

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 1, January 2015

Lucky Edge Labeling of Certain Graphs

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ABSTRACT: In this paper the lucky edge labeling of Ladder graph, Shell graph and Book with Triangular Pages are discussed and the lucky numbers of these graphs are found.

KEYWORDS: Lucky edge labeling, lucky number, Ladder graph, Shell graph and Book with Triangular Pages.

I. INTRODUCTION

All the graphs considered are simple and undirected. For the basic definitions of graph theory refer Douglas B. West [2] and Harary [4]. A graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions [5]. Rosa [1] in the year 1967 introduced the concept of labeling. The concept of labeling has much importance in graph theory as it is being used in various fields such as communication networks, coding theory, astronomy etc. In this paper the lucky edge labeling of Ladder graph, Shell graph and Book with Triangular Pages are discussed and the lucky numbers are found for these graphs. [2]

First, some basic definitions for the study of this paper are discussed in this section. [12]

Definition 1.1

Let G be a Simple Graph with Vertex set $V(G)$ and Edge set $E(G)$ respectively. Vertex set $V(G)$ are labeled arbitrary by positive integers and let $E(e)$ denote the edge label such that it is the sum of labels of vertices incident with edge e [13]. The labeling is said to be lucky edge labeling if the edge set $E(G)$ is a proper coloring of G , that is, if we have $E(e_1) \neq E(e_2)$ whenever e_1 and e_2 are adjacent edges [6]. The least integer k for which a graph G has a lucky edge labeling from the set $\{1, 2, \dots, k\}$ is the lucky number of G denoted by $\eta(G)$. A graph which admits lucky edge labeling is the lucky edge labeled graph. [3]

Definition 1.2

A ladder graph L_n [7] of order n is a planar undirected graph with $2n$ vertices and $n+2(n-1)$ edges.

Definition 1.3

The shell S_n is the graph obtained by taking $n-3$ concurrent chords in cycle C_n . The vertex at which all the chords are concurrent is called the apex vertex. The shell is also called fan f_{n-1} . Thus $S_n = f_{n-1} = P_{n-1} + K_1$ [6].

Definition 1.4

A triangular book is the complete tripartite graph $K_{1,1,p}$ [4]. It is a graph consisting of p triangles sharing a common edge.

II. MAIN RESULTS

Theorem 2.1:-

The lucky number of Ladder graph is $\eta(L_n) = 6, n \geq 3$. [8]

Proof:

Let the vertices of the Ladder graph be v_1, v_2, \dots, v_{2n} where v_{2n-1} and v_{2n} , $n = 1, 2, \dots$ are the vertices in each step of the ladder. Label the vertices v_{2n-1} , $n = 1, 2, 3, \dots$ with labels $\{1, 2, 3\}$ in cyclic order

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respectively[10]. And label the vertices v_{2n} , $n = 1,2,3,\dots$ with labels $\{1,2,3\}$ in cyclic order respectively. This induces a lucky edge labeling of L_n with $\eta(L_n) = 6$, $n \geq 3$ [12].

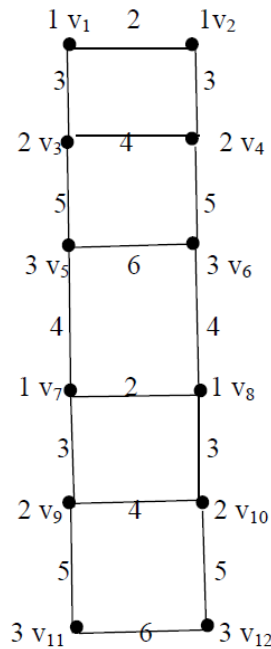


Fig. 1 : $\eta(L_6) = 6$

Special Case:-

Case 1:

The lucky number of L_1 is 2.

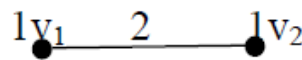


Fig. 2 : $\eta(L_1) = 2$

Case 2:

The lucky number of L_2 is 4.

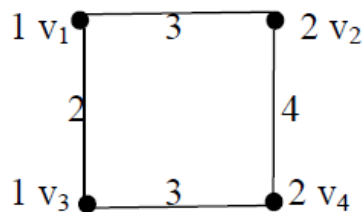


Fig. 3 : $\eta(L_2) = 4$

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Vol. 4, Issue 1, January 2015

Theorem 2.2 :-

The lucky number of shell graph is S_n is $\eta(S_n) = 2n-1$.

Proof:

Let the vertices of the shell graph be v_1, v_2, \dots, v_n and let the apex vertex at which the $n-3$ chords are concurrent be $v_{n[1]}$.

Assign labels to the vertices of the graph S_n such that label of the vertex v_i is i . This induces a lucky edge labeling of S_n with $\eta(S_n) = 2n-1$.

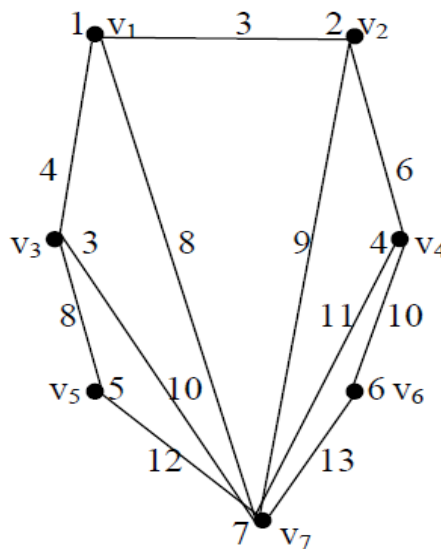


Fig. 4 : $\eta(S_7) = 13$

Theorem 2.3:-

The lucky number of a triangular book $K_{1,1,p}$ is $\eta(K_{1,1,p}) = p+4$.

Proof:

Let the common edge shared by p triangles have vertices u and v . Let the other p vertices of the graph be denoted by w_1, w_2, \dots, w_p . [5] Assign labels to the vertices of the graph $K_{1,1,p}$ as follows:

Label the vertices u and v with labels 1 and 2 respectively and label the vertices w_i with labels $i+2$. This induces a lucky edge labeling of the graph $K_{1,1,p}$. [7] Thus for the graph $K_{1,1,p}$ the lucky number is given by, $\eta(K_{1,1,p}) = p+4$.

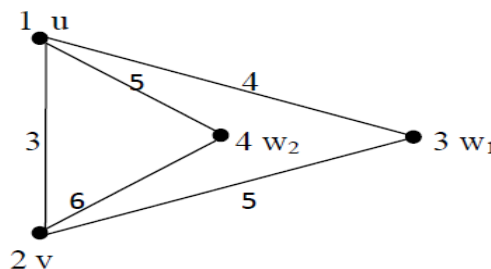


Fig.5 : $\eta(K_{1,1,2}) = 6$

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III. CONCLUSION

In this paper the lucky numbers are found for the Ladder graph, Shell graph and Book with Triangular Pages. With the increasing importance of labeling in graph theory the study can be extended for many other types of graphs and algorithms can also be developed for the same.

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