{Pr} \theta + H \frac{\partial \theta}{\partial z} \quad (2.10)$$

$$\frac{\partial C}{\partial t} - (1 + \varepsilon e^{-nt}) \frac{\partial C}{\partial z} = \frac{1}{Sc} \frac{\partial^2 C}{\partial z^2} + \frac{S_0}{Sc} \frac{\partial^2 \theta}{\partial z^2} - KrC \quad (2.11)$$

The boundary conditions are then given by the following dimensionless form

$$z = 0: \quad u = 1 + h \frac{\partial u}{\partial z}, v = h \frac{\partial v}{\partial z}, \frac{\partial \theta}{\partial z} = -1, \frac{\partial C}{\partial z} = -1$$

$$z \rightarrow \infty: \quad u \rightarrow 0, v \rightarrow 0, \theta \rightarrow 0, C \rightarrow 0$$

IV. SOLUTION OF THE PROBLEM

In order to reduce the above system of partial differential equations to a system of ordinary differential equations in dimensionless form we may represent the velocity, temperature and concentration as

$$u = u_0(z) + \varepsilon e^{-nt} u_1(z) + o(\varepsilon^2) \quad (3.1)$$

$$\theta = \theta_0(z) + \varepsilon e^{-nt} \theta_1(z) + o(\varepsilon^2) \quad (3.2)$$

$$C = C_0(z) + \varepsilon e^{-nt} C_1(z) + o(\varepsilon^2) \quad (3.3)$$

Substituting the equations (3.1) – (3.3) into the equations (2.9) – (2.11) and equating the harmonic and non harmonic terms, neglecting the coefficient of $O(\varepsilon^2)$, we get the following equations

$$u_0'' + u_0' - (N + \frac{M^2}{1+m^2})u_0 = NU - Gr\theta_0 - GcC_0 - 2EV_0 - \frac{M^2 \nu_0}{1+m^2} + \frac{M^2 U}{1+m^2} \quad (3.4)$$

$$u_1'' + u_1' - (N - n + \frac{M^2}{1+m^2})u_1 = -u_0' - Gr\theta_1 - GcC_1 - 2EV_1 - \frac{M^2mv_1}{1+m^2} \tag{3.5}$$

$$\theta_0'' + Pr(1+H)\theta_0' - R\theta_0 = 0 \tag{3.6}$$

$$\theta_1'' + Pr(1+H)\theta_1' - (nPr - R)\theta_1 = -Pr\theta_0' \tag{3.7}$$

$$C_0'' + ScC_0' - ScKrC_0 = -S_0\theta_0'' \tag{3.8}$$

$$C_1'' + ScC_1' + Sc(n - Kr)C_1 = -ScC_0' - S_0\theta_1'' \tag{3.9}$$

The corresponding boundary conditions can be written as

$$\begin{aligned} z = 0: \quad & u_0 = 1 + hu_0', \theta_0' = -1, \theta_1 = hu_1', \theta_1' = 0, C_1' = 0, \\ z \rightarrow \infty: \quad & u_0 \rightarrow 0, u_1 \rightarrow 0, \theta_0 \rightarrow 0, \theta_1 \rightarrow 0, C_0 \rightarrow 0, C_1 \rightarrow 0, \end{aligned} \tag{3.10}$$

From the equations (3.1) – (3.3), we obtain for velocity, temperature and concentration, as follows

$$\begin{aligned} u(y,t) = & A9e^{-r9z} + C8e^{-r5z} + (C7 + C9)e^{-r1z} \\ & + \epsilon e^{-nt} \{ A11e^{-r11z} - C17e^{-r12z} + C10e^{-r3z} + C11e^{-r1z} + C12e^{-r7z} \\ & + C13e^{-r5z} + C14e^{-r3z} + C15e^{-r1z} + C16e^{-r9z} + C17e^{-r10z} \\ & + C18e^{-r1z} + C19e^{-r5z} \} \end{aligned} \tag{3.11}$$

$$\theta(y,t) = A1e^{-r1z} + \epsilon e^{-nt} \{ A3e^{-r3y} + C1e^{-r1z} \} \tag{3.12}$$

$$\begin{aligned} C(y,t) = & A5e^{-r5z} + C2e^{-r1z} \\ & + \epsilon e^{-nt} \{ A7e^{-r7z} + A8e^{-r8z} + C3e^{-r5z} + C4e^{-r1z} + C5e^{-r3z} + C6e^{-r1z} \} \end{aligned} \tag{3.13}$$

SKIN FRICTION: Knowing the velocity field, the skin friction at the plate can be obtained, which in non-dimensional form is given by

$$\begin{aligned} C_f = \left(\frac{\partial u}{\partial z} \right)_{z=0} = & [-r9A9 - r5C8 - r1(C7 + C9)] \\ & + \epsilon e^{-nt} \{ -r11A11 + r12 - r3C10 - r1C11 - r7C12 - r5C13 \\ & - r3C14 - r1C15 - r9C16 - r10C17 - r1C18 - r5C19 \} \end{aligned} \tag{3.14}$$

NUSSELT NUMBER: Knowing the temperature field, the rate of heat transfer coefficient can be obtained, which in non-dimensional form is given in terms of the Nusselt number, is given by

$$Nu = \frac{1}{\theta(0,t)} = \frac{1}{A1 + \epsilon e^{-nt} (A3 + C1)} \tag{3.15}$$

SHERWOOD NUMBER: Knowing the concentration field, the rate of mass transfer coefficient can be obtained, which in non-dimensional form is given in terms of the Sherwood number, is given by

$$Sh = \frac{1}{C(0,t)} = \frac{1}{(A5 + C2) + \epsilon e^{-nt} (A7 + C3 + C5 + C4 + C6)} \tag{3.16}$$

V. RESULTS AND CONCLUSION

Increasing values of Sc, Pr, Kr results in decrease in velocity. Figure 6, velocity (u) profiles are plotted against z. We fix t = 1, Kr=0.2, Pr=0.71 and Sc = 0.61. It is observed that on increasing K, Gr, Gc and So, velocity increases. Temperature profiles are plotted against z in Figure 4 by fixing t = 1. From the figure we observe that as we increase the radiation our temperature drops as heat is being radiated while the effect of absorption is obvious, the temperature rises. Also, increase in H, which is source parameter, decreases the temperature. In Figure 5, effects of different parameters are studied on concentration (C), profiles are plotted against z fixing t = 1. We observe that on increasing the Schmidt number (Sc) our concentration drops, while concentration rises on increasing the Soret number (So) and H. It's interesting to note that when we increase R concentration decreases.

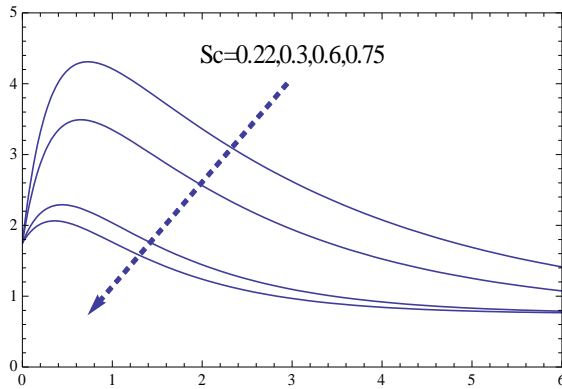


Figure 1: Velocity for different values of Sc

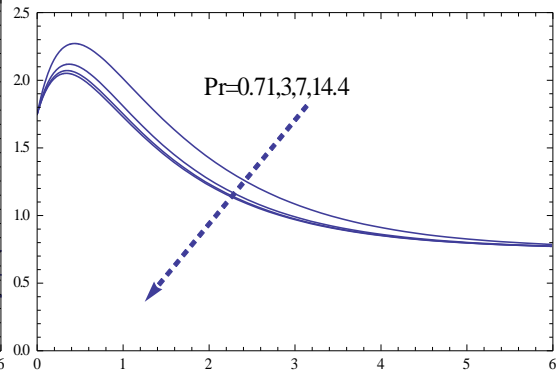


Figure 2: Velocity for different values of Pr

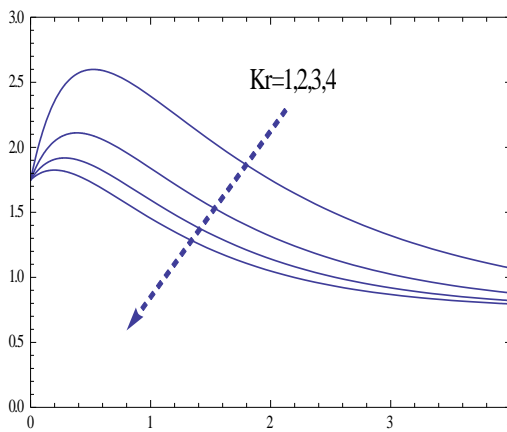


Figure 3: Velocity for different values of Kr

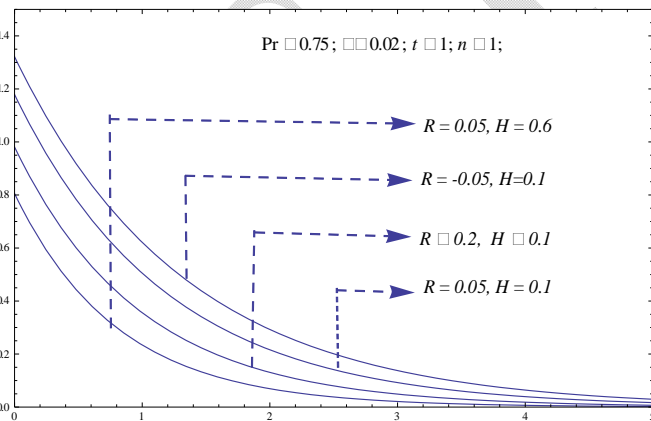


Figure 4: Temperature profiles plotted against z for different values of R and H

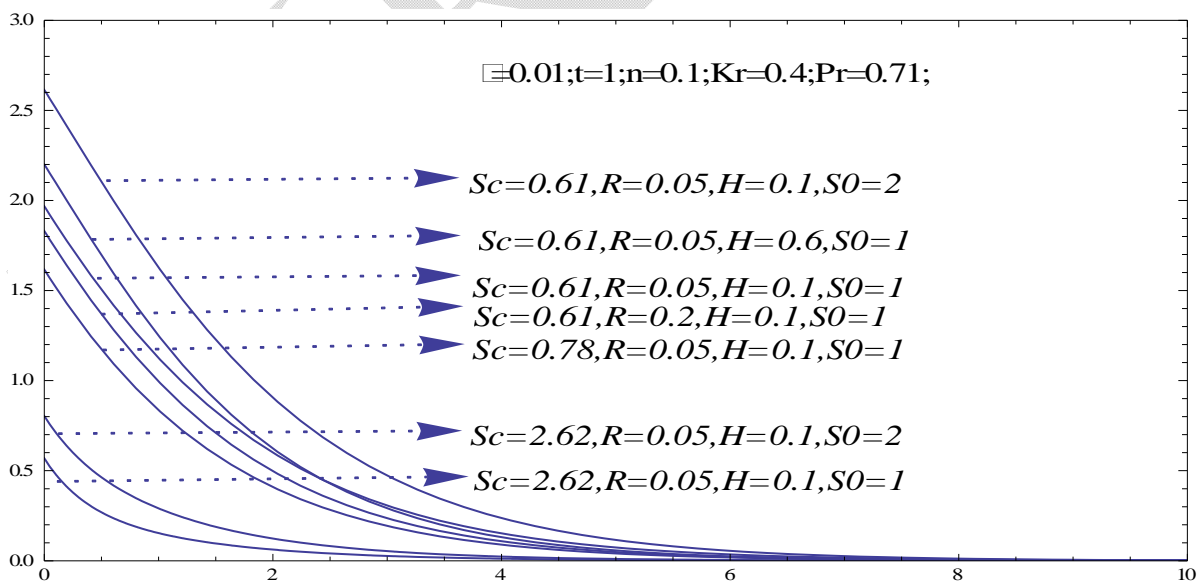


Figure 5: Concentration profiles plotted against z for different values of Sc, R, H, and S0

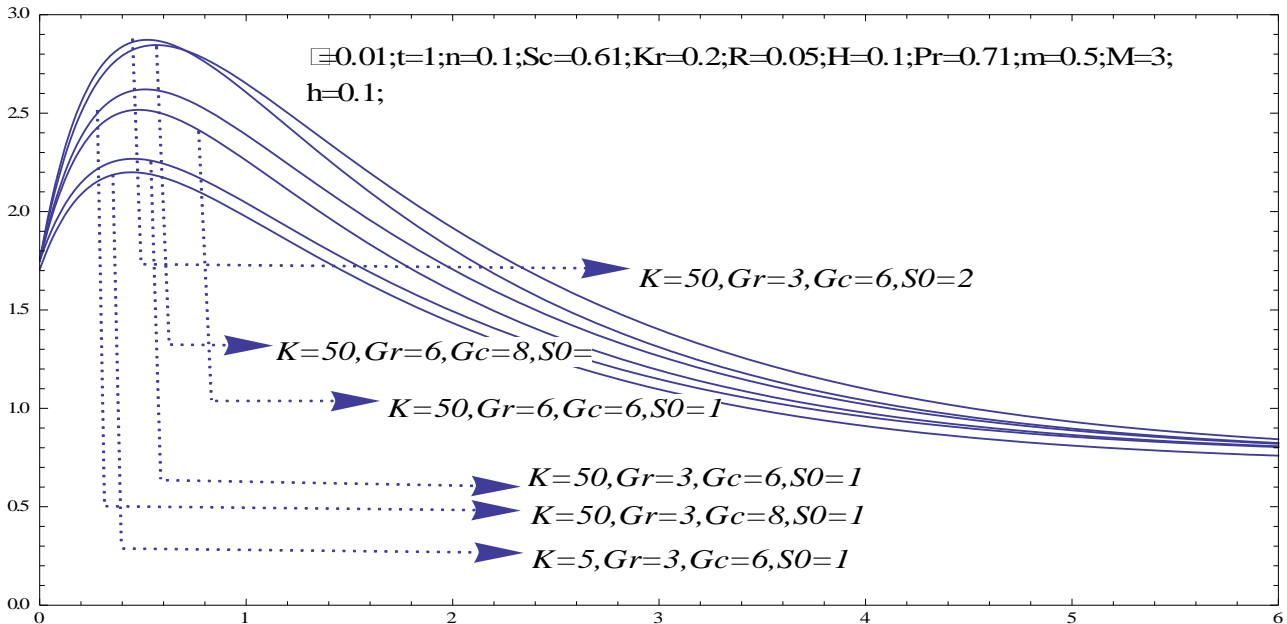


Figure 6: velocity profiles plotted against z for different values of K, Gr, Gc and So

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