

The Comparison of Domination Number, Bondage Number and the Average Distance of Circular-Arc Graphs

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ABSTRACT: Among the various applications of the theory of domination and the distance, the most often discussed is a communication network. This network consists of communication links all distance between affixed set of sites. Circular-arc graphs are rich in combinatorial structures and have found applications in several disciplines such as Biology, Ecology, Psychology, Traffic control, Genetics, Computer sciences and particularly useful in cyclic scheduling and computer storage allocation problems etc. The problem is to select the smallest set of sites at which transmitters are placed so that every site in the network that does not have a transmitters is joined by a direct communication link to the site, which has a transmitter then this problem reduces to that of finding a minimum dominating set in the graph corresponding to this network. Suppose communication network does not work due to link failure. Then the problem is what is the fewest number of communication links such that at least one additional transmitter would be required in order that communication with all sites as possible. This leads to the introducing of the concept of the bondage number and the average distance. In this paper the comparison of the bondage number and the average distance of circular - arc graph.

KEY WORDS: Circular-arc family, Circular-arc graph, dominating set, Bondage Number ,Average Distance.

I. INTRODUCTION

A graph $G=(V,E)$ is called a circular-arc graph or an intersecting graph for a finite family A of a non empty set if there is a one to one correspondence between A and B . Such that two sets in A have non empty intersection if and only if there corresponding vertices in V are adjacent to each other. We call A an intersection model of G for an intersection model A we use $G(A)$ to denote the intersection graph for G . If A is a family of arcs on a circle, then G is called circular-arc graph for A and A is called a circular-arc model of G [2]. Circular-arc graphs have many applications in different fields such as genetic research, traffic control, computer compiler design etc.

Let $A= \{A_1, A_2, A_3, \dots, A_n\}$ be a family of n arcs on a circle C . Each end point of the arcs in assigned a positive integer called a coordinate. The end point of each arc are located on the circumference are C in the ascending order of the values of the coordinates in the clock wise direction. For convenience, each arcs are $A_i, i=1,2,3, \dots, n$ is represented as (h_i, t_i) . Where h_i is the head point and t_i is the tail point respectively that starting and ending point_s of the arc when it is traversed in counter clock wise manner, starting with an arbitrary chosen point on C which is not an end of any arc in A . Without loss of generality, we will assume that the following condition

- No two arcs share a common end point.
- No single arc in A covers the entire circle C by itself otherwise the shortest path result becomes trivial and in this case the distance between any two arcs in either 1 or 2 unit.
- $\bigcup_{i=1}^n C_i = C$, otherwise the result becomes one on trivial graph.
- The end points of the arcs in A are already given and sorted, according to the order in which they are visited during clockwise traversal along C by starting at arc a_1 .

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- e) The arcs are sorted in increasing values of h_i 's that is $h_i > h_j$ for $i > j$.
- f) The family of arcs A is said to be canonical if h_i 's and t_i 's for $i = 1, 2, \dots, n$ are all distinct integers between 1 and $2n$ and the point 1 is the head of the arc a_1 .

And again alternatively, the circular arc graph can be defined as follows an undirected graph $G = (V, E)$ is a circular arc graph if and only if its vertices circularly indexed as V_1, V_2, \dots, V_n and $(V_i, V_j) \in E$ provided a_i and a_j intersect with each other, where V_i and V_j are the vertices in the graph G corresponding to the arcs a_i and a_j in A respectively.

A subset D of v is said to be a dominating set of G if every vertex not in D is adjacent to vertex in D . The domination number $\gamma(G)$ of a graph G is the minimum cardinality of a dominating set in G [1]. The bondage number $b(G)$ of a non empty graph G is the minimum cardinality[5,6] among all sets of edges E_1 for which $\gamma(G-E_1) > \gamma(G)$.

In this section we discuss about the computation of average distance of circular arc graph G . The average distance $\mu(G)$ of a connected circular arc graph is defined to be the average of all distances in G [3,4].

$$\mu(G) = \frac{1}{2^n C_2} \sum_{\substack{A_i, A_j \in v(G) \\ A_i \neq A_j}} \delta(A_i, A_j).$$

Where $\delta(x, y)$ denotes the length of shortest path joining the vertices x and y . The average distance can be used as a tool in analytic networks where the performance time is proportions to the distance between any two nodes. It is a measure of the time needed in the average case as opposed to the diameter, which indicates the maximum performance time. And also the formulated of the walk, length of a walk, eccentricity, radius and diameter of the graph.

A walk from v_0 to v_n is an alternating sequence $W = \{v_0, e_1, v_1, e_2, \dots, v_{n-1}, e_n, v_n\}$ of vertices and edges such that end points $(e_i) = \{v_{i-1}, v_i\}$; for $i = \{1, 2, \dots, n\}$.

In a simple graph, there is only one edge between two consecutive vertices of a walk, so one could abbreviate the walk as $W = \{v_0, e_1, v_1, e_2, \dots, v_{n-1}, e_n, v_n\}$.

The length of a walk or directed walk is the number of edge steps in the walk sequence. A walk of length zero, i.e with one vertex and no edges is called a trivial walk.

II. MAIN THEOREMS

Theorem.1: Let $A = \{A_1, A_2, \dots, A_n\}$ be a circular arc family and let G be a circular arc graph corresponding to circular arc family without isolated vertices of G . Then the domination number $\gamma(G)$, the average distance $\mu(G)$ and the bondage number $b(G)$ is

$$\mu(G) \geq \gamma(G) \geq b(G)$$

Proof: We consider the circular arc graph corresponding to circular arc family, In this it will arise three cases.

i. the domination number $\gamma(G)$

ii. The bondage number $b(G) = \gamma(G-e) > \gamma(G)$

iii. The average distance of G , $\mu(G) = \frac{1}{2^n C_2} \sum_{\substack{A_i, A_j \in v(G) \\ A_i \neq A_j}} \delta(A_i, A_j).$

Case i: Let $\gamma(G)$ be a minimal domination number of G . Suppose $V - \gamma(G)$ is not a domination set. Then there exist a vertex u such that u is not dominated by any vertex in $V - \gamma(G)$. Since G has no isolated vertices, u is dominated by at least one vertex $\gamma(G - e) - \{u\}$ thus $\gamma(G) - \{u\}$ is a domination number, which is contradicts the minimality of $\gamma(G)$. Thus every vertex in $\gamma(G)$ is adjacent with at least one vertex in $V - \gamma(G)$. Thus $V - \gamma(G)$ is a domination number. In this domination number we proved in circular-arc graph G .

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Case ii: Let $A = \{A_1, A_2, \dots, A_n\}$ be a circular arc family and let G be a circular arc graph corresponding to circular arc family A . Let (A_i, A_j) be any two arcs in A , which satisfy the hypothesis of the theorem. Then clearly $A_i \in \gamma(G)$. Where $\gamma(G)$ is a minimal domination number of a circular arc graph G because there is no other arc in A other than A_i that dominates A_j . Consider the edge $e = (A_i, A_j)$ in G . If we remove this edge from a circular arc graph G . Then A_j becomes an isolated in circular arc graph $G-e$, as there is no other vertex in G other than A_i . That is adjacent with A_j .

Hence $\gamma_1(G) = \gamma(G) \cup (A_j)$ becomes a domination number of a circular arc graph $G-e$ and since $\gamma(G)$ is a minimum dominating set it follows that $\gamma_1(G)$ is also a minimum domination number of circular arc graph $G-e$.

Therefore we get $|\gamma_1(G)| = \gamma(G-e) = |\gamma(G)| + 1 > |\gamma_1(G)|$

Thus the bondage number $b(G) = 1$. As follows circular-arc graph G .

Case iii: Now we will find the average distance $\mu(G)$ of a circular arc graph G , where $\mu(G)$ is a average distance of G , n is the number of arcs and $\delta(A_i, A_j)$ is the distance of the circular arc G . In this fact first we will discuss the distance of circular arc graph G for any two vertices (A_i, A_j) in a circular arc graph G , the distance from A_i to A_j is denoted by $\delta(A_i, A_j)$ and defined as the length of a shortest $A_i - A_j$ path in circular-arc graph G .

For connected a circular arc graph G the term distance we just defined satisfies all four of the following axioms,

1. $d(A_i, A_j) \geq 0$, for all $A_i, A_j \in V(G)$.
2. $d(A_i, A_j) = 0$, if and only if $A_i = A_j$.
3. $d(A_i, A_j) = d(A_j, A_i)$, for all $A_i, A_j \in V(G)$.
4. $d(A_i, A_k) \leq d(A_i, A_j) + d(A_j, A_k)$, for all $A_i, A_j, A_k \in V(G)$.

If graph G is not a connected and suppose G_1 and G_2 are two components of G , is denoted by $W(G)$.

Which are $W(G) = W(G_1) \cup W(G_2)$ and $E(W(G_1)) \cup E(W(G_2))$

And $W(G_1) \cap W(G_2) = \emptyset$

That is $V(W(G_1)) \cap V(W(G_2)) = \emptyset$ and also $E(W(G_1)) \cap V(W(G_2)) = \emptyset$

Then $d(A_i, A_j) = \infty$ for $A_i \in V(W(G_1))$ & $V \in V(W(G_2))$

Under this distance function the set $V(G)$ is a metric space. In this fact the circular-arc graph corresponding to circular-arc family $A = \{A_i, A_j, A_k, \dots, A_n\}$, where n is a number of vertices.

In this section we discuss about the computation of average distance of a circular-arc graph G . The average distance $\mu(G)$ of a connected a circular arc graph is defined to be the average of distance in G . Where

$$\mu(G) = \frac{1}{2 \binom{n}{2}} \sum_{\substack{A_i, A_j \in V(G) \\ A_i \neq A_j}} \delta(A_i, A_j).$$

Where $\mu(G) = 2.118$ from the circular-arc graph G .

Thus the theorem is hold.

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III. EXPERIMENTAL PROBLEM OF THEOREM.1

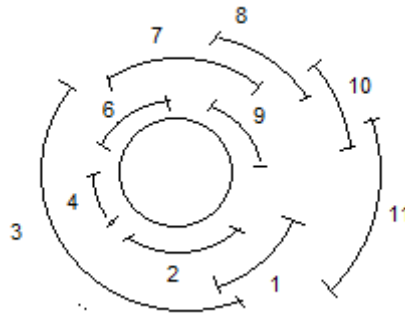


Fig. 1: Circular-arc family A

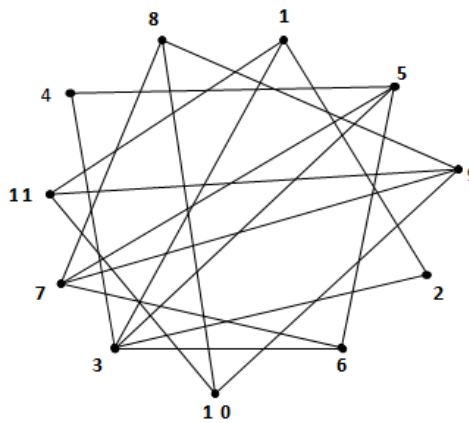


Fig. 2: Circular-arc graph G

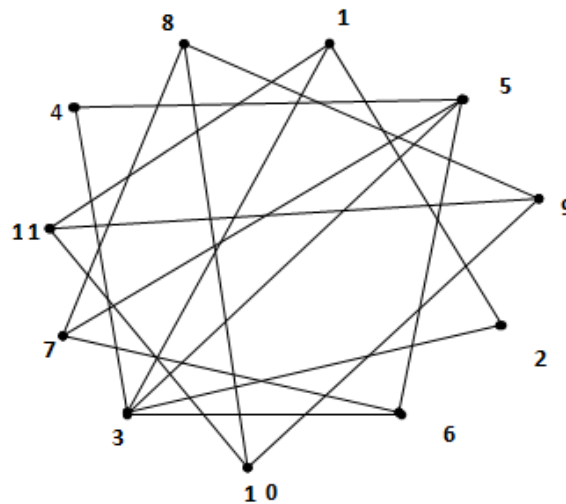


Fig. 3: Circular-arc graph (G-e)

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IV. TO FIND THE DISTANCES FROM G

d(1,1)=0	d(2,1)=1	d(3,1)=1	d(4,1)=2	d(5,1)=2	d(6,1)=2
d(1,2)=1	d(2,2)=0	d(3,2)=1	d(4,2)=2	d(5,2)=2	d(6,2)=2
d(1,3)=1	d(2,3)=1	d(3,3)=0	d(4,3)=1	d(5,3)=1	d(6,3)=1
d(1,4)=2	d(2,4)=2	d(3,4)=1	d(4,4)=0	d(5,4)=2	d(6,4)=2
d(1,5)=2	d(2,5)=2	d(3,5)=1	d(4,5)=2	d(5,5)=0	d(6,5)=1
d(1,6)=2	d(2,6)=2	d(3,6)=1	d(4,6)=2	d(5,6)=1	d(6,6)=0
d(1,7)=3	d(2,7)=3	d(3,7)=2	d(4,7)=3	d(5,7)=1	d(6,7)=1
d(1,8)=3	d(2,8)=4	d(3,8)=3	d(4,8)=4	d(5,8)=2	d(6,8)=2
d(1,9)=2	d(2,9)=3	d(3,9)=3	d(4,9)=4	d(5,9)=2	d(6,9)=2
d(1,10)=2	d(2,10)=3	d(3,10)=3	d(4,10)=4	d(5,10)=3	d(6,10)=3
d(1,11)=1	d(2,11)=2	d(3,11)=2	d(4,11)=3	d(5,11)=3	d(6,11)=3

d(7,1)=3	d(8,1)=3	d(9,1)=2	d(10,1)=2	d(11,1)=1
d(7,2)=3	d(8,2)=4	d(9,2)=3	d(10,2)=3	d(11,2)=2
d(7,3)=2	d(8,3)=3	d(9,3)=3	d(10,3)=3	d(11,3)=2
d(7,4)=3	d(8,4)=4	d(9,4)=4	d(10,4)=4	d(11,4)=3
d(7,5)=1	d(8,5)=2	d(9,5)=2	d(10,5)=3	d(11,5)=3
d(7,6)=1	d(8,6)=3	d(9,6)=2	d(10,6)=3	d(11,6)=3
d(7,7)=0	d(8,7)=1	d(9,7)=1	d(10,7)=2	d(11,7)=2
d(7,8)=1	d(8,8)=0	d(9,8)=1	d(10,8)=1	d(11,8)=2
d(7,9)=1	d(8,9)=1	d(9,9)=0	d(10,9)=1	d(11,9)=1
d(7,10)=2	d(8,10)=1	d(9,10)=1	d(10,10)=0	d(11,10)=1
d(7,11)=3	d(8,11)=2	d(9,11)=2	d(10,11)=1	d(11,11)=0

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V.TO FIND THE AVERAGE DISTANCE OF G

vertices	1	2	3	4	5	6	7	8	9	10	11
1	0	1	1	2	2	2	3	3	2	2	1
2	1	0	1	2	2	2	3	4	3	3	2
3	1	1	0	1	1	1	2	3	3	3	2
4	2	2	1	0	2	2	3	4	4	4	3
5	2	2	1	2	0	1	1	2	2	3	3
6	2	2	1	2	1	0	1	2	2	3	3
7	3	3	2	3	1	1	0	1	1	2	3
8	3	4	3	4	2	3	1	0	1	1	2
9	2	3	3	4	2	2	1	1	0	1	2
10	2	3	3	4	3	3	2	1	1	0	1
11	1	2	2	3	3	3	2	2	1	1	0
Total	19	23	18	27	19	20	19	23	20	23	22

Table-1

Therefore Average distance

$$\mu(G) = \frac{1}{2 \binom{n}{2}} \sum_{\substack{a_i, a_j \in V(G) \\ a_i \neq a_j}} \delta(a_i, a_j).$$

$$\mu(G) = \frac{1}{11 \times 10} \times 233$$

$$\mu(G) = 2.118$$

$$\therefore \mu(G) \geq b(G)$$

Therefore $\mu(G) \geq \gamma(G) \geq b(G)$.

In this graph we have $2.118 \geq 2 \geq 1$.

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VI. THE COMPARISON OF THE DOMINATION NUMBER, AVERAGE DISTANCE AND BONDAGE NUMBER OF $G^1 = G - e$

Theorem.2: Let G be a circular-arc graph corresponding to circular-arc family A without isolated vertices G^1 . Then the comparison of domination number, the bondage number and the average distance is $\gamma(G^1) \geq b(G^1) \geq \mu(G^1)$.

Proof: Let $A = \{A_1, A_2, \dots, A_n\}$ be a circular-arc family and let G be a circular arc graph corresponding to circular-arc family A . Suppose G^1 be a circular arc sub graph of G . Now we have to find the following conditions which are $\gamma(G^1) \geq b(G^1) \geq \mu(G^1)$. First we will prove that $\gamma(G^1) \geq b(G^1)$. Where $\gamma(G^1)$ is the domination number of G^1 and $b(G^1)$ is the bondage number of G^1 . Already we have proved the domination number $\gamma(G)$ and the bondage number $b(G)$ in theorem.1. In theorem.1. we proved $\gamma(G) \geq b(G)$. Again we to prove that the domination number and the average distance of G^1 . Hence $G^1 = G - e$ of G . In this connection the proof is trivially and we need not prove above said. Therefore $\gamma(G^1) \geq b(G^1)$. Again we have to prove that $b(G^1) \geq \mu(G^1)$. Where $b(G)$ is a bondage number of G^1 . We know that the bondage number is always greater than or equal to the average distance of G .

Hence $b(G^1)$ is greater than or equal to $\mu(G^1)$

Which implies that $\gamma(G^1) \geq b(G^1)$. Again which implies that $b(G^1) \geq \mu(G^1)$.

Therefore $\gamma(G^1) \geq b(G^1) \geq \mu(G^1)$.

Therefore the comparison of domination number, the average distance and the bondage number is $\gamma(G^1) \geq b(G^1) \geq \mu(G^1)$.

Hence the theorem is hold.

VII. EXPERIMENTAL PROBLEM OF THEOREM.2

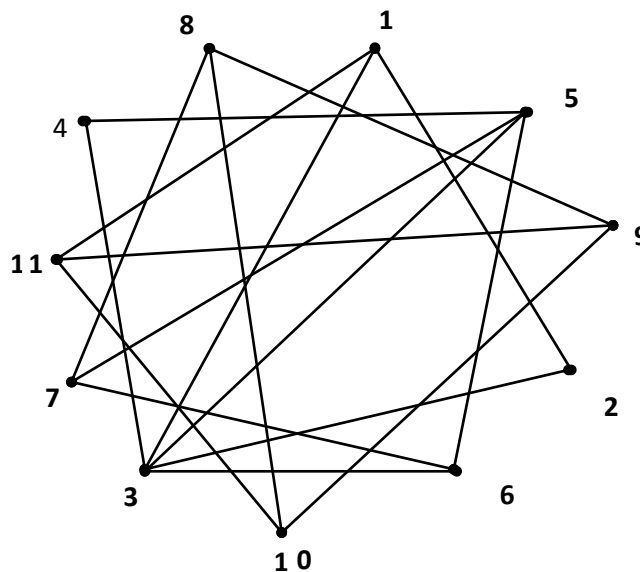


Fig. 4: Circular-arc graph G^1

Dominating set of $G^1 = \{3, 7, 10\}$

Therefore $\gamma(G^1) = 3$

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Remove the three edges from graph G^1-e

$$\text{i.e } G^1-e = G^1 - \{(3,2), (3,5), (4,5)\}$$

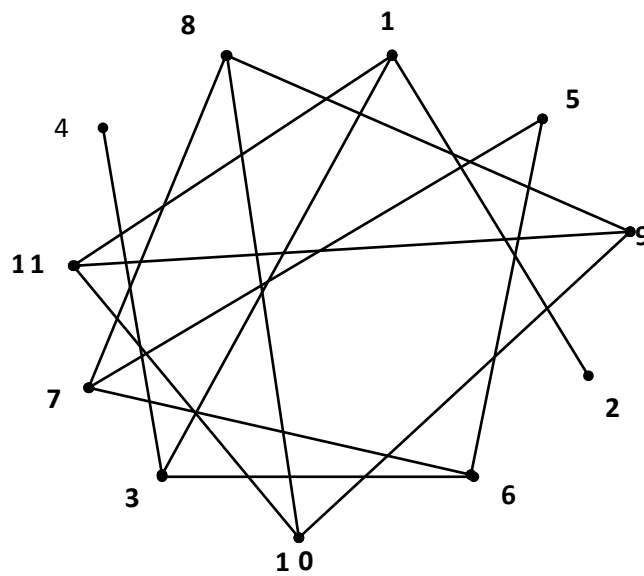


Fig. 5: Circular-arc graph (G^1-e)

Then the dominating set of $G^1-e = \{1,4,5,9\}$

Therefore $\gamma(G^1-e) = 4$

Therefore $\gamma(G^1-e) > \gamma(G^1)$.

The bondage number $b(G) = 3$

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VIII. TO FIND THE DISTANCES FROM G^1

d(1,1)=0	d(2,1)=1	d(3,1)=1	d(4,1)=2	d(5,1)=2	d(6,1)=2
d(1,2)=1	d(2,2)=0	d(3,2)=1	d(4,2)=2	d(5,2)=2	d(6,2)=2
d(1,3)=1	d(2,3)=1	d(3,3)=0	d(4,3)=1	d(5,3)=1	d(6,3)=1
d(1,4)=2	d(2,4)=2	d(3,4)=1	d(4,4)=0	d(5,4)=2	d(6,4)=2
d(1,5)=2	d(2,5)=2	d(3,5)=1	d(4,5)=2	d(5,5)=0	d(6,5)=2
d(1,6)=2	d(2,6)=2	d(3,6)=1	d(4,6)=2	d(5,6)=1	d(6,6)=0
d(1,7)=3	d(2,7)=3	d(3,7)=2	d(4,7)=3	d(5,7)=2	d(6,7)=1
d(1,8)=3	d(2,8)=4	d(3,8)=3	d(4,8)=4	d(5,8)=2	d(6,8)=2
d(1,9)=2	d(2,9)=3	d(3,9)=3	d(4,9)=4	d(5,9)=4	d(6,9)=3
d(1,10)=2	d(2,10)=3	d(3,10)=3	d(4,10)=4	d(5,10)=4	d(6,10)=3
d(1,11)=1	d(2,11)=2	d(3,11)=2	d(4,11)=3	d(5,11)=3	d(6,11)=3
d(7,1)=3	d(8,1)=3	d(9,1)=2	d(10,1)=2	d(11,1)=1	
d(7,2)=3	d(8,2)=4	d(9,2)=3	d(10,2)=3	d(11,2)=2	
d(7,3)=2	d(8,3)=3	d(9,3)=3	d(10,3)=3	d(11,3)=2	
d(7,4)=3	d(8,4)=4	d(9,4)=4	d(10,4)=4	d(11,4)=3	
d(7,5)=1	d(8,5)=2	d(9,5)=3	d(10,5)=3	d(11,5)=3	
d(7,6)=1	d(8,6)=2	d(9,6)=3	d(10,6)=3	d(11,6)=3	
d(7,7)=0	d(8,7)=1	d(9,7)=2	d(10,7)=2	d(11,7)=3	
d(7,8)=1	d(8,8)=0	d(9,8)=1	d(10,8)=1	d(11,8)=2	
d(7,9)=2	d(8,9)=1	d(9,9)=0	d(10,9)=1	d(11,9)=1	
d(7,10)=2	d(8,10)=1	d(9,10)=1	d(10,10)=0	d(11,10)=1	
d(7,11)=3	d(8,11)=2	d(9,11)=1	d(10,11)=1	d(11,11)=0	

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IX. TO FIND THE AVERAGE DISTANCE OF G^1

vertices	1	2	3	4	5	6	7	8	9	10	11
1	0	1	1	2	2	2	3	3	2	2	1
2	1	0	1	2	2	2	3	4	3	3	2
3	1	1	0	1	1	1	2	3	3	3	2
4	2	2	1	0	2	2	3	4	4	4	3
5	2	2	1	2	0	1	2	2	4	4	3
6	2	2	1	2	2	0	1	2	3	3	3
7	3	3	2	3	1	1	0	1	2	2	3
8	3	4	3	4	2	2	1	0	1	1	2
9	2	3	3	4	3	3	2	1	0	1	1
10	2	3	3	4	3	3	2	1	1	0	1
11	1	2	2	3	3	3	3	2	1	1	0
Total	19	23	18	27	21	20	22	23	24	24	21

Table-2

Therefore Average distance

$$\mu(G - e) = \frac{1}{2^n \binom{n}{2}} \sum_{\substack{a_i, a_j \in V(G-e) \\ a_i \neq a_j}} \delta(a_i, a_j).$$

$$\mu(G) = \frac{1}{11 \times 10} \times 242$$

$$\mu(G) = 2.2$$

Therefore $\gamma(G^1) \geq b(G^1) \geq \mu(G^1)$.

REFERENCES

- [1] Grandoni, F., A note on the complexity of minimum dominating set, Journal of discrete algorithms, Vol. 4, No. 2, p.p-209- 214, 2006.
- [2] Saha, A., Pal, M. and Pal, T.K., An optimal parallel algorithm for solving all-pairs shortest paths problem on circular-arc graphs, Journal of Applied Mathematics and Computing, 17(1+2), 2006.
- [3] Bienstock, D., and Gyon, E., Average distance in graphs with removed elements, J. Combin. Theory ser. B, 48(1990)140-142.
- [4] Anita Saha, Computation of Average distance, radius and centre of a circular-arc graph in parallel, Journal of physical sciences, Physical sciences, Vol. 10, 2006, 178-187.
- [5] J.F. Fink, M.S. Jacobson, L.F. Kinch, J. Roberts, The bondage number of a graph discrete mathematics, Vol. 86(1990), p.p-47- 57.
- [6] Dr. A. Sudhakaraiyah, K. Ramakrishna, T. Venkateswarlu, The comparison of bondage number and the average distance of an interval graph-IJIRSET- Vol. 6 Issue-10, Oct-2015.