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ψ^*g^*- Closed Sets in Topological Spaces

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ABSTRACT: The aim of this paper is to introduce and study a new class of generalized closed sets called ψ^*g^*-closed sets in topological spaces using ψg- closed sets and ψ-closed sets. We analyse the relations between ψ^*g^*-closed sets with already existing closed sets. The class of ψ^*g^*-closed sets is properly placed between the class of ψ- closed sets and the class of $g^*\psi$ - closed sets and a chain of relation is proved as follows.

r-closed $\rightarrow$ closed $\rightarrow$ ψ-closed $\rightarrow$ ψ^*g^*-closed $\rightarrow$ $g^*\psi$-closed $\rightarrow$ $\psi\hat{g}$-closed $\rightarrow$ ψg-closed $\rightarrow$ gsp-closed

KEYWORDS: g-closed sets, ψ-closed sets, ψg-closed sets and ψ^*g^*-closed sets

I. INTRODUCTION

Levine [7] introduced the concepts of generalized closed sets denoted by g-closed sets in topological spaces and studied their basic properties. Veerakumar [18] introduced and analysed ψ-closed sets in topological spaces. Veerakumar [22] introduced the concepts of $g^*\psi$-closed sets using g-closed sets in topological spaces. Ramya and Parvathi [15] introduced a new concept of generalized closed sets called $\psi\hat{g}$-closed sets and ψg-closed sets in topological spaces. The purpose of this paper is to introduce a new class of generalized closed sets called ψ^*g^*-closed sets in topological spaces.

II. PRELIMINARIES

Throughout this paper (X, τ) represents non-empty topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ), cl(A) and int(A) denote the closure of A and the interior of A respectively.

Definition 2.1

A subset A of a topological space (X,τ) is called
(i) Regular open set [16] if A=int(cl(A))
(ii) Semi-open set [6] if $A \subseteq cl(int(A))$
(iii) α-open set[14] if $A \subseteq int(cl(int(A)))$
(iv) Pre-open set [12] if $A \subseteq int(cl(A))$
(v) semi pre-open set [2] if $A \subseteq cl(int(clA)))$

The complements of the above mentioned sets are called regular closed, semi closed, α-closed, pre-closed and semi pre-closed sets respectively.

The intersection of all regular closed subsets of (X, τ) containing A is called the regular closure of A and is denoted by rcl(A). Similarly scl(A)-semi closure of A, αcl(A) - α closure of A, pcl(A)-pre closure of A and spcl(A) - semi pre closure of A are defined.

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Definition 2.2

A subset A of a topological space (X, τ) is called

(a) generalized closed set (g-closed) [7] if $cl(A)\subseteq U$ whenever $A\subseteq U$ and U is open in (X, τ).

(b) semi-generalized closed set (sg-closed) [3] if $scl(A)\subseteq U$ whenever $A\subseteq U$ and U is semi-open in (X, τ).

(c) generalized α-closed set(gα-closed) [8] if $\alpha cl(A)\subseteq U$ whenever $A\subseteq U$ and U is α-open in (X, τ).

(d) α-generalized closed set (αg-closed) [9] if $\alpha cl(A)\subseteq U$ whenever $A\subseteq U$ and U is open in (X, τ).

(e) generalized semi-pre -closed set (gsp-closed) [5] if $spcl(A)\subseteq U$ whenever $A\subseteq U$ and U is open in (X, τ).

(f) g^*-closed set[19] if $cl(A)\subseteq U$ whenever $A\subseteq U$ and U is g-open in (X, τ).

(g) $\hat{g}$ - closed set [20] if $cl(A)\subseteq U$ whenever $A\subseteq U$ and U is semi-open in (X, τ).

(h) gp-closed set [10] if $pcl(A)\subseteq U$ whenever $A\subseteq U$ and U is open in (X, τ).

(i) g*p-closed set [21] if $pcl(A)\subseteq U$ whenever $A\subseteq U$ and U is g-open in (X, τ).

(j) $\alpha\hat{g}$ - closed set [1] if $\alpha cl(A)\subseteq U$ whenever $A\subseteq U$ and U is $\hat{g}$ -open in (X, τ).

(k) αg^* - closed set [13] if $cl(A)\subseteq U$ whenever $A\subseteq U$ and U is α-open in (X, τ).

(l) $s\alpha g^*$ - closed set [11] if $\alpha cl(A)\subseteq U$ whenever $A\subseteq U$ and U is g^*-open in (X, τ).

(m) wgα-closed set [23] if $\alpha cl(int(A))\subseteq U$ whenever $A\subseteq U$ and U is α-open in (X, τ).

(n) wαg-closed set [23] if $\alpha cl(int(A))\subseteq U$ whenever $A\subseteq U$ and U is open in (X, τ).

(o) ψ-closed set [18] if $scl(A)\subseteq U$ whenever $A\subseteq U$ and U is sg-open in (X, τ).

(p) ψg-closed set [15] if $\psi cl(A)\subseteq U$ whenever $A\subseteq U$ and U is open in (X, τ).

(q) $g^*\psi$-closed set [22] if $\psi cl(A)\subseteq U$ whenever $A\subseteq U$ and U is g-open in (X, τ).

(r) $\psi\hat{g}$ - closed set [15] if $\psi cl(A)\subseteq U$ whenever $A\subseteq U$ and U is $\hat{g}$ -open in (X, τ).

(s) $\alpha\psi$ - closed set [4] if $\psi cl(A)\subseteq U$ whenever $A\subseteq U$ and U is α-open in (X, τ).

The complements of the above mentioned sets are called their respective open-sets.

Remark 2.3

r-closed(r-open) $\rightarrow$ closed (open) $\rightarrow$ α-closed(α-open) $\rightarrow$ semi-closed(semi-open) $\rightarrow$ ψ -closed(ψ -open) $\rightarrow$ semi pre-closed(semi pre-open).

III. ψ^*g^*- CLOSED SETS

Definition: 3.1 A subset A of a topological space (X, τ) is said to be **ψ^*g^*-closed set** if $\psi cl(A)\subseteq U$ whenever $A\subseteq U$ and U is ψg-open in (X, τ).

The class of all ψ^*g^*-closed sets of (X,τ) is denoted by $\psi^*g^*C(X,\tau)$.

Definition: 3.2 A subset A of a topological space (X, τ) is said to be **g#ψ -closed set** if $\psi cl(A)\subseteq U$ whenever $A\subseteq U$ and U is ψ-open in (X, τ).

Preposition 3.3 Every closed set in (X, τ) is ψ^*g^*-closed but not conversely.

Proof: Let A be a closed set of (X, τ). Let U be any ψg-open set containing A. By remark 2.3 every closed set is ψ-closed, $\psi cl(A)\subseteq cl(A)=A\subseteq U$. Therefore A is ψ^*g^*- closed.

Example 3.4 Let $X=\{a,b,c\}$ with $\tau=\{\varphi,X,\{a\},\{a,b\}\}$. Then the subset $\{b\}$ is ψ^*g^* -closed but not closed in (X, τ).

Preposition 3.5 Every regular closed set in (X, τ) is ψ^*g^*-closed but not conversely.

Proof: As every regular closed set is closed and by preposition 3.3 the proof follows.

Example 3.6 Let $X=\{a,b,c\}$ with $\tau=\{\varphi,X,\{a\},\{b\},\{a,b\}\}$. Then the subset $\{c\}$ is ψ^*g^* -closed but not regular closed in (X, τ).

Preposition 3.7 Every α -closed set in (X, τ) is ψ^*g^*-closed but not conversely.

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Proof: Let A be an α-closed set of (X, τ). Let U be any ψg-open set containing A in X. Since A is α-closed, $\alpha cl(A) = A$ and by remark 2.3 every α-closed set is ψ-closed, $\psi cl(A) \subseteq \alpha cl(A) = A \subseteq U$. Therefore A is $\psi{*}g{*}$-closed.

Example 3.8 Let $X = \{a, b, c\}$ with $\tau = \{\varphi, X, \{a, b\}\}$. Then the subset {b, c} is $\psi{*}g{*}$-closed but not α-closed in(X, τ).

Preposition 3.9 Every ψ-closed set in (X, τ) is $\psi{*}g{*}$-closed but not conversely.

Proof: Let A be a ψ- closed set of (X, τ). Let U be any ψg-open set containing A in X. Since A is ψ-closed, $\psi cl(A) = A$. Therefore $\psi cl(A) = A \subseteq U$ and hence A is $\psi{*}g{*}$-closed.

Example 3.10 Let $X = \{a, b, c\}$ with $\tau = \{\varphi, X, \{a\}, \{a, b\}\}$. Then the subset {a,c} is $\psi{*}g{*}$ -closed but not ψ-closed in (X, τ).

Preposition 3.11 Every $\psi{*}g{*}$-closed set in (X, τ) is ψg-closed but not conversely.

Proof: Let A be a $\psi{*}g{*}$-closed set and U be any open set containing A in X. Since every open set is ψg-open and A is $\psi{*}g{*}$-closed, $\psi cl(A) \subseteq U$. Hence A is ψg-closed.

Example 3.12 Let $X = \{a, b, c, d\}$ with $\tau = \{\varphi, X, \{a\}, \{a, b\}\{a, b, c\}\}$. Then the subset {a, b, d} is ψg- closed but not $\psi{*}g{*}$-closed in (X, τ).

Preposition 3.13 Every $\psi{*}g{*}$-closed set in (X, τ) is gsp- closed but not conversely.

Proof: Let A be a $\psi{*}g{*}$-closed set and U be any open set containing A in X. Since every open set is ψg-open and A is $\psi{*}g{*}$-closed, $\psi cl(A) \subseteq U$. For every subset A of X, $spcl(A) \subseteq \psi cl(A)$ and so $spcl(A) \subseteq U$. Hence A is gsp-closed.

Example 3.14 Let $X = \{a, b, c\}$ with $\tau = \{\varphi, X, \{a, b\}\}$. Then the subset {a} is gsp-closed but not $\psi{*}g{*}$-closed in (X, τ).

Lemma 3.15 Every $\hat{g}$-closed closed set in (X, τ) is ψg-closed.

Proof: Let A be a $\hat{g}$-closed set and U be any open set containing A in X. Since every open set is semi open and A is $\hat{g}$-closed, $cl(A) \subseteq U$. For every subset A of X, $\psi cl(A) \subseteq cl(A)$ and so $\psi cl(A) \subseteq U$. Hence A is ψg-closed.

Preposition 3.16 Every $\psi{*}g{*}$-closed set in (X, τ) is $\psi\hat{g}$-closed but not conversely.

Proof: Let A be a $\psi{*}g{*}$-closed set and U be any $\hat{g}$ open set containing A in X. By **Lemma 3.15** every $\hat{g}$-open set is ψg-open and A is $\psi{*}g{*}$-closed, $\psi cl(A) \subseteq U$. Hence A is $\psi\hat{g}$-closed.

Example 3.17 Let $X = \{a, b, c, d\}$ with $\tau = \{\varphi, X, \{a\}, \{a, b, c\}\}$. Then the subset {a, b, d} is $\psi\hat{g}$-closed but not $\psi{*}g{*}$-closed in (X, τ).

Lemma 3.18 Every g-closed set in (X, τ) is ψg-closed.

Proof: Let A be a g- closed set and U be any open set containing A in X. Since A is g- closed, $cl(A) \subseteq U$. For every subset A of X, $\psi cl(A) \subseteq cl(A)$ and so $\psi cl(A) \subseteq U$. Hence A is ψg-closed.

Preposition 3.19 Every $\psi{*}g{*}$-closed set in (X, τ) is $g{*}\psi$-closed but not conversely.

Proof: Let A be a $\psi{*}g{*}$-closed set and U be any g-open set containing A in X. By **lemma 3.18** every g-open set is ψg -open and A is $\psi{*}g{*}$-closed, so $\psi cl(A) \subseteq U$. Hence A is $g{*}\psi$-closed.

Example 3.20 Let $X = \{a, b, c, d\}$ with $\tau = \{\varphi, X, \{a\}, \{a, b\}\}$. Then the subset {a, c, d} is $g{*}\psi$-closed but not $\psi{*}g{*}$-closed in (X, τ).

Preposition 3.21 Every $\psi{*}g{*}$-closed set in (X, τ) is $\alpha\psi$-closed but not conversely.

Proof: Let A be a $\psi{*}g{*}$-closed set and U be any α-open set containing A in X. Since every α-open set is ψg-open and A is $\psi{*}g{*}$-closed, $\psi cl(A) \subseteq U$. Hence A is $\alpha\psi$-closed.

Example 3.22 Let $X = \{a, b, c\}$ with $\tau = \{\varphi, X, \{a\}, \{a, c\}\}$. Then the subset {a, b} is $\alpha\psi$-closed but not $\psi{*}g{*}$ -closed in (X, τ).

Preposition 3.23 Every $\psi{*}g{*}$-closed set in (X, τ) is $g\#\psi$- closed but not conversely.

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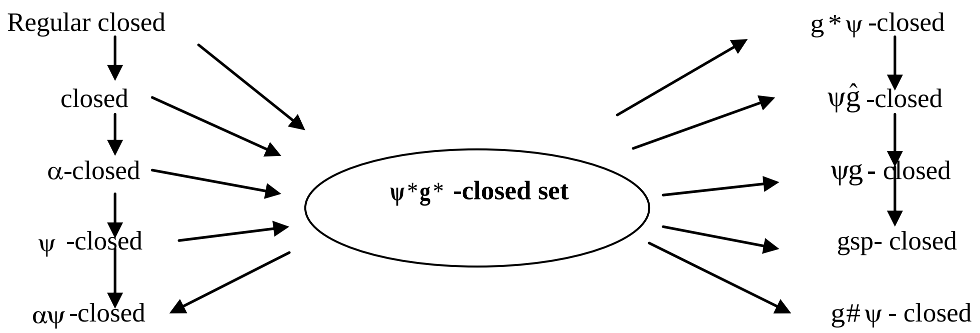
Proof: Let A be a ψ^*g^*-closed set and U be any ψ-open set containing A in X. Since every ψ-open set is ψg- open and A is a ψ^*g^*-closed set, $\psi cl(A) \subseteq U$. Hence A is $g\#\psi$-closed.

Example 3.24 Let $X = \{a, b, c\}$ with $\tau = \{\varphi, X, \{a\}, \{b, c\}\}$. Then the subset {a, b} is $g\#\psi$-closed but not ψ^*g^*-closed in (X, τ).

Remark 3.25 The following diagram gives the dependence of ψ^*g^*-closed set with already existing various closed sets.



Where A ⟶ B represents A implies B.

Remark 3.26 The following examples show that ψ^*g^*- closedness is independent from pre-closedness, g-closedness, $\hat{g}$-closedness, gp-closedness, g*p-closedness, αg- closedness, gα-closedness, $\alpha\hat{g}$ - closedness, αg*-closedness, sαg*- closedness, wαg- closedness and wgα-closedness.

Example 3.27 Let $X = \{a, b, c\}$ with $\tau = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$. Then the subset {b} is ψ^*g^*-closed but not pre-closed, g-closed, $\hat{g}$ - closed, gp- closed, g*p- closed, αg-closed, gα-closed, $\alpha\hat{g}$ - closed αg*- closed, sαg*- closed, wαg-closed and wgα-closed in (X,τ).

Example 3.28 Let $X = \{a, b, c\}$ with $\tau = \{\varphi, X, \{a\}, \{b, c\}\}$. Then the subset {a, b} is pre- closed, g-closed, $\hat{g}$ - closed, gp- closed, g*p- closed, αg-closed, gα-closed, $\alpha\hat{g}$ - closed αg*- closed, sαg*- closed, wαg-closed and wgα-closed but not ψ^*g^*-closed in (X, τ).

Remark 3.29 The following examples show that ψ^*g^*-closedness is independent from g*-closedness.

Example 3.30 Let $X = \{a, b, c, d\}$ with $\tau = \{\varphi, X, \{a\}, \{a, b, c\}\}$. Then the subset {a, b, d} is g*-closed but not ψ^*g^*-closed and the subset {c} is ψ^*g^*-closed but not g*-closed in (X, τ).

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