

# Abel-Type Method of Summability in Non-Archimedean Fields

R.Deepa

Department of Mathematics, Faculty of Science and Humanities, Bharath University, Chennai, Tamil Nadu, India

**ABSTRACT:** The aim of this paper is to introduce Abel-type method of summability in complete, non-trivially valued, non-archimedean fields and prove some interesting properties.

**KEYWORDS:** Non-Archimedean, Abel-type method, Regular[1]

## I. INTRODUCTION

Throughout this paper,  $K$  denotes complete, non-trivially valued, non-archimedean fields. Infinite matrices and sequences considered in the sequel have entries in  $K$ . A general reference for the study of summability methods in classical one is [4]. For analysis, in non-archimedean fields, see [5].

If  $A = (a_{n,k})$ ,  $a_{n,k} \in K$ ,  $n, k = 0, 1, 2, \dots$  is an infinite matrix, then the  $A$ -transform of a sequence  $x = \{x_k\}$ ,  $x_k \in K$ ,  $k = 0, 1, 2, \dots$ , is the sequence, where

$$(Ax)_n = \sum_{k=0}^n a_{nk} x_k, \quad n = 0, 1, 2, \dots, \text{ it being assumed that the series on the right converge.}$$

In the sequel, the following definitions and theorems are needed.

**Definition 1.** Let  $\{x_k\}$  be a sequence in  $K$  and  $x \in K$ . We say that  $\lim_{k \rightarrow \infty} x_k = x$  if for each  $\epsilon > 0$ , the set  $\{|x - x_k| \geq \epsilon\}$  is finite. In such a case, we say that  $x$  is the limit of  $\{x_k\}$ .

**Definition 2.** Let  $A = (a_{n,k})$ ,  $a_{n,k} \in K$ ,  $n, k = 0, 1, 2, \dots$ , if  $\lim_{n \rightarrow \infty} (Ax)_n$  exists whenever  $\lim_{n \rightarrow \infty} x_k$  exists, then

$A$  is said to be convergence preserving [2]. Further, if  $\lim_{n \rightarrow \infty} (Ax)_n = s$

whenever  $\lim_{n \rightarrow \infty} x_k = s$ , we say that  $A$  is regular.

**Theorem 1.**  $A = (a_{n,k})$  is a regular if and only if

(i)  $\sup_{n,k} |a_{n,k}| < \infty$

(ii)  $\lim_{n \rightarrow \infty} a_{n,k} = 0, \quad k = 0, 1, \dots,$

(iii)  $\lim_{n \rightarrow \infty} \sum_{k=0}^{\infty} a_{n,k} = 1$

We introduce Abel-type method of summability in  $K$  and prove some interesting properties [3].

## International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 1, January 2015

**Definition 3.** Let  $q \in K$  be such that  $|q| < 1$ . The Abel-type method of order  $q$  or the  $[S, q]$  method is given by the matrix  $(s_{n,k}^{(q)})$  as follows [1].

If  $q \neq 0$

$$s_{n,k}^{(q)} = \begin{cases} 0 & k > n; \\ (n+k)C_k q^k (1-q)^{n+1} & k \leq n. \end{cases}$$

**Remark 1.** We note that  $q \neq 0$ , since  $|q| < 1$

### II. MAIN RESULT

**Theorem 2.** The  $[s, p]$  method is regular if  $|q| < 1$ .

**Proof .** Let  $\sum_{k=0}^{\infty} s_{n,k}^{(q)} = \sum_{k=0}^n s_{n,k}^{(q)}$

$$\begin{aligned} &= \sum_{k=0}^n (n+k)C_k q^k (1-q)^{n+1} \\ &= \sum_{k=0}^n (n+k)C_k q^k (1-q)^{(n+k)-k} (1-q) \\ &= (1-q) \sum_{k=0}^n (n+k)C_k q^k (1-q)^{(n+k)-k} \\ &= (1-q) \sum_{k=0}^n (n+k)C_k q^k (1-q)^{(n+k)-k}, \\ &= (1-q)(\overline{1-q} + q)^{n+1} \\ &= 1 \text{ for } |1-q| = 1, |q| < 1 \end{aligned}$$

That is  $\lim_{n \rightarrow \infty} \sum_{k=0}^{\infty} s_{n,k}^{(q)} = 1$

Now,  $s_{n,k}^{(q)} = 0$ , if  $k > n$  implies that

$$\lim_{n \rightarrow \infty} s_{n,k}^{(q)} = 0, k = 0, 1, 2, \dots,$$

When  $s_{n,k}^{(q)} \neq 0$ ,

**International Journal of Innovative Research in Science,  
Engineering and Technology**

*(An ISO 3297: 2007 Certified Organization)*

**Vol. 4, Issue 1, January 2015**

$$\begin{aligned} \left|s_{n,k}^{(q)}\right| &= \left|(n+k)C_k q^k (1-q)^{n+1}\right| \\ &\leq \left|(n+k)C_k\right| \left|q^k\right| \left|1-q\right|^{n+1} \\ &\leq \left|(n+k)C_k\right| \left|1-q\right|^{n+1}, \text{ since } |q| < 1 \\ &= \left|n+kC_k\right|, \text{ since } |1-q| = 1 \\ &= \left|n+kC_k\right| \leq 1 \end{aligned}$$

So that  $\sup_{n,k} \left|s_{n,k}^{(q)}\right| < 1$

Thus the  $[S, q]$  method is regular.

**REFERRANCES**

1. Leena Sankari S., Masthan K.M.K., AravindhaBabu N., Bhattacharjee T., Elumalai M., "Apoptosis in cancer - an update", Asian Pacific Journal of Cancer Prevention, ISSN : 1513-7368, 13(10) (2012) PP. 4873-4878.
2. Subhashree A.R., Parameaswari P.J., Shanthi B., Revathy C., Parijatham B.O., "The reference intervals for the haematological parameters in healthy adult population of Chennai, Southern India", Journal of Clinical and Diagnostic Research, ISSN : 0973 - 709X, 6(10) (2012), PP. 1675-1680.
3. Ramaswamy S., Sengottuvelu S., HajaSherief S.H., Jaikumar S., Saravanan R., Prasadkumar C., Sivakumar T., "Gastroprotective activity of ethanolic extract of Trachyspermum fruit", International Journal of Pharma and Bio Sciences, ISSN :0975-6299, 1(1) (2010) Art-no.17.
4. Kumar S.S., Karunakaran C.M., Rao M.R.K., Balasubramanian M.P., "Inhibitory effects of Indigofera aspalathoides on 20-methylcholanthrene-induced chemical carcinogenesis in rats", Journal of Carcinogenesis, ISSN : 1477-3163, 10(1) (2011) Art.no.1.
5. Kumaravel A., Meetei O.N., "An application of non-uniform cellular automata for efficient cryptography", 2013 IEEE Conference on Information and Communication Technologies, ICT 2013, ISSN : 1681-8016, (2013) pp.1200-1205.