Robust Design Optimization of Bridge Pier including Parameter Uncertainty in Monte Carlo Simulation Framework

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ABSTRACT: The paper explores Robust Design Optimization (RDO) application under uncertainty of reinforced concrete (RC) bridge pier structure subjected to live and seismic load. This has been well recognized now-a-days that optimization disregarding uncertainty may invite catastrophic failure, especially for important structures like bridges. An RDO is a modern optimization technique which simultaneously optimizes expected value and standard deviation of the performance function satisfying a target reliability index for limit states of failure. However, application of RDO on bridge pier under seismic force is observed to be scarce in the literature. Thus, the aim of the present study is to undertake RDO of bridge pier incorporating parameter uncertainty. The seismic force in the pier is judiciously evaluated by the Moving Least Squares based Response Surface approach. Then, parameter uncertainty is incorporated by the direct Monte Carlo Simulation approach. The results of the RDO imply that considerable reliability and robustness in design can be ensured by the RDO by an affordable increment in cost.

KEYWORDS: Robust Design Optimization, Monte Carlo Simulation, Reinforced concrete bridge pier, target reliability, parameter uncertainty

INTRODUCTION

The RC bridge pier is an important structural element. The design optimization of bridge pier has been attempted by Structural Engineering research community since last three decades [1],[2], [3].However, the majority of these applications are in deterministic format. That is, the parameters involved in optimization process are considered to be of fixed value. But, it is well established since last decade that uncertainty is always present in a structural system and optimization disregarding uncertainty may invite catastrophic failure [4], especially for such important and mega-structures like bridges. It can be noted also that the partial safety factor based approach may either very conservative or even sometime unsafe. Thus, an attempt has been taken in this study to undertake optimization of bridge pier incorporating uncertainty in the parameters. The Robust Design Optimization (RDO), [5] a modern tool of optimization under uncertainty has been applied in the present paper. The RDO study on RC bridge pier structure is not yet observed in the literature survey. An RDO scheme simultaneously optimizes mean and variance of the performance function satisfying a target reliability index for its limit states of failure. The quality or the feasibility of constraint satisfaction i.e. the robustness in constraint is ensured by adding suitable penalty term and through a target reliability index. The RDO is framed and solved by Sequential Quadratic Programming (SQP). The seismic load has also been considered in the optimization process. The uncertainty in the involved parameters is incorporated by a Monte Carlo Simulation (MCS). A numerical study has been undertaken to study the novelty of the RDO approach. In doing so, in section 2, the RDO scheme is presented followed by numerical study in section 3 and conclusion in section 4.
II. THE RDO SCHEME

The undesirable deviation of system performance caused by presence of uncertainty in the input parameters indicates a poor quality and added life-cycle cost of structure, including inspection, repair and other maintenance costs in the perspective of entire life span of the structure. To decrease such deviation, one possible way is to reduce or even to eliminate the source of uncertainty in system, which may either be practically impossible or adds much to the total cost of the structure. Rather, it is better to perform an RDO, by which the system becomes so healthy to be minimally affected from the adverse effect of system uncertainty. This is achieved by minimizing standard deviation (SD) of the objective function and enhancing constraint satisfaction feasibility. Conventional Reliability Based Design Optimization (RBDO) approach will bring specified target reliability of structure performance, but the design may be still sensitive to input parameter variations. Moreover when the probability density function is unavailable and stochastic system parameters are modelled like uncertain but bounded type RDO becomes an attractive alternate to RBDO.

The RDO is fundamentally concerned with minimizing the effect of uncertainty in the DVs and the DPs. The definition of RDO is “A product or process is said to be robust when it is insensitive to the effects of sources of variability, even though sources themselves have not been eliminated”. In recent years, substantial work is envisaged in the field of RDO in the international perspective [6]. Mathematically, an RDO problem is stated as [7],

\[
\begin{align*}
\minimize & \quad \phi(u) = \alpha \mu_f + (1-\alpha) \sigma_f, \quad 0 \leq \alpha \leq 1 \\
\text{subject to} & \quad \mu_{g_j} + k_j \sigma_{g_j} \leq 0 & j = 1, 2, \ldots, J \\
& \quad x^L_i \leq x_i \leq x^U_i & i = 1, 2, \ldots, K.
\end{align*}
\]

(1)

In above, \(\mu_f\) and \(\sigma_f\) are the mean and the SD of the performance function, respectively; \(\phi(u)\) is a new objective function, called desirability function; the parameter \(\alpha\) serves as a weighting factor; \(\mu^*_g\) and \(\sigma^*_g\) are the optimal values of \(\mu_g\) and \(\sigma_g\) obtained for \(\alpha\) values one and zero, respectively. Larger value of \(\alpha\) reckons that designer puts more significance on the conventional objective over its variance. In Eq. (1), \(\mu_{g_j}\) and \(\sigma_{g_j}\) are the mean and the SD of the \(j^{th}\) constraint \(g_j\), respectively. The designer specified penalty factor, \(k_j\) is used to enhance the feasibility of the \(j^{th}\) constraint and can be obtained from, \(k_j = \Phi^{-1}(P_{oj})\), where \(\Phi^{-1}(.)\) is the inverse of the cumulative density function of a standard normal variate and \(P_{oj} = 1 - P_f\). \(k_j\) can also be represented as the target reliability index. The mean and SD of the functions are estimated by a direct MCS. Once, the RDO problem is formulated the solution is done by SQP by Matlab.

III. NUMERICAL STUDY

In this section, the RDO is presented for a Railway bridge pier (Fig. 1) of inter-span span 32 m. The bridge is of three spans simply supported with steel superstructure with open web girder and ballast less track. The train load is Heavy Mineral type (HM loading). The site soil at is of hard type (Type I). The seismic zone is varied from zone III to zone V. Damping is considered to be 5%. The concrete and steel grade is M40 and Fe415, respectively. The plan section of concrete pier is taken as taken as 2.4 m x 2 m. The Dead load, Live load and seismic load have been considered in the study. The seismic load has been calculated according to IS [8] and IITK-RDSO guideline [9] and the structure is analyzed by a finite element software SAP 2000 [10]. The Deterministic Optimization (DO) is formulated to minimize the cost of the pier subjected to combined bending and axial stress constraint under seismic loading. Live load, bending stress, axial stress in the pier, and the allowable stress have been obtained from [11-14]. The grade of concrete, steel, live load, unit cost of concreting and zone factor, response reduction factor, spectral acceleration for calculation of seismic load by Response Spectrum approach are considered to be uncertain parameter with random normal distribution. The uncertainty in steel grade is taken at 5% coefficient of variation (cov) and all other parameters are taken at 10 to 40% cov. The uncertainty is incorporated by MCS within SQP execution process. It has been observed that 10,000 simulations
are required to achieve converge of the result. The design variable is considered to be area of steel of pier. The RDO is formulated according to Eq. (1). The value of $\alpha$ in Eq.(1) is taken to be 0.5. $k_j$ is taken from 3.0 that represents $Pf$ of 0.135%. The optimization problem is solved by MATLAB by SQP.

![Fig. 1 A Reinforced concrete bridge pier](image)

It may be noted here that the process would be extremely time consuming to optimize the structure by direct simulation only where for 400 iterations of optimization it will take 400x10,000 runs of SAP analysis. Thus, a Response Surface method (RSM) based expression has been evaluated to approximate the force on the pier caused by seismic force after only around 100 run of SAP analysis adopting a Redundant Design of Experiment which takes only axial points. In this paper a Moving Least Squares based RSM [15] is explored in place of conventional Least-Squares based RSM to improve the accuracy. Then, by MCS, the mean, SD of the combined bending and axial stress developed on the pier for dead load, live load and RSM predicted seismic force is evaluated to form robust constraint of Eq.(1).

The vertical load on the pier for different grade of concrete and seismic zone is shown in Fig. 2(a) and the area of steel required by the DO is shown in Fig. 2(b). It may be observed from Fig. 2(b) that reinforcing steel area required is 46% more for seismic zone V in comparison to zone III.

![Fig. 2. (a) vertical load and (b) Area of reinforcing steel required by the DO by reinforced concrete bridge pier](image)

The results of RDO are depicted through Figs. 3(a)-3(f) for varying live load on the bridge and range of uncertainty in live load. Fig. 3 shows RDO results for varying Live load and its uncertainty. Fig. 3(a) shows area of reinforcing steel required in seismic zone V. From Fig. 3(a) it is clear that as the uncertainty is introduced to 20%, 30% and 40% cov, the increase in the requirement of steel reinforcement is 17%, 39% and 63% respectively comparing to DO results. Fig. 3(b) and (c) depicts mean cost in seismic zone III and V, respectively. Comparing Fig. 3(b) and (c), it is observed that mean cost by RDO increases 35% to 50% as seismic zone changes from zone III to V. Effect of uncertainty is more prominent for higher live load level. Fig. 3(d), (e) and (f) represent cov of cost for seismic zone III, IV and V. The maximum cov is observed to be 2.78%, 2.9% and 3.01% for 20%, 30% and 40% uncertainty in design load of 2000 KN. It is to underline...
here that for all the cases, the cov of mean cost remain almost constant over the varying uncertainty levels in load. Thus, the RDO yields solutions, which are insensitive to the variation of input uncertainty, even when the load uncertainty is raised up to an extent of 40%.

![Graphs showing RDO results](image)

**Fig. 3** The RDO results by varying live load and uncertainty in live load for (a) Area of reinforcing steel, (b) mean cost in seismic zone II, (c) mean cost in seismic zone V, (d) cov in zone III, (e) cov in zone IV, (f) cov in zone V

**IV. CONCLUSION**

The RDO of a RC bridge pier under seismic load is presented. The uncertainty in the various parameters is explicitly incorporated using a direct MCS. Since, it will involve interlinking of finite element software and prohibitive computational time to solve one RDO problem by MCS, an RSM approach is used to approximate the seismic effect on the bridge pier. In doing so, a MLSM based RSM is explored to improve accuracy of the process. Then, the MCS is adopted to incorporate uncertainty in the load and parameters and to calculate mean, standard deviation of the reliability.
based robust constraint function. The resulted optimization problem is solved by SQP. The DO is framed so as to minimize the cost of the pier subjected to constraints of maximum combined axial and bending stress. Subsequently, the requirement of robustness of objective function and constraints are imposed. The RDO results depict that considerable increase of mean cost by the RDO in seismic zone V in comparison to seismic zone III. The mean cost also increases with increase in uncertainty. It has been observed that the maximum cov is observed to be 3.01% for 20%, 30% and 40% uncertainty in design load of 2000 KN. It is to underline here that for all the cases, the cov of mean cost remain almost constant over the varying uncertainty levels in load. Thus, the present approach produces solutions, which are insensitive to the variation of input uncertainty, that is, the solutions are robust, even when the load uncertainty is raised up to an extent of 40%. Also, the designer gets a high level of reliability by the RDO. Although, the cost will increase by the RDO in comparison to the DO; that much increment is felt to be affordable for the sake of high reliability and robust behavior of the structural performance for such very important structural element, like bridge piers.

REFERENCES


BIography

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