

# Relation of Generalized Two Dimensional Fractional Cosine Transform with Other Transform

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**ABSTRACT:** Fractional Cosine Transform (FRCT) is a generalization of the ordinary cosine transform and it has similar relationship with Fractional Fourier Transform (FRFT). In actual computations of fractional cosine transform (FCT) and fractional sine transform (FST), the basic integrations are performed with quadratures. Because the data are sampled and the duration is finite, most of the quadratures can be implemented via matrix computations. In this paper we obtained some relations between fractional cosine and sine transform with other transforms.

**KEYWORDS:** fractional cosine transform, fractional sine transform, fractional Fourier transforms, generalized function, two dimensional fractional cosine transform.

## I. INTRODUCTION

The FrCT, which is the generalization of cosine transform. The real part of FrFT Kernel was chosen as the kernel for Fractional Cosine Transform as in case of (CT) where real part of FT is chosen as (CT) kernel. The FrFT belongs to the class of time frequency representation that has been extensively used by the signal processing community. In all the time frequency representation, one normally uses a plane with two orthogonal axes corresponding to time frequency. If we consider a signal  $x(t)$  to be represented along the time axis and its ordinary FT  $X(F)$  to be represented along the frequency axis, then the FT operator (denoted by F) can be visualized as change in representation of signal corresponding to counter clock wise rotation of the axis by an angle  $\frac{\pi}{2}$ . This is consistent with some of absorb properties of FT. For example, two successive rotations of a signal through  $\frac{\pi}{2}$  will result in an inversion of time axis. Moreover, four successive rotations will leave the signal unaltered since a rotation through  $2\pi$  of the signal should leave the signal unaltered. The FrFT is a linear operator that corresponds to rotation of signal through an angle which is not multiple of  $\frac{\pi}{2}$  that is it is the representation of the signal along the axis  $u$  making an angle  $\alpha$  with time axis.

In addition to the FT, the cosine transform (CT), which are based on half range expansion of a function over cosine basis function, are also important tools in signal processing. Despite of some lack of elegance in their properties compared to the FT, CT has their own areas of applications. The idea of fractionalization of the CT was proposed in [2]. The real part of the FrFT kernel was chosen as the kernels for an FrCT as in the case of CT where real part of FT is chosen as a CT kernel. Thus FrFT and FrCT with parameter 'a' are finding its place in many applications where FT and CT are found to be useful like beam forming, image compression, noise removal and signal restoration. In our previous work we already defined following definitions:

### 1.1. Generalized two dimensional fractional Cosine transform

Two dimensional fractional Cosine transform with parameter  $\alpha$   $f(x, y)$  denoted by  $F_c^\alpha(x, y)$  perform a linear operation given by the integral transform.

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$$F_c^\alpha \{f(x, y)\}(u, v) = \int_0^\infty \int_0^\infty f(x, y) K_\alpha(x, y, u, v) dx dy \dots \dots \dots (1.1)$$

Where the kernel,

$$K_c^\alpha(x, y, u, v) = \sqrt{\frac{1-icota}{2\pi}} e^{\frac{i(x^2+y^2+u^2+v^2)cota}{2}} \cos(\text{coseca}.ux) . \cos(\text{coseca}.vy) \dots \dots \dots (1.2)$$

### 1.2. The test function space E

An infinitely differentiable complex valued function  $\phi$  on  $R^n$  belongs to  $E(R^n)$  if for each compact set  $I \subset S_{a,b}$ , where,

$$S_{a,b} = \{x, y: x, y \in R^n, |x| \leq a, |y| \leq b, a > 0, b > 0\}, I \in R^n$$

$$\gamma_{E,p,q}(\phi) = \sup_{x,y} |D_{x,y}^{p,q} \phi(x, y)| < \infty \text{ Where, } p, q = 1, 2, 3, \dots$$

Thus  $E(R^n)$  will denote the space of all  $\phi \in E(R^n)$  with support contained in  $S_{a,b}$

Note that: the space E is complete and therefore a Frechet space. Moreover, we say that f is a fractional Cosine transformable, if it is a member of  $E^*$ , the dual space of E.

In this paper we have to find out some relation between fractional cosine and sine transform with other transform and defined distributional two-dimensional fractional Cosine transform

## II. DISTRIBUTIONAL TWO-DIMENSIONAL FRACTIONAL COSINE TRANSFORM

The two dimensional distributional fractional Cosine transform of  $f(x, y) \in E^*(R^n)$  defined by

$$F_c^\alpha \{f(x, y)\} = F^\alpha(u, v) = \langle f(x, y), K_\alpha(x, y, u, v) \rangle \dots \dots \dots (2.1)$$

$$K_c^\alpha(x, y, u, v) = \sqrt{\frac{1-icota}{2\pi}} e^{\frac{i(x^2+y^2+u^2+v^2)cota}{2}} \cos(\text{coseca}.ux) . \cos(\text{coseca}.vy) \dots \dots (2.2)$$

Where , RHS of equation (2.1) has a meaning as the application of  $f \in E^*$  to  $K_\alpha(x, y, u, v) \in E$

## III. RELATIONS OF TWO DIMENSIONAL FRACTIONAL COSINE TRANSFORM WITH OTHER TRANSFORMS

### 3.1 Relation between two dimensional fractional Fourier transform and two dimensional fractional sine and cosine transform.

If two dimensional fractional Fourier transform is

$$F\{f(x, y)\}(u, v) = \int_{-\infty}^\infty \int_{-\infty}^\infty \sqrt{\frac{1-icota}{2\pi}} e^{\frac{i}{2\text{sin}\alpha}[(x^2+y^2+u^2+v^2)\text{cosa} - 2(xu+yv)]} f(x, y) dx dy$$

Then FrFT changes to FrCT and FrST

**Solution:** Let  $A = \sqrt{\frac{1-icota}{2\pi}}$

$$F\{f(x, y)\}(u, v) = \int_{-\infty}^\infty \int_{-\infty}^\infty A e^{\frac{i}{2\text{sin}\alpha}[(x^2+y^2+u^2+v^2)\text{cosa}]} e^{\frac{i}{2\text{sin}\alpha}(-2(xu+yv))} f(x, y) dx dy$$

$$F\{f(x, y)\}(u, v) = \int_{-\infty}^\infty \int_{-\infty}^\infty A e^{\frac{i}{2}cota [(x^2+y^2+u^2+v^2)]} e^{-icoseca(xu+yv)} f(x, y) dx dy$$

Let  $B = e^{\frac{i}{2}cota [(x^2+y^2+u^2+v^2)]}$

$$F\{f(x, y)\}(u, v) = \int_{-\infty}^\infty \int_{-\infty}^\infty A B e^{-icosecaxu} e^{-icosecayv} f(x, y) dx dy$$

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$$\begin{aligned}
 F\{f(x, y)\}(u, v) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A B (\cos(\csc axu) - i \sin(\csc axu)) \\
 &(\cos(\csc ayv) - i \sin(\csc ayv)) f(x, y) dx dy \\
 F\{f(x, y)\}(u, v) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A B (\cos(\csc axu) \cdot \cos(\csc ayv)) f(x, y) dx dy \\
 &- e^{-i(\alpha - \frac{\pi}{2})} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\alpha - \frac{\pi}{2})} A B (\sin(\csc axu) \cdot \sin(\csc ayv)) f(x, y) dx dy \\
 &- i \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A B (e^{i \csc axu} - i \sin(\csc axu)) \sin(\csc ayv) f(x, y) dx dy \\
 &- i \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{i} A B (e^{i \csc axu} - \cos(\csc axu)) \cdot \cos(\csc ayv) f(x, y) dx dy
 \end{aligned}$$

Here we take

$$(e^{i \csc axu} - i \sin(\csc axu)) = \cos(\csc axu), \frac{1}{i} (e^{i \csc axu} - \cos(\csc axu)) = \sin(\csc axu)$$

$$\begin{aligned}
 F\{f(x, y)\}(u, v) &= F_{\alpha}^c [f(x, y)](u, v) - e^{-i(\alpha - \frac{\pi}{2})} F_{\alpha}^s [f(x, y)](u, v) \\
 &- i \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A B (e^{i \csc axu} \sin(\csc ayv) f(x, y) dx dy - i \sin(\csc axu) \sin(\csc ayv) f(x, y) dx dy) \\
 &- i \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{i} A B (e^{i \csc axu} \cos(\csc ayv) f(x, y) dx dy - \cos(\csc axu) \cos(\csc ayv) f(x, y) dx dy)
 \end{aligned}$$

$$\begin{aligned}
 F\{f(x, y)\}(u, v) &= F_{\alpha}^c [f(x, y)](u, v) - e^{-i(\alpha - \frac{\pi}{2})} F_{\alpha}^s [f(x, y)](u, v) \\
 &- i \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A B e^{i \csc axu} \sin(\csc ayv) f(x, y) dx dy \\
 &- e^{-i(\alpha - \frac{\pi}{2})} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\alpha - \frac{\pi}{2})} A B \sin(\csc axu) \sin(\csc ayv) f(x, y) dx dy \\
 &- \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A B e^{i \csc axu} \cos(\csc ayv) f(x, y) dx dy + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A B \cos(\csc axu) \cos(\csc ayv) f(x, y) dx dy
 \end{aligned}$$

$$\begin{aligned}
 F\{f(x, y)\}(u, v) &= F_{\alpha}^c [f(x, y)](u, v) - e^{-i(\alpha - \frac{\pi}{2})} F_{\alpha}^s [f(x, y)](u, v) \\
 &- i \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A B e^{i \csc axu} \sin(\csc ayv) f(x, y) dx dy - e^{-i(\alpha - \frac{\pi}{2})} F_{\alpha}^s [f(x, y)](u, v) \\
 &- \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A B e^{i \csc axu} \cos(\csc ayv) f(x, y) dx dy + F_{\alpha}^c [f(x, y)](u, v)
 \end{aligned}$$

$$\begin{aligned}
 F\{f(x, y)\}(u, v) &= 2F_{\alpha}^c [f(x, y)](u, v) - 2e^{-i(\alpha - \frac{\pi}{2})} F_{\alpha}^s [f(x, y)](u, v) - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A B e^{i \csc axu} (\cos(\csc ayv) \\
 &+ i \sin(\csc ayv)) f(x, y) dx dy
 \end{aligned}$$

$$F\{f(x, y)\}(u, v) = 2F_{\alpha}^c [f(x, y)](u, v) - 2e^{-i(\alpha - \frac{\pi}{2})} F_{\alpha}^s [f(x, y)](u, v) - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A B e^{i \csc axu} e^{i \csc ayv} f(x, y) dx dy$$

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$$F\{f(x, y)\}(u, v) = 2F_\alpha^c[f(x, y)](u, v) - 2e^{-i(\alpha-\frac{\pi}{2})}F_\alpha^s[f(x, y)](u, v) - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A B e^{i(xu+vy)\text{cosec}\alpha} f(x, y) dx dy$$

$$F\{f(x, y)\}(u, v) = 2F_\alpha^c[f(x, y)](u, v) - 2e^{-i(\alpha-\frac{\pi}{2})}F_\alpha^s[f(x, y)](u, v) - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sqrt{\frac{1-icota}{2\pi}} e^{\frac{i}{2}cota[(x^2+y^2+u^2+v^2)]} e^{-i(xu+vy)\text{cosec}\alpha} e^{2i(xu+vy)\text{cosec}\alpha} f(x, y) dx dy$$

$$F\{f(x, y)\}(u, v) = 2F_\alpha^c[f(x, y)](u, v) - 2e^{-i(\alpha-\frac{\pi}{2})}F_\alpha^s[f(x, y)](u, v) - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sqrt{\frac{1-icota}{2\pi}} e^{\frac{i}{2\sin\alpha}[(x^2+y^2+u^2+v^2)\text{cosec}\alpha - 2(xu+vy)]} e^{2i(xu+vy)\text{cosec}\alpha} f(x, y) dx dy$$

$$F\{f(x, y)\}(u, v) = 2F_\alpha^c[f(x, y)](u, v) - 2e^{-i(\alpha-\frac{\pi}{2})}F_\alpha^s[f(x, y)](u, v) - F_\alpha[e^{2i(xu+vy)\text{cosec}\alpha} f(x, y)]$$

Here  $F_\alpha$  is fractional Fourier transform and  $F_\alpha^c$   $F_\alpha^s$  are fractional cosine and sine transform respectively.

3.2 Relation between two dimensional fractional offset Fourier transform and two dimensional fractional offset Sine and Cosine transform.

If two dimensional offset fractional Fourier transform is

$$F_\alpha^{\tau, \eta, \xi, \gamma}\{f(x, y)\}(u - \eta, v - \gamma) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sqrt{\frac{1-icota}{2\pi}} e^{i(u\tau+v\xi)} e^{\frac{i}{2\sin\alpha}[(x^2+y^2+(u-\eta)^2+(v-\gamma)^2)\text{cosec}\alpha - 2(x(u-\eta)+y(v-\gamma))]} f(x, y) dx dy$$

Then offset FrFT changes to offset FrCT and FrST

**Solution:** Let  $A = \sqrt{\frac{1-icota}{2\pi}}$

$$F_\alpha^{\tau, \eta, \xi, \gamma}\{f(x, y)\}(u - \eta, v - \gamma) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A e^{i(u\tau+v\xi)} e^{\frac{i}{2\sin\alpha}[(x^2+y^2+(u-\eta)^2+(v-\gamma)^2)\text{cosec}\alpha]} e^{\frac{i}{2\sin\alpha}(-2(x(u-\eta)+y(v-\gamma)))} f(x, y) dx dy$$

$$F_\alpha^{\tau, \eta, \xi, \gamma}\{f(x, y)\}(u - \eta, v - \gamma) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A e^{i(u\tau+v\xi)} e^{\frac{i}{2}cota[(x^2+y^2+(u-\eta)^2+(v-\gamma)^2)]} e^{-icosec\alpha(x(u-\eta)+y(v-\gamma))} f(x, y) dx dy$$

Let  $B = e^{\frac{i}{2}cota[(x^2+y^2+u^2+v^2)]} e^{i(u\tau+v\xi)}$

$$F_\alpha^{\tau, \eta, \xi, \gamma}\{f(x, y)\}(u - \eta, v - \gamma) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A B e^{-icos\text{ec}\alpha(u-\eta)} e^{-icos\text{ec}\alpha(v-\gamma)} f(x, y) dx dy$$

$$F_\alpha^{\tau, \eta, \xi, \gamma}\{f(x, y)\}(u - \eta, v - \gamma) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A B (\cos(\text{csc}\alpha x(u - \eta)) - i\sin(\text{csc}\alpha x(u - \eta))) (\cos(\text{csc}\alpha y(v - \gamma)) - i\sin(\text{csc}\alpha y(v - \gamma))) f(x, y) dx dy$$

$$F_\alpha^{\tau, \eta, \xi, \gamma}\{f(x, y)\}(u - \eta, v - \gamma) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A B (\cos(\text{csc}\alpha x(u - \eta)) \cdot \cos(\text{csc}\alpha y(v - \gamma))) f(x, y) dx dy - e^{-i(\alpha-\frac{\pi}{2})} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\alpha-\frac{\pi}{2})} A B (\sin(\text{csc}\alpha x(u - \eta)) \cdot \sin(\text{csc}\alpha y(v - \gamma))) f(x, y) dx dy$$

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$$\begin{aligned}
 & -i \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A B \left( e^{i \operatorname{cosec} ax (u-\eta)} - i \sin(\operatorname{csc} ax (u-\eta)) \right) \sin(\operatorname{csc} ay (v-\gamma)) f(x, y) dx dy \\
 & -i \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{i} A B \left( e^{i \operatorname{cosec} ax (u-\eta)} - \cos(\operatorname{csc} ax (u-\eta)) \right) \cdot \cos(\operatorname{csc} ay (v-\gamma)) f(x, y) dx dy
 \end{aligned}$$

Here we take

$$\left( e^{i \operatorname{cosec} ax (u-\eta)} - i \sin(\operatorname{csc} ax (u-\eta)) \right) = \cos(\operatorname{csc} ax (u-\eta)),$$

$$\frac{1}{i} \left( e^{i \operatorname{cosec} ax (u-\eta)} - \cos(\operatorname{csc} ax (u-\eta)) \right) = \sin(\operatorname{csc} ax (u-\eta))$$

$$F_{\alpha}^{\tau, \eta, \xi, \gamma} \{f(x, y)\}(u-\eta, v-\gamma) = F_{\alpha}^{c, \tau, \eta, \xi, \gamma} [f(x, y)](u-\eta, v-\gamma) - e^{-i(\alpha-\frac{\pi}{2})} F_{\alpha}^{s, \tau, \eta, \xi, \gamma} [f(x, y)](u-\eta, v-\gamma)$$

$$\begin{aligned}
 & -i \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A B \left( e^{i \operatorname{cosec} ax (u-\eta)} \sin(\operatorname{csc} ay (v-\gamma)) f(x, y) dx dy \right. \\
 & \quad \left. - i \sin(\operatorname{csc} ax (u-\eta)) \sin(\operatorname{csc} ay (v-\gamma)) f(x, y) dx dy \right)
 \end{aligned}$$

$$\begin{aligned}
 & -i \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{i} A B \left( e^{i \operatorname{cosec} ax (u-\eta)} \cos(\operatorname{csc} ay (v-\gamma)) f(x, y) dx dy \right. \\
 & \quad \left. - \cos(\operatorname{csc} ax (u-\eta)) \cos(\operatorname{csc} ay (v-\gamma)) f(x, y) dx dy \right)
 \end{aligned}$$

$$F_{\alpha}^{\tau, \eta, \xi, \gamma} \{f(x, y)\}(u-\eta, v-\gamma) = F_{\alpha}^{c, \tau, \eta, \xi, \gamma} [f(x, y)](u-\eta, v-\gamma) - e^{-i(\alpha-\frac{\pi}{2})} F_{\alpha}^{s, \tau, \eta, \xi, \gamma} [f(x, y)](u-\eta, v-\gamma)$$

$$\begin{aligned}
 & -i \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A B e^{i \operatorname{cosec} ax (u-\eta)} \sin(\operatorname{csc} ay (v-\gamma)) f(x, y) dx dy \\
 & \quad - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A B \sin(\operatorname{csc} ax (u-\eta)) \sin(\operatorname{csc} ay (v-\gamma)) f(x, y) dx dy \\
 & \quad - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A B e^{i \operatorname{cosec} ax (u-\eta)} \cos(\operatorname{csc} ay (v-\gamma)) f(x, y) dx dy \\
 & \quad + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A B \cos(\operatorname{csc} ax (u-\eta)) \cos(\operatorname{csc} ay (v-\gamma)) f(x, y) dx dy
 \end{aligned}$$

$$F_{\alpha}^{\tau, \eta, \xi, \gamma} \{f(x, y)\}(u-\eta, v-\gamma) = F_{\alpha}^{c, \tau, \eta, \xi, \gamma} [f(x, y)](u-\eta, v-\gamma) - e^{-i(\alpha-\frac{\pi}{2})} F_{\alpha}^{s, \tau, \eta, \xi, \gamma} [f(x, y)](u-\eta, v-\gamma)$$

$$\begin{aligned}
 & -i \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A B e^{i \operatorname{cosec} ax (u-\eta)} \sin(\operatorname{csc} ay (v-\gamma)) f(x, y) dx dy - e^{-i(\alpha-\frac{\pi}{2})} F_{\alpha}^{s, \tau, \eta, \xi, \gamma} [f(x, y)](u-\eta, v-\gamma) \\
 & \quad - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A B e^{i \operatorname{cosec} ax (u-\eta)} \cos(\operatorname{csc} ay (v-\gamma)) f(x, y) dx dy + F_{\alpha}^{c, \tau, \eta, \xi, \gamma} [f(x, y)](u-\eta, v-\gamma)
 \end{aligned}$$

$$\begin{aligned}
 & F_{\alpha}^{\tau, \eta, \xi, \gamma} \{f(x, y)\}(u-\eta, v-\gamma) \\
 & = 2F_{\alpha}^{c, \tau, \eta, \xi, \gamma} [f(x, y)](u-\eta, v-\gamma) - 2e^{-i(\alpha-\frac{\pi}{2})} F_{\alpha}^{s, \tau, \eta, \xi, \gamma} [f(x, y)](u-\eta, v-\gamma) \\
 & \quad - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A B e^{i \operatorname{cosec} ax (u-\eta)} (\cos(\operatorname{csc} ay (v-\gamma)) + i \sin(\operatorname{csc} ay (v-\gamma))) f(x, y) dx dy
 \end{aligned}$$

$$\begin{aligned}
 & F_{\alpha}^{\tau, \eta, \xi, \gamma} \{f(x, y)\}(u-\eta, v-\gamma) \\
 & = 2F_{\alpha}^{c, \tau, \eta, \xi, \gamma} [f(x, y)](u-\eta, v-\gamma) - 2e^{-i(\alpha-\frac{\pi}{2})} F_{\alpha}^{s, \tau, \eta, \xi, \gamma} [f(x, y)](u-\eta, v-\gamma) \\
 & \quad - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A B e^{i \operatorname{cosec} ax (u-\eta)} e^{i \operatorname{cosec} ay (v-\gamma)} f(x, y) dx dy
 \end{aligned}$$

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$$\begin{aligned}
 F_{\alpha}^{\tau, \eta, \xi, \gamma} \{f(x, y)\}(u - \eta, v - \gamma) &= 2F_{\alpha}^{c, \tau, \eta, \xi, \gamma} [f(x, y)](u - \eta, v - \gamma) - 2e^{-i(\alpha - \frac{\pi}{2})} F_{\alpha}^{s, \tau, \eta, \xi, \gamma} [f(x, y)](u - \eta, v - \gamma) \\
 &\quad - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A B e^{i(x(u-\eta)+y(v-\gamma))\operatorname{cosec}\alpha} f(x, y) dx dy \\
 F_{\alpha}^{\tau, \eta, \xi, \gamma} \{f(x, y)\}(u - \eta, v - \gamma) &= 2F_{\alpha}^{c, \tau, \eta, \xi, \gamma} [f(x, y)](u - \eta, v - \gamma) - 2e^{-i(\alpha - \frac{\pi}{2})} F_{\alpha}^{s, \tau, \eta, \xi, \gamma} [f(x, y)](u - \eta, v - \gamma) \\
 &\quad - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sqrt{\frac{1 - i\cot\alpha}{2\pi}} e^{\frac{i}{2}\cot\alpha [(x^2+y^2+(u-\eta)^2+(v-\gamma)^2)]} e^{-i(x(u-\eta)+y(v-\gamma))\operatorname{cosec}\alpha} e^{2i(x(u-\eta)+y(v-\gamma))\operatorname{cosec}\alpha} f(x, y) dx dy \\
 F_{\alpha}^{\tau, \eta, \xi, \gamma} \{f(x, y)\}(u - \eta, v - \gamma) &= 2F_{\alpha}^{c, \tau, \eta, \xi, \gamma} [f(x, y)](u - \eta, v - \gamma) - 2e^{-i(\alpha - \frac{\pi}{2})} F_{\alpha}^{s, \tau, \eta, \xi, \gamma} [f(x, y)](u - \eta, v - \gamma) \\
 &\quad - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sqrt{\frac{1 - i\cot\alpha}{2\pi}} e^{\frac{i}{2\sin\alpha} [(x^2+y^2+(u-\eta)^2+(v-\gamma)^2)\cos\alpha - 2(x(u-\eta)+y(v-\gamma))] } e^{2i(x(u-\eta)+y(v-\gamma))\operatorname{cosec}\alpha} f(x, y) dx dy \\
 F_{\alpha}^{\tau, \eta, \xi, \gamma} \{f(x, y)\}(u - \eta, v - \gamma) &= 2F_{\alpha}^{c, \tau, \eta, \xi, \gamma} [f(x, y)](u - \eta, v - \gamma) - 2e^{-i(\alpha - \frac{\pi}{2})} F_{\alpha}^{s, \tau, \eta, \xi, \gamma} [f(x, y)](u - \eta, v - \gamma) - \\
 F_{\alpha}^{\tau, \eta, \xi, \gamma} [e^{2i(x(u-\eta)+y(v-\gamma))\operatorname{cosec}\alpha} f(x, y)] &
 \end{aligned}$$

Here  $F_{\alpha}^{\tau, \eta, \xi, \gamma}$  is offset fractional Fourier transform and  $F_{\alpha}^{c, \tau, \eta, \xi, \gamma}, F_{\alpha}^{s, \tau, \eta, \xi, \gamma}$  are offset fractional cosine and sine transformed respectively.

### 3.3 Relation between two dimensional canonical cosine-cosine transform and two dimensional fractional cosine transform

Two dimensional canonical cosine-cosine transform is

$$\{2DCCCT(f(x, y))\}(u, v) = \sqrt{\frac{1}{2\pi ib}} \sqrt{\frac{1}{2\pi ib}} e^{\frac{i}{2}(\frac{d}{b})u^2} e^{\frac{i}{2}(\frac{d}{b})v^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cos\left(\frac{u}{b}x\right) \cos\left(\frac{v}{b}y\right) e^{\frac{i}{2}(\frac{a}{b})x^2} e^{\frac{i}{2}(\frac{a}{b})y^2} f(x, y) dx dy$$

If  $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  then 2DCCCT changes to FrCT

**Solution:** if  $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\therefore a = \cos\alpha, b = \sin\alpha, c = -\sin\alpha, d = \cos\alpha$

Consider two dimensional canonical cosine-cosine transform

$$\{2DCCCT(f(x, y))\}(u, v) = \sqrt{\frac{1}{2\pi ib}} \sqrt{\frac{1}{2\pi ib}} e^{\frac{i}{2}(\frac{d}{b})u^2} e^{\frac{i}{2}(\frac{d}{b})v^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cos\left(\frac{u}{b}x\right) \cos\left(\frac{v}{b}y\right) e^{\frac{i}{2}(\frac{a}{b})x^2} e^{\frac{i}{2}(\frac{a}{b})y^2} f(x, y) dx dy$$

Putting above values in 2DCCCT

$$\begin{aligned}
 \{2DCCCT(f(x, y))\}(u, v) &= \sqrt{\frac{1}{2\pi i\sin\alpha}} \sqrt{\frac{1}{2\pi i\sin\alpha}} e^{\frac{i}{2}(\frac{\cos\alpha}{\sin\alpha})u^2} e^{\frac{i}{2}(\frac{\cos\alpha}{\sin\alpha})v^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cos\left(\frac{u}{\sin\alpha}x\right) \cos\left(\frac{v}{\sin\alpha}y\right) e^{\frac{i}{2}(\frac{\cos\alpha}{\sin\alpha})x^2} e^{\frac{i}{2}(\frac{\cos\alpha}{\sin\alpha})y^2} f(x, y) dx dy
 \end{aligned}$$

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$$\{2DCCCT(f(x,y))\}(u,v) = \sqrt{\frac{1}{2\pi i \sin \alpha}} \sqrt{\frac{1}{2\pi i \sin \alpha}} e^{\frac{i}{2}(\cot \alpha)u^2} e^{\frac{i}{2}(\cot \alpha)v^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cos(\frac{u}{\sin \alpha} x) \cos(\frac{v}{\sin \alpha} y) e^{\frac{i}{2}(\cot \alpha)x^2} e^{\frac{i}{2}(\cot \alpha)y^2} f(x,y) dx dy$$

$$\{2DCCCT(f(x,y))\}(u,v) = \sqrt{\frac{1}{2\pi i \sin \alpha}} \sqrt{\frac{1}{2\pi i \sin \alpha}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{\frac{i}{2}(x^2+y^2+u^2+v^2)\cot \alpha} \cos(\frac{u}{\sin \alpha} x) \cos(\frac{v}{\sin \alpha} y) f(x,y) dx dy$$

$$(e^{i\alpha})^{\frac{1}{2}} \{2DCCCT(f(x,y))\}(u,v) = \sqrt{\frac{\cos \alpha + i \sin \alpha}{2\pi i \sin \alpha}} \sqrt{\frac{1}{2\pi i \sin \alpha}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{\frac{i}{2}(x^2+y^2+u^2+v^2)\cot \alpha} \cos(\frac{u}{\sin \alpha} x) \cos(\frac{v}{\sin \alpha} y) f(x,y) dx dy$$

$$(e^{i\alpha})^{\frac{1}{2}} \{2DCCCT(f(x,y))\}(u,v) = \sqrt{\frac{1 - i \cot \alpha}{2\pi}} \sqrt{\frac{-i \csc \alpha}{2\pi}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{\frac{i}{2}(x^2+y^2+u^2+v^2)\cot \alpha} \cos(\frac{u}{\sin \alpha} x) \cos(\frac{v}{\sin \alpha} y) f(x,y) dx dy$$

$$(e^{i\alpha})^{\frac{1}{2}} \{2DCCCT(f(x,y))\}(u,v) = \sqrt{\frac{-i \csc \alpha}{2\pi}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sqrt{\frac{1 - i \cot \alpha}{2\pi}} e^{\frac{i}{2}(x^2+y^2+u^2+v^2)\cot \alpha} \cos(\frac{u}{\sin \alpha} x) \cos(\frac{v}{\sin \alpha} y) f(x,y) dx dy$$

$$(e^{i\alpha})^{\frac{1}{2}} \{2DCCCT(f(x,y))\}(u,v) = \sqrt{\frac{-i \csc \alpha}{2\pi}} F_{\alpha}^c \{f(x,y)\}(u,v)$$

### 3.4 Relation between two dimensional canonical sine-sine transform and two dimensional fractional sine transform

Two dimensional canonical sine-sine transform is

$$\{2DCSST(f(x,y))\}(u,v) = (-1) \sqrt{\frac{1}{2\pi i b}} \sqrt{\frac{1}{2\pi i b}} e^{\frac{i}{2}(\frac{d}{b})u^2} e^{\frac{i}{2}(\frac{d}{b})v^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sin(\frac{u}{b} x) \sin(\frac{v}{b} y) e^{\frac{i}{2}(\frac{a}{b})x^2} e^{\frac{i}{2}(\frac{a}{b})y^2} f(x,y) dx dy$$

If  $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  then 2DCSST changes to FrST

**Solution:** if  $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\therefore a = \cos \alpha, b = \sin \alpha, c = -\sin \alpha, d = \cos \alpha$

Consider two dimensional canonical sine-sine transform

$$\{2DCSST(f(x,y))\}(u,v) = (-1) \sqrt{\frac{1}{2\pi i b}} \sqrt{\frac{1}{2\pi i b}} e^{\frac{i}{2}(\frac{d}{b})u^2} e^{\frac{i}{2}(\frac{d}{b})v^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sin(\frac{u}{b} x) \sin(\frac{v}{b} y) e^{\frac{i}{2}(\frac{a}{b})x^2} e^{\frac{i}{2}(\frac{a}{b})y^2} f(x,y) dx dy$$

Putting above values in 2DCCCT

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$$\begin{aligned} & \{2DCSST(f(x, y))\}(u, v) \\ &= (-1) \sqrt{\frac{1}{2\pi i \sin \alpha}} \sqrt{\frac{1}{2\pi i \sin \alpha}} e^{\frac{i}{2}(\frac{\cos \alpha}{\sin \alpha})u^2} e^{\frac{i}{2}(\frac{\cos \alpha}{\sin \alpha})v^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sin\left(\frac{u}{\sin \alpha} x\right) \sin\left(\frac{v}{\sin \alpha} y\right) e^{\frac{i}{2}(\frac{\cos \alpha}{\sin \alpha})x^2} e^{\frac{i}{2}(\frac{\cos \alpha}{\sin \alpha})y^2} \\ & \quad f(x, y) dx dy \\ & \{2DCSST(f(x, y))\}(u, v) \\ &= (-1) \sqrt{\frac{1}{2\pi i \sin \alpha}} \sqrt{\frac{1}{2\pi i \sin \alpha}} e^{\frac{i}{2}(\cot \alpha)u^2} e^{\frac{i}{2}(\cot \alpha)v^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sin(csc \alpha \cdot u x) \sin(csc \alpha v y) e^{\frac{i}{2}(\cot \alpha)x^2} e^{\frac{i}{2}(\cot \alpha)y^2} \\ & \quad f(x, y) dx dy \\ & \{2DCSST(f(x, y))\}(u, v) \\ &= (-1) \sqrt{\frac{1}{2\pi i \sin \alpha}} \sqrt{\frac{1}{2\pi i \sin \alpha}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{\frac{i}{2}(x^2+y^2+u^2+v^2)\cot \alpha} \sin(csc \alpha \cdot u x) \sin(csc \alpha v y) f(x, y) dx dy \\ & (e^{i\alpha})^{\frac{1}{2}} \{2DCSST(f(x, y))\}(u, v) \\ &= (-1) \sqrt{\frac{\cos \alpha + i \sin \alpha}{2\pi i \sin \alpha}} \sqrt{\frac{1}{2\pi i \sin \alpha}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{\frac{i}{2}(x^2+y^2+u^2+v^2)\cot \alpha} \sin(csc \alpha \cdot u x) \sin(csc \alpha v y) f(x, y) dx dy \\ & (e^{i\alpha})^{\frac{1}{2}} \{2DCSST(f(x, y))\}(u, v) \\ &= (-1) \sqrt{\frac{1 - i \cot \alpha}{2\pi}} \sqrt{\frac{-i csc \alpha}{2\pi}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{\frac{i}{2}(x^2+y^2+u^2+v^2)\cot \alpha} \sin(csc \alpha \cdot u x) \sin(csc \alpha v y) f(x, y) dx dy \\ & (e^{i\alpha})^{\frac{1}{2}} \{2DCSST(f(x, y))\}(u, v) \\ &= (-1) e^{-i(\alpha - \frac{\pi}{2})} \sqrt{\frac{-i csc \alpha}{2\pi}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sqrt{\frac{1 - i \cot \alpha}{2\pi}} e^{\frac{i}{2}(x^2+y^2+u^2+v^2)\cot \alpha} e^{i(\alpha - \frac{\pi}{2})} \sin(csc \alpha \cdot u x) \sin(csc \alpha v y) f(x, y) dx dy \\ & (e^{i\alpha})^{\frac{1}{2}} \{2DCSST(f(x, y))\}(u, v) = (-1) e^{-i(\alpha - \frac{\pi}{2})} \sqrt{\frac{-i csc \alpha}{2\pi}} F_{\alpha}^S \{f(x, y)\}(u, v) \\ & (e^{i\alpha})^{\frac{1}{2}} \{2DCSST(f(x, y))\}(u, v) = e^{-i(\alpha - \frac{\pi}{2})} \sqrt{\frac{-i^5 csc \alpha}{2\pi}} F_{\alpha}^S \{f(x, y)\}(u, v) \end{aligned}$$

### IV. CONCLUSION

In this paper we have extended two-dimensional fractional Cosine transform in the distributional generalized sense and its relationship between fractional Fourier transform and canonical sine and cosine transform

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