

# Problems Based On Rational and Irrational Numbers in Real Number System

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**ABSTRACT:** In this paper, we define some problems based on rational and irrational numbers using real numbers.

**KEYWORDS:** rational number, irrational number.

## I. INTRODUCTION

Several methods are used to introduce real numbers. One method starts with the positive integers 1,2,3,..... as undefined concepts and uses them to build a largest system[4], the positive rational numbers (quotients of positive integers), their negatives, and zero. The rational numbers, in turn, are then used to construct the irrational numbers, real numbers like  $\sqrt{2}$  and  $\pi$  which are not rational[5]. The rational and irrational numbers together constitute the real-number system[3].

**Definition 1.1.** Quotients of integers  $a/b$  (where  $b \neq 0$ ) are called rational numbers.

For example,  $1/2$ ,  $-7/5$ , and  $6$  are rational numbers. The set of rational numbers, which we denote by  $\mathbf{Q}$ , contains  $\mathbf{Z}$  as a subset.

Another example, if  $a$  and  $b$  are rational, their average  $(a + b)/2$  is also rational and lies between  $a$  and  $b$ [6]. Therefore between any two rational numbers there are infinitely many rational numbers, which implies that if we are given a certain rational number we cannot speak of the “next largest” rational number[2].

**Definition 1.2.** Real numbers that are not rational are called irrational[1].

For example, the numbers  $\sqrt{2}$ ,  $e$ ,  $\pi$  and  $e^\pi$  are irrational[7].

**Theorem 1.1.** If  $n$  is a positive integer which is not a perfect square, then  $\sqrt{n}$  is irrational.

*Proof.* Suppose first that  $n$  contains no square factor  $> 1$ . We assume that  $\sqrt{n}$  is rational and obtain a contradiction[8].

Let  $\sqrt{n} = a/b$ , where  $a$  and  $b$  are integers having no factor in common. Then  $nb^2 = a^2$  and, since the left of this equation is a multiple of  $n$ , so too is  $a^2$ . However, if  $a^2$  is a multiple of  $n$ ,  $a$  itself must be a multiple of  $n$ , since  $n$  has no square factors  $> 1$ [8].

This means that  $a = cn$ , where  $c$  is some integer. Then the equation  $nb^2 = a^2$  becomes  $nb^2 = c^2 n^2$  or  $b^2 = nc^2$ . The same argument shows that  $b$  must also be a multiple of  $n$ . Thus  $a$  and  $b$  are both multiples of  $n$ , which contradicts the fact that they have no factor in common. This completes the proof if  $n$  has no square factor  $> 1$ .

## II. MAIN RESULTS

In this section. We prove some problems based on rational and irrational numbers using real number system[9].

**Problem 2.1.** Find the rational number whose decimal expansion is 0.3344444444....

*Proof.* Let  $x = 0.3344444444\dots$ , then

$$x = \frac{3}{10} + \frac{3}{10^2} + \frac{4}{10^3} + \dots + \frac{4}{10^n} + \dots + \dots \text{ Where } n \geq 3$$

$$= \frac{33}{10^2} + \frac{4}{10^3} \left( 1 + \frac{1}{10} + \dots + \frac{1}{10^n} + \dots \right)$$

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$$\begin{aligned}
 &= \frac{33}{10^2} + \frac{4}{10^3} \left( \frac{1}{1-\frac{1}{10}} \right) \\
 &= \frac{33}{10^2} + \frac{4}{900} \\
 &= \frac{301}{900}
 \end{aligned}$$

**Problem 2.2.** Prove that the decimal expansion of  $x$  will end in zeros (or in nines) if, and only if,  $x$  is a rational number whose denominator is of the form  $2^n 5^m$ , where  $m$  and  $n$  are nonnegative integers [10].

*Proof.* Suppose that  $x = \frac{k}{2^n 5^m}$ , if  $n \geq m$ , we have  

$$\frac{k 5^{n-m}}{2^n 5^m} = \frac{5^{n-m} k}{10^n}.$$

So, the decimal expansion of  $x$  will end in zeros. Similarly for  $m \geq n$ .

Suppose that the decimal expansion of  $x$  will end in zeros (or in nines).

For case,  $x = a_0.a_1 a_2 \dots a_n$ . Then

$$x = \frac{\sum_{k=0}^n 10^{n-k} a_k}{10^n} = \frac{\sum_{k=0}^n 10^{n-k} a_k}{2^n 5^n}.$$

For case,  $x = a_0.a_1 a_2 \dots a_n 999999 \dots$ . Then

$$\begin{aligned}
 x &= \frac{\sum_{k=0}^n 10^{n-k} a_k}{2^n 5^n} + \frac{9}{10^{n+1}} + \dots + \frac{9}{10^{n+m}} + \dots \\
 &= \frac{\sum_{k=0}^n 10^{n-k} a_k}{2^n 5^n} + \frac{9}{10^{n+1}} \sum_{j=0}^{\infty} 10^{-j} \\
 &= \frac{\sum_{k=0}^n 10^{n-k} a_k}{2^n 5^n} + \frac{1}{10^n} \\
 &= \frac{1 + \sum_{k=0}^n 10^{n-k} a_k}{2^n 5^n}.
 \end{aligned}$$

So, in both case, we prove that  $x$  is a rational number whose denominator is of the form  $2^n 5^m$ , where  $m$  and  $n$  are nonnegative integers

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