

Heat Conduction Problem in a Thick Circular Plate and its Thermal Stresses due to Ramp Type Heating

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ABSTRACT: This paper deal with the study of unsteady state thermal stresses in a thick circular plate subjected to arbitrary heat supply at upper surface of the thick circular plate while the lower surface and circular boundary of the thick circular plate insulated the initially temperature of thick circular plate is kept at zero degree temperature the governing heat conduction equation is solved by using integral transform technique and the result illustrated graphically with the help of special case.

KEYWORDS: Quasi-static, Transient, Thermoelastic problem, Thermal Stresses, Axisymmetric Thermal Stresses.

I. INTRODUCTION

Recently Ruhi et al (2005) [7] did thermoelastic analysis of thick walled finite length cylinders of functionally graded materials and obtained the results for stress, strain and displacement components through the thickness and along the length are presented due to uniform internal pressure and thermal loading. Nowacki (1957) [5] has determined steady-state thermal stresses in circular plate subjected to an axisymmetric temperature distribution on the upper face with zero temperature on the lower face and the circular edge. Roy Choudhary (1972) (1973) [8] and Wankhede (1982) [11] determined Quasi static thermal stresses in thin circular plate. Gogulwar and Deshmukh (2005) [2] determined thermal stresses in thin circular plate with heat sources. Also Tikhe and Deshmukh (2005) [1] studied transient thermoelastic deformation in a thin circular plate, whereas Qian & Batra (2004) [6] studied transient thermoelastic deformation of thick functionally thick plate under lateral loads and obtained the results for radial and axial displacements and temperature change moreover Sharma et al (2004) [9] studied the behavior of thermoelastic thick plate under lateral loads and obtained the results for radial and axial displacements and temperature change have been computed numerically and illustrated graphically for different theories of generalized thermoelasticity. Also Nasser M. EI-Maghray (2004) (2005) [3] solved two-dimensional problems of thick plate with heat sources in generalized thermoelasticity. V.S. Kulkarni, K.C. Deshmukh et al [10] considered a thick annular disc which is subjected to a transient axisymmetric temperature field on the radial and axial directions of the cylindrical coordinate system and determined the expression for temperature, displacement and stress functions due to arbitrary heat flux on the upper and lower surface.

In this paper we consider Quasi-static thermal stresses in thick circular plate due to ramp type heating. Studied by Deshmukh K.C. and Kulkarni V.S. and discuss the thermal stresses. This paper deal with the study of unsteady state thermal stresses in a thick circular plate subjected to asymmetric arbitrary heat supply at upper surface of the thick circular plate while the lower surface ($z = -h/2$) and circular boundary of the thick circular plate insulated the initially temperature of thick circular plate is kept at zero degree temperature the governing heat conduction equation is solved by using integral transform technique and the result in illustrated graphically with the help of special case.

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II. STATEMENT OF THE PROBLEM

Consider a thick circular plate of radius a and thickness h occupying space D defined by $0 \leq r \leq a, -h/2 \leq z \leq +h/2$ the plate is kept at zero initial temperature the circular region $D1 : b \leq r \leq a$ of upper surface ($z = +h/2$) subjected to temperature distribution as follows

$$[T(r, z, t)] = \frac{T_0}{t_0} [H(r-b) - H(r-a)]t \quad \text{for } 0 \leq t \leq t_0$$

$$= T_0 [H(r-b) - H(r-a)] \quad \text{for } t \geq t_0$$

where $H(r-b)$ is the Heaviside function. The lower surface ($z = -h/2$) and the circular edge ($r = a$) are thermally insulated. Assume the circular boundary of the thick circular plate is free from traction under these more realistic prescribed condition the quasi-static thermal stresses in a thick clamped circular plate are required to be determined.

Heat Condition Equation:

The temperature of the plate at time t satisfies the Heat Condition equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k} \frac{\partial T}{\partial t} \tag{1}$$

with the condition

$$T = \frac{T_0}{t_0} f(r)t \quad \text{for } 0 \leq t \leq t_0$$

$$T = f(r)T_0 \quad \text{for } t \geq t_0 \quad \text{at } z = \frac{h}{2} \quad b \leq r \leq a \tag{2}$$

$$\frac{\partial T}{\partial z} = 0 \quad \text{at } z = \frac{-h}{2} \quad b \leq r \leq a \tag{3}$$

$$\frac{\partial T}{\partial r} = 0 \quad \text{at } r = a \quad \frac{-h}{2} \leq z \leq \frac{h}{2} \tag{4}$$

and initial conditions

$$T = 0 \quad \text{at } t = 0 \tag{5}$$

where k is thermal diffusivity of the material of the plate and $f(r) = [H(r-b) - H(r-a)]$

Displacement Potential and Thermal Stresses:

The differential equation and governing the displacement potential function $\phi(r, z, t)$

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = k\tau \tag{6}$$

where k is restraint coefficient and temperature change $\tau = T - T_i$, T_i is the initial temperature the displacement potential function ϕ is known as Goodier's thermoelastic displacement potential the displacement function in the cylindrical coordinate system are represented by Michell's function

$$U_r = \frac{\partial \phi}{\partial r} - \frac{\partial^2 M}{\partial r \partial z} \tag{7}$$

$$U_z = \frac{\partial \phi}{\partial z} + 2(1-\nu)\nabla^2 M - \frac{\partial^2 M}{\partial z^2} \tag{8}$$

the Michell's function M must satisfy

$$\nabla^2 \nabla^2 M = 0 \tag{9}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \tag{10}$$

the Component stresses are represented by thermoelastic displacement potential ϕ an Michell's function M as

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$$\sigma_{rr} = 2G \left[\frac{\partial^2 \phi}{\partial r^2} - k\tau + \frac{\partial}{\partial z} \left(\nu \nabla^2 M \frac{\partial^2 M}{\partial r^2} \right) \right] \quad (11)$$

$$\sigma_{\theta\theta} = 2G \left[\frac{1}{r} \frac{\partial \phi}{\partial r} - k\tau + \frac{\partial}{\partial z} \left(\nu \nabla^2 - \frac{1}{r} \frac{\partial M}{\partial r} \right) \right] \quad (12)$$

$$\sigma_{zz} = 2G \left[\frac{\partial^2 \phi}{\partial r^2} - k\tau + \frac{\partial}{\partial z} \left((2-\nu) \nabla^2 M - \frac{\partial^2 M}{\partial z^2} \right) \right] \quad (13)$$

and

$$\sigma_{rz} = 2G \left[\frac{\partial^2 \phi}{\partial r \partial z} - k\tau + \frac{\partial}{\partial z} \left((1-\nu) \nabla^2 M - \frac{\partial^2 M}{\partial z^2} \right) \right] \quad (14)$$

where G and ν are shear modulus and Poisson's ratio respectively.

For traction free surface stress function.

$$\sigma_{rr} = \sigma_{rz} = 0 \quad \text{at} \quad r = a \quad (15)$$

the equation (1) to (15) constitute mathematical formulation of the problem.

III. SOLUTION OF THE PROBLEM

Taking the Laplace transformation of the equation (1) to (4) w.r.t and using (5) one obtains,

$$\frac{\partial^2 \bar{T}}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{T}}{\partial r} + \frac{\partial^2 \bar{T}}{\partial z^2} = \frac{P}{k} \bar{T} \quad (16)$$

with the condition

$$\bar{T} = \frac{T_0}{t_0} \left(\frac{1 - e^{-pt_0}}{P^2} \right) f(r) \quad \text{at} \quad z = \frac{h}{2} \quad b \leq r \leq a \quad (17)$$

$$\frac{\partial \bar{T}}{\partial z} = 0 \quad \text{at} \quad z = -\frac{h}{2}, \quad b \leq r \leq a \quad (18)$$

and

$$\frac{\partial \bar{T}}{\partial r} = 0 \quad \text{at} \quad r = a \quad -\frac{h}{2} \leq z \leq \frac{h}{2} \quad (19)$$

p is Laplace transform parameter.

Now Assume

$$\bar{T} = \sum_{m=1}^{\infty} A_m J_0(\alpha_m r) \text{Cosh} \left[\beta_m \left(z + \frac{h}{2} \right) \right] \quad (20)$$

where $J_n(x)$ in the Bessel function of the first kind of order n and $\alpha_1 \alpha_2 \dots$ are the roots of the equation

$$J_1(\alpha_n a) = 0 \quad (21)$$

Substitute equation (20) in equation (16) one obtain.

$$\beta_m = \sqrt{\left(\alpha_m^2 + \frac{P}{k} \right)} \quad n = 1, 2, \dots, \infty \quad (22)$$

Now the equation (17) in equation (20)

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$$\frac{T_0}{t_0} \left(\frac{1 - e^{-pt_0}}{P^2} \right) f(r) = \sum_{m=1}^{\infty} A_m J_0(\alpha_m r) \text{Cosh}[\beta_m h]$$

Multiplying the above equation $rJ_0(\alpha_m r)$ on both sides and integration from 0 to a

$$\int_0^a \frac{T_0}{t_0} \left(\frac{1 - e^{-pt_0}}{P^2} \right) f(r) r J_0(\alpha_m r) dr = \int_0^a A_m r J_0^2(\alpha_m r) \text{Cosh}[\beta_m h] dr$$

$$f(r) = H(r - b) - H(r - a) = \begin{cases} 1 & b \leq r \leq a \\ 0 & r > a \end{cases}$$

and

$$\int_0^a r J_0^2(\alpha_m r) dr = \left(\frac{a^2}{2} \right) J_0^2(\alpha_m a)$$

one obtain

$$A_m = \left(\frac{2bT_0}{a^2 t_0} \right) \sum_{m=1}^{\infty} \frac{J_1(\alpha_m b) (1 - e^{-pt_0})}{\alpha_m J_0^2(\alpha_m a) p^2 \text{Cosh}[\beta_m h]} \tag{23}$$

Using equation (22) and (23) in the equation (20) one obtain.

$$\bar{T}(r, z, p) = \left(\frac{-2bT_0}{a^2 t_0} \right) \sum_{m=1}^{\infty} \frac{J_0(\alpha_m r) J_1(\alpha_m b)}{\alpha_m J_0^2(\alpha_m a)} \left[\frac{\text{Cosh} \left[\left(\sqrt{\alpha_m^2 + \frac{P}{k}} \right) \left(z + \frac{h}{2} \right) (1 - e^{-pt_0}) \right]}{P^2 \text{Cosh} \left[\sqrt{\alpha_m^2 + \frac{P}{k}} (h) \right]} \right] \tag{24}$$

Hence the temperature distribution function is given by

$$T = \sum_{n=1}^{\infty} - \left(\frac{2bT_0}{a^2 t_0} \right) \frac{J_0(\alpha_m r) J_1(\alpha_m b)}{\alpha_m J_0^2(\alpha_m a)} \cdot \frac{(2n+1) \text{Cos} \left[(2n+1) \frac{\pi}{2} (z + h) \right]}{h \text{Sin} \left[(2n+1) \frac{\pi}{2} h \right]} \int_0^t [uH(u) - (u - t_0)H(u - t_0)] e^{-k \left[(2n+1)^2 \frac{\pi^2}{2} + \alpha_m^2 \right] u} du \tag{25}$$

Since the initial temperature of the plate $T_i = 0$ the temperature change

$$\tau = t - T_i \quad \Rightarrow \quad \tau = T \tag{26}$$

Michell's function M

Suitable form of M satisfying (10) is given by

$$M = \frac{2bT_0 k}{a^2 t_0} \sum_{m=1}^{\infty} \frac{J_0(\alpha_m r) J_1(\alpha_m b)}{\alpha_m J_0(\alpha_m a)} \left[H_{mn} \text{Sinh} \left[\alpha_m \left(z + \frac{h}{2} \right) \right] + R_{mn} \alpha_m \left(z + \frac{h}{2} \right) \text{Cosh} \left[\alpha_m \left(z + \frac{h}{2} \right) \right] \right] \tag{27}$$

H_{mn} and R_{mn} are arbitrary constants

Goodiers Thermoelastic Displacement Function ϕ

Assuming displacement function $\phi(r, z, t)$ which satisfies (6) as

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$$\phi(r, z, t) = \left(\frac{2kbT_0}{a^2t_0} \right) \sum_{n=1}^{\infty} \left(\frac{J_0(\alpha_m r) J_1(\alpha_m b)}{\alpha_m J_0(\alpha_m a)} \right) \left[\frac{\cos \left[(2n+1) \frac{\pi}{2} \left(z + \frac{h}{2} \right) \right]}{\left(\alpha_m^2 + \frac{(2n+1)^2 \pi^2}{4} \right)} \right] \frac{(2n+1)\pi}{h \sin \left[(2n+1) \frac{\pi}{2} h \right]} \int_0^t [uH(u) - (u-t_0)H(u-t_0)] e^{-k \left[\frac{(2n+1)^2 \pi^2}{4} + \alpha_m^2 \right] (t-u)} du \quad (28)$$

Displacement and Thermal Stresses

Using the equation (25), (26), (27) and (28) in the equation (7), (8) and as (11) to (14) one obtain the expression for displacement and stresses respectively.

$$U_r = \left(\frac{2kbT_0}{a^2t_0} \right) \sum_{n=1}^{\infty} \frac{J_1(\alpha_m r) J_1(\alpha_m b)}{\alpha_m J_0^2(\alpha_m a)} \left\{ \frac{(-\alpha_m) \cos \left[(2n+1) \left(z + \frac{h}{2} \right) \frac{\pi}{2} \right] (2n+1)\pi}{\left(\alpha_m^2 + \frac{(2n+1)^2 \pi^2}{4} \right) h \sin \left[(2n+1) \frac{\pi}{2} h \right]} \int_0^t [uH(u) - (u-t_0)H(u-t_0)] e^{-k \left((2n+1)^2 \frac{\pi^2}{4} + \alpha_m^2 \right) (d-u)} + \alpha_m H_{mn} \cosh \left[\alpha_m \left(z + \frac{h}{2} \right) \right] + \alpha_m R_{mn} \left\langle \cosh \left[\alpha_m \left(z + \frac{h}{2} \right) \right] + \alpha_m \left(z + \frac{h}{2} \right) \sinh \left[\alpha_m \left(z + \frac{h}{2} \right) \right] \right\rangle \right\} \quad (29)$$

$$U_z = \left(\frac{2kbT_0}{a^2t_0} \right) \sum_{n=1}^{\infty} \left(\frac{J_1(\alpha_m r) J_1(\alpha_m r_0)}{\alpha_m J_0^2(\alpha_m a)} \right) \left\{ \frac{-\sin \left[(2n+1) \frac{\pi}{2} \left(z + \frac{h}{2} \right) \right] (2n+1)^2 \frac{\pi^2}{2}}{\left(\alpha_m^2 + \frac{(2n+1)^2 \pi^2}{4} \right) h \sin \left[(2n+1) \frac{\pi}{2} h \right]} \int_0^t [uH(u) - (u-t_0)H(u-t_0)] e^{-k \left((2n+1)^2 \frac{\pi^2}{4} + \alpha_m^2 \right) (t-u)} du - \alpha_m^2 H_{mn} \sinh \left[\alpha_m \left(z + \frac{h}{2} \right) \right] + \alpha_m^2 R_{mn} \left\langle 2(1-2\nu) \sinh \left[\alpha_m \left(z + \frac{h}{2} \right) \right] - \alpha_m \left(z + \frac{h}{2} \right) \cosh \left[\alpha_m \left(z + \frac{h}{2} \right) \right] \right\rangle \right\} \quad (30)$$

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$$\begin{aligned}
 \sigma_{rr} = & \left(\frac{4GkbT_0}{a^2 t_0} \right) \sum_{m=1}^{\infty} \left(\frac{J_1(\alpha_m b)}{\alpha_m J_0^2(\alpha_m a)} \right) \cdot \left\{ \left(-\alpha_m J_0(\alpha_m r) + \frac{J_1(\alpha_m r)}{r} \right) \right. \\
 & \left. \left[\frac{\cos \left[(2n+1) \frac{\pi}{2} \left(z + \frac{h}{2} \right) \right]}{\left(\alpha_m^2 \frac{(2n+1)^2 \pi^2}{4} \right)} \right] \cdot \frac{(2n+1)\pi}{h \sin \left[(2n+1) \frac{\pi}{2} h \right]} \right. \\
 & \left. \cdot \int_0^t [u H(u) - (u-t_0) H(u)] \cdot e^{-k \left((2n+1)^2 \frac{\pi^2}{4} + \alpha_m^2 \right) (t-u)} du \right\} + \\
 & J_0(\alpha_m r) \frac{\cos \left[(2n+1) \frac{\pi}{2} (z+h) \right] (2n+1)\pi}{h \sin \left[(2n+1) \frac{\pi}{2} h \right]} \cdot \\
 & \int_0^t [u H(u) - (u-t_0) H(u-t_0)] \cdot e^{-k \left((2n+1)^2 \frac{\pi^2}{4} + \alpha_m^2 \right) (t-u)} du \\
 & + \alpha_m^2 H_{mn} \left(\alpha_m J_0(\alpha_m r) - \frac{J_1(\alpha_m r)}{r} \right) \cosh \left[\alpha_m \left(z + \frac{h}{2} \right) \right] + \\
 & \alpha_m^2 R_{mn} \left(-\alpha_m J_0(\alpha_m r) + \frac{J_1(\alpha_m r)}{r} \right) \\
 & \left(\cosh \left[\alpha_m \left(z + \frac{h}{2} \right) \right] + \alpha_m \left(z + \frac{h}{z} \right) \sinh \left[\alpha_m \left(z + \frac{h}{2} \right) \right] + \right. \\
 & \left. 2v J_0(\alpha_m r) \alpha_m \cosh \left[\alpha_m \left(z + \frac{h}{2} \right) \right] \right) \left. \right\} \tag{31}
 \end{aligned}$$

$$\begin{aligned}
 \sigma_{\theta\theta} = & \left(\frac{4GkbT_0}{a^2 t_0} \right) \sum_{m=1}^{\infty} \left(\frac{J_1(\alpha_m b)}{\alpha_m J_0^2(\alpha_m a)} \right) \cdot \\
 & \left\{ \left(\frac{-J_1(\alpha_m r)}{r} \frac{\cos \left[(2n+1) \frac{\pi}{2} \left(z + \frac{h}{2} \right) \right] (2n+1)\pi}{\left(\alpha_m^2 + \frac{(2n+1)^2 \pi^2}{4} \right) h \sin \left[(2n+1) \frac{\pi}{2} h \right]} \right) \right. \\
 & \left. \int_0^t [u H(u) - (u-t_0) H(u-t_0)] \cdot e^{-k \left((2n+1)^2 \frac{\pi^2}{4} + \alpha_m^2 \right) (t-u)} du \right\} + \\
 & J_0(\alpha_m r) \frac{\cos \left[(2n+1) \frac{\pi}{2} \left(z + \frac{h}{2} \right) \right] (2n+1)\pi}{h \sin \left[(2n+1) \frac{\pi}{2} h \right]} \cdot \\
 & \int_0^t [u H(u) - (u-t_0) H(u-t_0)] \cdot e^{-k \left((2n+1)^2 \frac{\pi^2}{4} + \alpha_m^2 \right) (t-u)} du
 \end{aligned}$$

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$$\begin{aligned}
 & +\alpha_m^2 H_{mn} \left(\frac{J_1(\alpha_m r)}{r} \right) \text{Cosh} \left[\alpha_m \left(z + \frac{h}{2} \right) \right] + \\
 & \alpha_m^2 R_{mn} \left\langle 2v\alpha_m J_0(\alpha_m r) \text{Cosh} \left[\alpha_m \left(z + \frac{h}{2} \right) \right] + \right. \\
 & \left. \frac{J_1(\alpha_m r)}{r} \text{Cosh} \left[\alpha_m \left(z + \frac{h}{2} \right) \right] + \frac{\alpha_m J_1(\alpha_m r)}{r} \left(z + \frac{h}{2} \right) \text{Sinh} \left[\alpha_m \left(z + \frac{h}{2} \right) \right] \right\rangle \quad (32)
 \end{aligned}$$

$$\begin{aligned}
 \sigma_{zz} = & \left(\frac{4GkbT_0}{a^2 t_0} \right) \sum_{n=1}^{\infty} \left(\frac{J_0(\alpha_m r) J_1(\alpha_m b)}{\alpha_m J_0^2(\alpha_m a)} \right) \\
 & \left\{ \frac{-\text{Cosh} \left[(2n+1) \frac{\pi}{2} \left(z + \frac{h}{2} \right) \right] (2n+1)^3 \pi^3}{\left(\alpha_m^2 + \frac{(2n+1)^2 \pi^2}{4} \right) h \text{Sin} \left[(2n+1) \frac{\pi}{2} h \right]} \right. \\
 & \int_0^t \left[u H(u) - (u-t_0) H(u-t_0) \right] \cdot e^{-k \left((2n+1)^2 \frac{\pi^2}{2} + \alpha_m^2 \right) (t-u)} du + \\
 & \frac{\text{Cos} \left[(2n+1) \frac{\pi}{2} \left(z + \frac{h}{2} \right) \right] (2n+1) \pi}{h \text{Sinh} \left[(2n+1) \frac{\pi}{2} h \right]} \cdot \\
 & \int_0^t \left[u H(u) - (u-t_0) H(u-t_0) \right] \cdot e^{-k \left((2n+1)^2 \frac{\pi^2}{2} + \alpha_m^2 \right) (t-u)} du \\
 & -\alpha_m^3 H_{mn} \text{Cosh} \left[\alpha_m \left(z + \frac{h}{2} \right) \right] + \alpha_m^3 R_{mn} \\
 & \left. \left\langle (1-2v) \text{Cosh} \left[\alpha_m \left(z + \frac{h}{2} \right) \right] - \alpha_m \left(z + \frac{h}{2} \right) \text{Sinh} \left[\alpha_m \left(z + \frac{h}{2} \right) \right] \right\rangle \right\} \quad (33)
 \end{aligned}$$

$$\begin{aligned}
 \sigma_{rz} = & \left(\frac{4GkbT_0}{a^2 t_0} \right) \sum_{n=1}^{\infty} \left(\frac{J_1(\alpha_m r) J_1(\alpha_m b)}{\alpha_m J_0^2(\alpha_m a)} \right) \\
 & \left\{ \frac{(\alpha_m) \text{Sin} \left[(2n+1) \frac{\pi}{2} \left(z + \frac{h}{2} \right) \right] (2n+1) \pi}{\left(\alpha_m^2 + \frac{(2n+1)^2 \pi^2}{4} \right) h \text{Sin} \left[(2n+1) \frac{\pi}{2} h \right]} \right. \\
 & \int_0^t \left[u H(u) - (u-t_0) H(u-t_0) \right] \cdot e^{-k \left(\alpha_m^2 + \frac{(2n+1)^2 \pi^2}{4} \right) (t-u)} du + \\
 & \alpha_m^3 H_{mn} \text{Sinh} \left[\alpha_m \left(z + \frac{h}{2} \right) \right] + \\
 & \left. \alpha_m^3 R_{mn} \left\langle (4-2v) \text{Sinh} \left[\alpha_m \left(z + \frac{h}{2} \right) \right] - \alpha_m \left(z + \frac{h}{2} \right) \text{Cosh} \left[\alpha_m \left(z + \frac{h}{2} \right) \right] \right\rangle \right\} \quad (34)
 \end{aligned}$$

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Determination of Unknown Arbitrary function H_{mn} and R_{mn}

In order to satisfy the condition (15) and solving the equation (31) and (34) one obtain

$$H_{mn} = \frac{1}{\alpha_m^3 \text{Cosh} \left[\alpha_m \left(z + \frac{h}{2} \right) \right]} \left[\frac{\text{Cos} \left[(2n+1) \frac{\pi}{2} \left(z + \frac{h}{2} \right) \right] (2n+1)\pi}{h \text{Sin} \left[(2n+1) \frac{\pi}{2} h \right] \left[\alpha_m^2 + \frac{(2n+1)^2 \pi^2}{4} \right]} - \frac{\text{Cos} (2n+1) \frac{\pi}{2} \left(z + \frac{h}{2} \right)}{h \text{Sin} \left[(2n+1) \frac{\pi}{2} h \right]} \right] \int_0^t [u H(u) - (u-t_0) H(u-t_0)] \cdot e^{-k \left((2n+1)^2 \frac{\pi^2}{4} + \alpha_m^2 \right) u} du \quad (35)$$

$$R_{mn} = 0 \quad (36)$$

IV. NUMERICAL RESULTS, DISCUSSION AND REMARKS

The numerical calculation have been carried out for steel (SN 50C) plate with parameters $a = 1m$, $b=a/2=0.5$, $h=1m$, $z=h/2$ Thermal diffusivity $k=15.9 \times 10^{-6} (m^2 s^{-1})$. with $\alpha_1 = 3.8317$, $\alpha_2 = 7.0156$, $\alpha_3 = 10.1735$, $\alpha_4 = 13.3237$, $\alpha_5 = 16.470$, $\alpha_6 = 19.6159$, $\alpha_7 = 22.7601$, $\alpha_8 = 25.9037$, $\alpha_9 = 29.0468$, $\alpha_{10} = 32.18$ are the roots of transcendental equation $J_1(\alpha_n a) = 0$. In order to examine the influence of ramp-type heating on the upper surface on thick circular plate, one performed the numerical $z = -0.25, 0, 0.25, 0.5$ Numerical variations in radial directions are shown in the figures with the help of computer programme.

V. CONCLUSION

In this chapter, a thick circular plate is considered and determined the expressions for temperature, displacement and stress function due to ramp-type heating on the upper surface as a special case mathematical model is constructed for $f(r) = [H(r-0.5) - H(r-1)]$ and performed numerical calculations. The thermoelastic behavior is examined such as temperature, displacement and stresses.

In figure 1 : For $z = 0$ Temperature is gradually is decrease and increases and then suddenly drop till $r = 0.7$ and again increase till $r = 0.8$ after than maintain same status till 1.0. For $z = 0.25$ Temperature is negative and gradually increases from $r = 0$ to $r = 0.65$ after that it again increase and has a value positive from $r = 0.65$ to 1.0. For $z = 0.5$ Temperature is negative from $r = 0$ to $r = 0.65$ with damping nature. After that it will become positive damping status till $r = 1.0$.

In figure 2 : Goodier potential function ϕ gradually increases from $r = 0$ to $r = 1$. For $z = 0.25$ to 0.5. The displacement function is ϕ the gradually decreases increases in radius of the thick circular plate.

In figure 3 : For $z = 0$ the radial displacement U_r decreases from $r = 0$ to $r = 0.5$ and then it increase till $r = 1$. For $z = 0.25$ to $z = 0.5$. The radial displacement U_r gradually increase from $r = 0$ to $r = 0.5$ and the decreases to $r = 1$. The radial displacement U_r has maximum value at $r = 0.5$.

In figure 4 : For $z = 0$ the axial displacement U_z is constant throughout the circular plate. The axial displacement U_z is increases from $r = 0$ to $r = 0.3$.

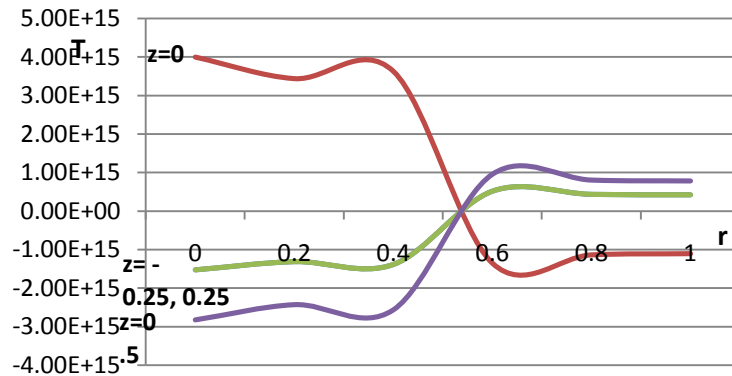
In figure 5 : For $z = 0$ the stress function σ_{rr} is decreases from $r = 0$ to $r = 1$. For $z = 0.25$ to $z = 0.5$ the stress function σ_{rr} is increases from $r = 0$ to $r = 1$.

In figure 6 : For $z = 0$ the stress function $\sigma_{\theta\theta}$ gradually increases from $r = 0$ to $r = 1$.

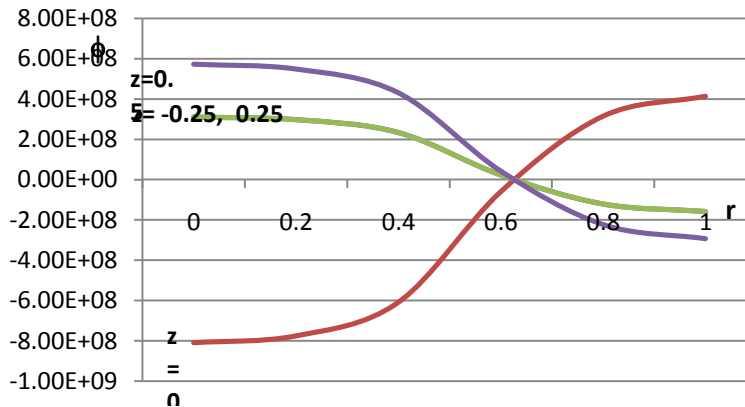
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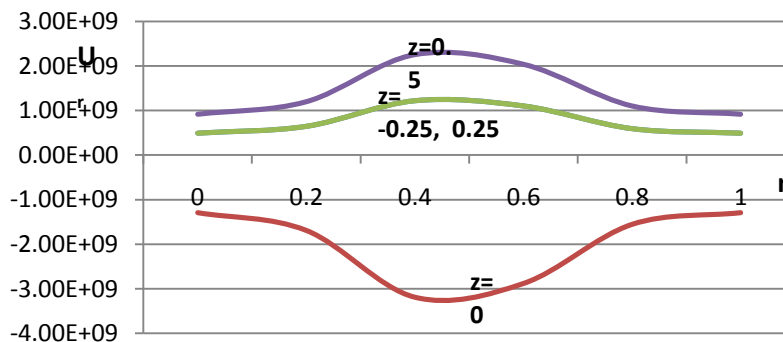
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In Figure 1 Temperature T versus r for different values of z at $z = -0.25, 0 \dots 0.5$ (Steel SN50C). Temperature of the circular plate varies with the radius and thickness of the circular plate.



In Figure 2 Displacement function ϕ versus r for different values of z at $z = -0.25, 0 \dots 0.5$ (Steel SN50C). The displacement function ϕ has the maximum displacement at the top of the circular plate. It is vary with the thickness and radius of the circular plate.

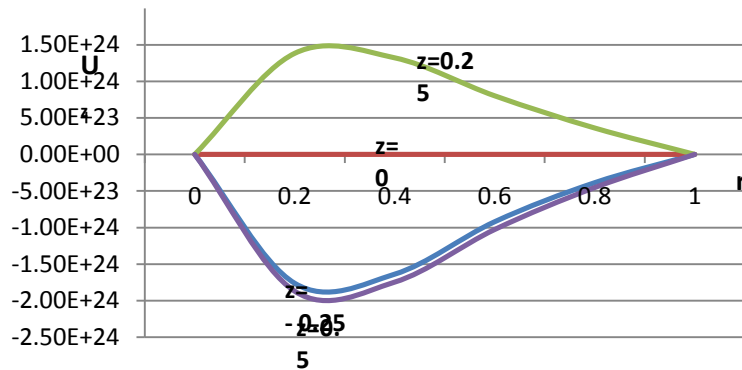


In Figure 3 Radial Displacement U_r versus r for different values of z at $z = -0.25, 0 \dots 0.5$ (Steel SN50C). The radial displacement is directly varies with the thickness and radius of the circular plate.

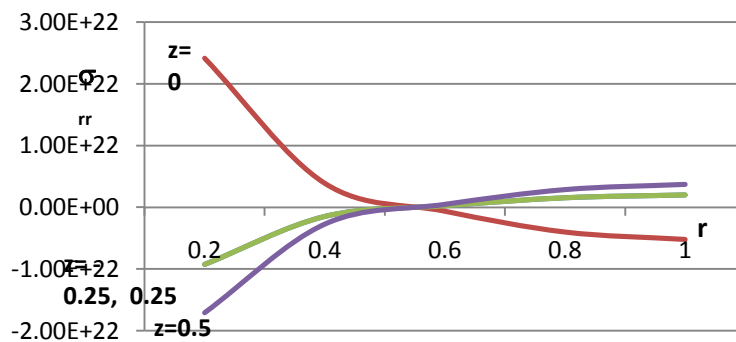
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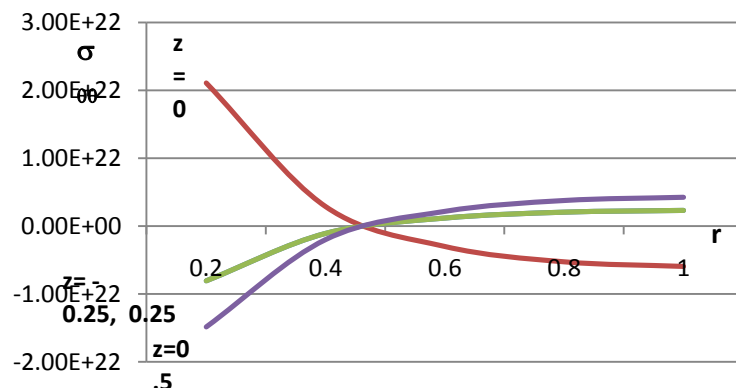
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In Figure 4 Axial Displacement U_z versus r for different values of z at $z = -0.25, 0 \dots 0.5$ (Steel SN50C). The axial displacement is vary with the thickness and radius of circular plate and has constant value at $r = 0$ as well as $r = 1$.



In Figure 5 Stress Function σ_{rr} versus r for different values of z at $z = -0.25, 0 \dots 0.5$ (Steel SN50C) ($r=0.2, 0.4 \dots 1$). The stress function σ_{rr} is decrease at zero with the thickness whereas at $r = 1$ it will increase with thickness of the circular plate.



In Figure 6 Stress Function $\sigma_{\theta\theta}$ versus r for different values of z at $z = -0.25, 0 \dots 0.5$ (Steel SN50C). The stress function $\sigma_{\theta\theta}$ decreases at the centre of the circular plate where as it will increase with radius and thickness of the circular plate.

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