

# (a, d) – Edge-antimagic Total labeling in Extended Duplicate Graph of Path

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**ABSTRACT:** In this paper, we prove that the Extended Duplicate Graph of a path is (a,d)-edge-antimagic total  
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**KEYWORDS :** Graph labeling, (a,d)-edge-antimagic total labeling, Extended Duplicate Graphs

## I. INTRODUCTION

In 1967, the concept of graph labeling was introduced by Rosa [3]. Graph labeling is an assignment of integers to the edges or vertices or both subject to certain conditions. Labeled graphs serve as useful models in a broad range of applications such as circuit design, communication network addressing, X-ray crystallography, radar, astronomy, data base management and coding theory. Over the past three decades various labeling of graphs such as cordial labeling, prime labeling, binary labeling, magic labeling, anti-magic labeling, bi-magic labeling, mean labeling, arithmetic labeling, graceful labeling, harmonious labeling etc., have been investigated in the literature [1].

Simanjuntak, et al introduced (a, d)-edge antimagic total labeling. They proved that  $C_{2n}$  has  $(2n+2, 1)$ - and  $(3n+2, 1)$ -edge-antimagic total labeling;  $C_{2n}$  has  $(4n+2, 2)$ - and  $(4n+3, 2)$ -edge-antimagic total labelings;  $C_{2n+1}$  has  $(3n+4, 3)$ - and  $(3n+5, 3)$ -edge-antimagic total labelings;  $P_{2n+1}$  has  $(3n+4, 2)$ -  $(3n+4, 3)$ -,  $(2n+4, 4)$ -,  $(5n+4, 2)$ -,  $(3n+5, 2)$ - and  $(2n+6, 4)$ -edge-antimagic total labelings;  $P_{2n}$  has  $(6n, 1)$ - and  $(6n+2, 2)$ -edge-antimagic total labelings. Ngurah proved that every odd cycle  $C_{2k+1}$  has a  $(4k+4, 2)$ -edge-antimagic total labeling and a  $(4k+5, 2)$ -edge-antimagic total labeling.

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Sudarsana et al prove: for every  $n \geq 2$ ,  $P_n \cup P_{n+1}$  has a  $(6n+1, 1)$ - and a  $(4n+3, 3)$ -edge-antimagic total labeling.

Though (a,d)-edge-antimagic total labeling have been studied for different kinds of graphs, (a,d)-edge-antimagic total labeling for duplicate graphs have not been investigated. In this paper we prove that Extended Duplicate Graph of path graphs are (a,d)-edge-antimagic[2].

## II. PRELIMINARIES

In this section, we give the basic notions relevant to this paper. Let  $G = G(V, E)$  be a finite, simple and undirected graph. By a labeling we mean one-to-one mapping that carries a set of graph elements onto a set of numbers, called labels (usually the set of integers). In this paper we deal labeling with domain as the set of all vertices[3].

**Definition 2.1** A function  $f$  is said to be (a,d)-edge-antimagic total labeling of a graph  $G$  if  $f : V \cup E \rightarrow \{1, 2, \dots, |V \cup E|\}$  such that the set  $\{f(v_i) + f(v_j) + f(v_i, v_j) \mid v_i, v_j \in E\}$  is  $\{a, a+d, a+2d, \dots, a+(|E|-1)d\}$ .

**Definition 2.2** Let  $G(V, E)$  be a simple graph. A duplicate graph of  $G$  is  $DG = (V_1, E_1)$  where the vertex set  $V_1 = V \cup V'$  and  $V \cap V' = \phi$  and  $f : V \rightarrow V'$  is bijective (for  $v \in V$ , we write  $f(v) = v'$  for convenience) and the edge set  $E_1$  of  $DG$  is defined as follows: The edge  $ab$  is in  $E$  if and only if both  $ab'$  and  $a'b$  are edges in  $E_1$ .

Clearly the duplicate graph of the path graph is disconnected. We give the following definition from [6].

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**Definition 2.3** Let  $DG = (V_1, E_1)$  be a duplicate graph of the path graph  $G(V, E)$ . We add an edge between any one vertex from  $V$  to any other vertex in  $V'$ , except the terminal vertices of  $V$  and  $V'$ [4]. For convenience, let us take  $v_2 \in V$  and  $v_2' \in V'$  and thus the edge  $(v_2, v_2')$  is formed. This graph is called the Extended Duplicate Graph of the path  $P_m$  and it is denoted by  $EDG(P_m)$ .

### III. MAIN RESULTS

In this section, we present an algorithm and prove the existence of  $(a,d)$ -edge-antimagic total labeling for  $EDG(P_m)$ .

We assign the name to all edges in the following manner:

If the paths are even like  $P_2, P_4, P_6, \dots$ , edges in the form of  $(v_1, v_2'), (v_2', v_3), (v_3, v_4'), \dots, (v_m', v_{m+1})$  are named as  $e_1, e_2, e_3, \dots, e_m$ ; the edge  $(v_2, v_2')$  is named as  $e_{m+1}$  and the edges in the form of  $(v_1', v_2), (v_2, v_3'), (v_3', v_4), \dots, (v_m, v_{m+1}')$  are named as  $e_1', e_2', e_3', \dots, e_m'$ .

In case of odd paths like  $P_3, P_5, P_7, \dots$ , the edges are in the form of  $(v_1, v_2'), (v_2', v_3), (v_3, v_4'), \dots, (v_m, v_{m+1}')$  are named as  $e_1, e_2, e_3, \dots, e_m$ ; the edge  $(v_2, v_2')$  is named as  $e_{m+1}$  and the edges  $(v_1', v_2), (v_2, v_3'), (v_3', v_4), \dots, (v_m', v_{m+1})$  are named as  $e_1', e_2', e_3', \dots, e_m'$ .

**Algorithm:**

**Input :**  $EDG(P_m)$  with  $2m+2$  vertices and  $2m+1$  edges.

**Step 1:** Denote the  $2m + 2$  vertices as  $V = \{v_1, v_2, \dots, v_m, v_{m+1}, v_1', v_2' \dots v_m', v_{m+1}'\}$ .

**Step 2 :** If 'm' is even, then  $m = 2n$ ;  $n \in \mathbb{N}$ . The  $EDG(P_m)$  is of the form  $EDG(P_{2n})$ ;  $n \in \mathbb{N}$ .

Define  $f: V \cup E \rightarrow \{1, 2, \dots, |V \cup E|\}$  such that

For  $1 \leq i \leq (m+2)/2$

$$f(v_{2i-1}) = 2i-1, \quad f(v_{2i-1}') = (2m+4)-2i$$

For  $1 \leq i \leq m/2$

$$f(v_{2i}) = (2m+3)+2i, \quad f(v_{2i}') = 2i,$$

$$f(e_{2i-1}) = (2m+1)+2i, \quad f(e_{2i-1}') = (4m+5)-2i,$$

$$f(e_{2i}) = (2m+2)+2i, \quad f(e_{2i}') = (4m+4)-2i,$$

and  $f(e_{m+1}) = 3m+3$

**Step 3:** If 'm' is odd, then  $m = 2n+1$ ;  $n \in \mathbb{N}$ . The  $EDG(P_m)$  is of the form  $EDG(P_{2n+1})$ ;  $n \in \mathbb{N}$ .

Define  $f: V \cup E \rightarrow \{1, 2, \dots, |V \cup E|\}$  such that

For  $1 \leq i \leq (m+1)/2$

$$f(v_{2i-1}) = 2i-1, \quad f(v_{2i}) = (2m+3)-2i,$$

$$f(v_{2i-1}') = (2m+4)-2i, \quad f(v_{2i}') = 2i,$$

$$f(e_{2i-1}) = (2m+1)+2i, \quad f(e_{2i-1}') = (4m+5)-2i,$$

$$f(e_{2i}) = (2m+2)+2i, \quad f(e_{2i}') = (4m+4)-2i,$$

and  $f(e_{m+1}) = 3m+3$

**Step 4:** Define  $f^*: E \rightarrow \mathbb{N}$  such that  $f^*(v_i, v_j) = |f(v_i) + f(v_j) + f(v_i, v_j)|$

**Output :** Labeled  $EDG(P_m)$

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**Theorem :** The Extended Duplicate Graph of path  $P_m$ ,  $m \geq 2$  admits (a,d)-edge-antimagic total labeling.[6]

**Proof:** Let  $EDG(P_m)$  be an Extended duplicate graph of path  $P_m$ . Clearly  $EDG(P_m)$  has  $p$  vertices and  $q$  edges where  $p = 2m+2$ ,  $q = 2m+1$ ;  $m \in \mathbb{N}$ .

Denote the set of vertices as  $V = (v_1, v_2, \dots, v_m, v_{m+1}, v'_1, v'_2, \dots, v'_m, v'_{m+1})$

**Case 1:** In this case, we prove the theorem for even paths. Let  $P_m$  be a path, where  $m=2n$ ;  $n \in \mathbb{N}$ .

Consider the paths of the type  $P_{2n}$ ;  $n \in \mathbb{N}$ , then we get path graphs  $P_2, P_4, P_6, \dots$

To get an (a,d)-edge-antimagic total labeling, define a map  $f: V \cup E \rightarrow \{1, 2, \dots, |V \cup E|\}$  as given in step 2 of the above algorithm.

Therefore the vertices  $v_1, v_3, v_5, \dots, v_{m+1}, v_m, v_{m-2}, \dots, v_2$  receives consecutive odd numbers such as 1, 3, 5, ...,  $q$  and  $v'_2, v'_4, v'_6, \dots, v'_m, v'_{m+1}, v'_{m-1}, v'_{m-3}, \dots, v'_1$  receives consecutive even numbers such as 2, 4, 6, ...,  $p$ .

Thus all the  $(2m+2)$  vertices are labeled.[7]

The edges of  $e_1, e_2, e_3, \dots, e_m, e_{m+1}$  receive consecutive numbers such as  $2m+3, 2m+4, 2m+5, \dots, 3m+2, 3m+3$  and  $e'_m, e'_{m-1}, e'_{m-2}, \dots, e'_3$  receive consecutive numbers such as  $3m+4, 3m+5, \dots, 4m+3$ .

Define induced function  $f^*: E \rightarrow \mathbb{N}$  such that

$$f^*(v_i, v_j) = |f(v_i) + f(v_j) + f(v_i, v_j)|$$

Now the edges are labeled as follows: The edges are in the following five forms namely  $(v_{2i-1}, v'_{2i}), (v'_{2i-1}, v_{2i})$  where  $1 \leq i \leq m/2$ ;  $(v_{2i}, v'_{2j-1}), (v'_{2i}, v_{2j-1})$  where  $j=i+1, 1 \leq i \leq m/2$  and  $(v_2, v'_2)$ .

For any  $i$ , where  $1 \leq i \leq m/2$ ,

$$\begin{aligned} f^*(v_{2i-1}, v'_{2i}) &= |f(v_{2i-1}) + f(v'_{2i}) + f(v_{2i-1}, v'_{2i})| \\ &= |f(v_{2i-1}) + f(v'_{2i}) + f(e_{2i-1})| \\ &= (2i-1) + 2i + \{(2m+1)+2i\} = |2m+6i| \end{aligned}$$

$$\begin{aligned} f^*(v'_{2i-1}, v_{2i}) &= |f(v'_{2i-1}) + f(v_{2i}) + f(v'_{2i-1}, v_{2i})| \\ &= |f(v'_{2i-1}) + f(v_{2i}) + f(e'_{2i-1})| \\ &= \{(2m+4)-2i\} + \{(2m+3)-2i\} + \{(4m+5)-2i\} \\ &= 8m+12-6i \end{aligned}$$

For any  $j$ , where  $j = i+1$  and  $1 \leq i \leq m/2$ ,

$$\begin{aligned} f^*(v'_{2i}, v_{2j-1}) &= |f(v'_{2i}) + f(v_{2j-1}) + f(v'_{2i}, v_{2j-1})| \\ &= |f(v'_{2i}) + f(v_{2j-1}) + f(e_{2i})| \\ &= 2i + (2j-1) + \{(2m+2)+2i\} = 2m+3+6i \end{aligned}$$

$$\begin{aligned} f^*(v_{2i}, v'_{2j-1}) &= |f(v_{2i}) + f(v'_{2j-1}) + f(v_{2i}, v'_{2j-1})| \\ &= |f(v_{2i}) + f(v'_{2j-1}) + f(e'_{2i})| \\ &= \{(2m+3)-2i\} + \{(2m+4)-2j\} + \{(4m+4)-2i\} \\ &= (8m+9)-6i \end{aligned}$$

For the edge  $(v_2, v'_2)$ ,

$$\begin{aligned} f^*(v_2, v'_2) &= |f(v_2) + f(v'_2) + f(v_2, v'_2)| \\ &= |f(v_2) + f(v'_2) + f(e_{m+1})| \end{aligned}$$

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$$= |(2m+1) + 2 + 3m + 3| = 5m + 6$$

That is  $f^*(E) = \{a, a+d, a+2d, \dots, a+ (|E|-1)d\}$  where  $a = 2m+2, d = 3$ , which are all distinct and hence EDG  $(P_m)$  is  $(a, d)$ -edge-antimagic total.

**Case (2):** In this case we prove the theorem for odd paths. Let  $P_m$  be a path, where  $m = 2n+1; n \in \mathbb{N}$ .

Consider the paths of the type  $P_{2n+1}; n \in \mathbb{N}$ , then we get path graphs  $P_3, P_5, P_7, \dots$

To get an  $(a,d)$ -edge-antimagic total labeling, define a map  $f: V \cup E \rightarrow \{1, 2, \dots, |V \cup E|\}$  as given in step 3 of the above algorithm.,[8]

Therefore the vertices  $v_1, v_3, v_5, \dots, v_{m+1}, v_m, v_{m-2}, \dots, v_2$  receives consecutive odd numbers such as 1, 3, 5, ...,  $q$  and  $v'_2, v'_4, v'_6, \dots, v'_m, v'_{m+1}, v'_{m-1}, v'_{m-3}, \dots, v'_1$  receives consecutive even numbers such as 2, 4, 6, ...,  $p$ .

Thus all the  $(2m+2)$  vertices are labeled.

The edges of  $e_1, e_2, e_3, \dots, e_m, e_{m+1}$  receive consecutive numbers such that  $2m+3, 2m+4, 2m+5, \dots, 3m+2, 3m+3$  and  $e'_m, e'_{m-1}, e'_{m-2}, \dots, e'_1$  receive consecutive numbers such as  $3m+4, 3m+5, \dots, 4m+3$ .

Define induced function  $f^*: E \rightarrow \mathbb{N}$  such that

$$f^*(v_i, v_j) = |f(v_i) + f(v_j) + f(v_i, v_j)|$$

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For any  $i$ , where  $1 \leq i \leq (m+1)/2$ ,

$$\begin{aligned} f^*(v_{2i-1}, v'_{2i}) &= |f(v_{2i-1}) + f(v'_{2i}) + f(v_{2i-1}, v'_{2i})| \\ &= |f(v_{2i-1}) + f(v'_{2i}) + f(e_{2i-1})| \\ &= (2i-1) + 2i + \{(2m+1) + 2i\} = |2m + 6i| \end{aligned}$$

$$\begin{aligned} f^*(v'_{2i-1}, v_{2i}) &= |f(v'_{2i-1}) + f(v_{2i}) + f(v'_{2i-1}, v_{2i})| \\ &= |f(v'_{2i-1}) + f(v_{2i}) + f(e'_{2i-1})| \\ &= \{(2m+4)-2i\} + (2m+3)-2i + \{(4m+5)-2i\} \\ &= 8m+12-6i \end{aligned}$$

For any  $j$ , where  $j = i+1$  and  $1 \leq i \leq (m-1)/2$ ,

$$\begin{aligned} f^*(v'_{2i}, v_{2j-1}) &= |f(v'_{2i}) + f(v_{2j-1}) + f(v'_{2i}, v_{2j-1})| \\ &= |f(v'_{2i}) + f(v_{2j-1}) + f(e_{2i})| \\ &= 2i + (2j - 1) + \{(2m+2)+2i\} = 2m + 3 + 6i \end{aligned}$$

$$\begin{aligned} f^*(v_{2i}, v'_{2j-1}) &= |f(v_{2i}) + f(v'_{2j-1}) + f(v_{2i}, v'_{2j-1})| \\ &= |f(v_{2i}) + f(v'_{2j-1}) + f(e'_{2i})| \\ &= \{(2m+3)-2i\} + \{(2m+4)-2j\} + \{(4m+4)-2i\} \\ &= (8m+9)-6i \end{aligned}$$

For the edge  $(v_2, v'_2)$ ,

$$\begin{aligned} f^*(v_2, v'_2) &= |f(v_2) + f(v'_2) + f(v_2, v'_2)| \\ &= |f(v_2) + f(v'_2) + f(e_{m+1})| \end{aligned}$$

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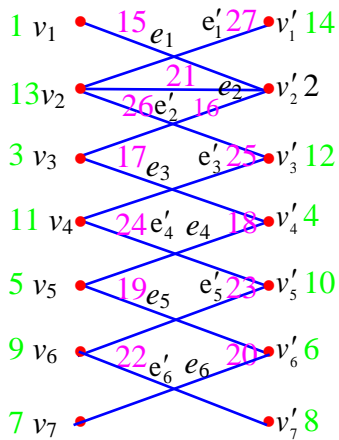
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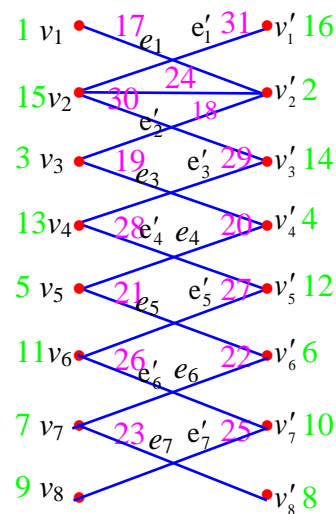
$$= |(2m+1) + 2 + 3m + 3| = 5m + 6$$

That is  $f^*(E) = \{a, a+d, a+2d, \dots, a+ (|E|-1)d\}$  where  $a = 2m+2, d = 3$ , which are all distinct and hence EDG  $(P_m)$  is  $(a, d)$ -edge-antimagic total.

**Illustration:**  $(a,d)$ -edge-antimagic total labeling for the graph EDG  $(P_6)$  and EDG  $(P_7)$  are shown in figure.[10]



**EDG  $(P_6)$**



**EDG  $(P_7)$**

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