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Objective Linear Programming Approach for Travelling Salesman Problem Fuzzy Ranking Method

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ABSTRACT: The assignment problem is a special type of linear programming problem in which our objectives is to a minimum cost (time). The mathematical formulation of the problem suggests that this is a **0 – 1** programming problem and is highly degenerate. All the algorithms developed to find optimal solution of transportation problems are applicable to assignment problem. However, due to its highly degeneracy nature, a specially designed algorithm widely known as Hungarian method proposed by khun is used for its solution. Examples of these types of problems are assigning men to offices, crew (drivers and conductors) to buses, trucks to delivery routes etc.

I. METHODS FOR SOLVING FUZZY ASSIGNMENT PROBLEMS AND FUZZY TRAVELLING SALESMAN PROBLEMS

Mukherjee and Basu proposed a method for solving fuzzy assignment problems. Kumar and Gupta chosen some fuzzy assignment problems and fuzzy travelling salesman problems, which cannot be solved y using the fore-mentioned method for solving such type of fuzzy assignment problems and fuzzy travelling salesman problems. In this chapter, the exiting method is presented.

Definition 1

The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function μ such that the value assigned to the element of the universal set X fall within specified range i.e, $\mu : X \rightarrow [0,1]$. The assigned value indicate the membership grade of the element in the set A.

Definition 2

A fuzzy set \tilde{A} defined on universal set of real numbers R, is said to be fuzzy number if its membership function has the following characteristics:

- i. is convex i.e, $\mu_{\tilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \geq \text{minimum}(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)) \forall x_1, x_2 \in R \forall \lambda \in [0,1]$
- ii. \tilde{A} is normal i.e, $x_o \in R$ such that $\mu_{\tilde{A}}(x_o) = 1$
- iii. $\mu_{\tilde{A}}$ is piecewise continuous.

Definition 3

Let $\tilde{A} = (m, n, \alpha, \beta)_{LR}$ be an LR flat fuzzy number and λ be a real number in the interval $[0, 1]$ then the crisp set

$$A_\lambda = \{x \in X | \mu_{\tilde{A}}(x) \geq \lambda\} = [m - \alpha L^{-1}(\lambda), n + \beta R^{-1}(\lambda)]$$

is said to be λ - cut of \tilde{A} .

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II. A NEW METHOD FOR SOLVING FUZZY ASSIGNMENT PROBLEMS

Consider a consultant company, which provides different services for customers for predetermined charges in each case. The quality of the job (case), i.e. the satisfaction of the customer or the job performance of the worker (consultant) doing that case, may assume to be positively correlated to the input time (cost) of the worker. The highest quality of the case is different because of the difference among individual workers. Furthermore, the manager of the company usually restricts the total labor hours or costs to a range as his fuzzy goal. Hence, we assume a minimum cost for a worker to perform a job, and greater cost spent may result in higher quality until it reaches an upper bound, after that an increase in cost does not increase the quality. In this case, the costs are no longer deterministic numbers and we will denote them as \widetilde{c}_{ij} . Further, we define as α_{ij} , the least cost associated with the worker i performing the job j and as β_{ij} , the least cost associated with worker i performing the job j at the highest quality. Without loss of generality, we may assume that $\beta_{ij} > \alpha_{ij} > 0$. We further define the quality matrix $[q_{ij}]$, where q_{ij} represents the highest quality associated with the worker i performing the job j . In most real cases, we have $0 < q_{ij} < 1$. The membership function of \widetilde{c}_{ij} is given below:

$$\mu_{ij}(c_{ij}) = \begin{cases} q_{ij} & : \text{if } c_{ij} \geq \beta_{ij}; x_{ij} = 1 \\ \frac{q_{ij}(c_{ij} - \alpha_{ij})}{(\beta_{ij} - \alpha_{ij})} & : \text{if } \alpha_{ij} \leq c_{ij} \leq \beta_{ij}; x_{ij} = 1 \\ 0 & : \text{otherwise} \end{cases}$$

We use the notation $\langle \alpha_{ij}, \beta_{ij} \rangle$ to \widetilde{c}_{ij} . The matrix $[\widetilde{c}_{ij}] = [\langle \alpha_{ij}, \beta_{ij} \rangle]$. In addition, all the α_{ij} 's form the matrix $[\alpha_{ij}]$ and β_{ij} 's form the matrix $[\beta_{ij}]$. From the definition of \widetilde{c}_{ij} , any expense exceeding β_{ij} is useless since the quality (worker's job performance or customer's satisfaction) can no longer be increased at its upper limit q_{ij} .

Furthermore, let \widetilde{c}_T denote the total cost, which related to the job performance of the manager, and numbers a and b are defined as the lower and upper bounds of total cost, respectively. We define the membership function of \widetilde{c}_T as the linear monotonically decreasing function and use the notation $\langle a, b \rangle$ to denote fuzzy interval \widetilde{c}_T numbers a and b are constants and are subjectively chosen by the manager. Using the idea from Werners, a number less than or equal to the minimum assignment of matrix $[\alpha_{ij}]$ as a and a number larger than or equal to the maximum assignment of matrix $[\beta_{ij}]$ as b are taken. The membership function of \widetilde{c}_T is given below:

$$\mu(c_T) = \mu\left(\sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ij}\right) = \begin{cases} 1 & : \text{if } c_T \leq a \\ \frac{b - c_T}{(\beta_{ij} - \alpha_{ij})} & : \text{if } a \leq c_T \leq b \\ 0 & : \text{if } c_T \geq b \end{cases}$$

The main objective of the FAP is to find an assignment to the given FAP which minimize the total cost of workers and maximize the job performance of the manager.

The above FAP was modeled by Lin and Wen as a 0 – 1 linear fractional programming problem as follows:

$$(FP) \text{ Maximize } f = \frac{b - \sum_{i=1}^n \sum_{j=1}^n \alpha_{ij} x_{ij}}{b - a + \sum_{i=1}^n \sum_{j=1}^n \gamma_{ij} x_{ij}}$$

Subject to

$$\sum_{j=1}^n x_{ij} = 1, \quad i = 1, 2, \dots, n \quad \dots \dots (1)$$

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$$\sum_{i=1}^n x_{ij} = 1, \quad j = 1, 2, \dots, n \quad \dots \dots (2)$$

$$x_{ij} = \{0, 1\}; i = 1, 2, \dots, n \quad j = 1, 2, \dots, n \quad \dots \dots (3)$$

where

$$\gamma_{ij} = \frac{\beta_{ij} - \alpha_{ij}}{q_{ij}}$$

We now, need the following result related to FAP which can be found in Lin and Wen.

Now, the linear programming model of the FAP, (LP) is given below:

$$(LP) \text{ Maximize } - \sum_{i=1}^n \sum_{j=1}^n \alpha_{ij} y_{ij} + bt$$

Subject to

$$\sum_{j=1}^n y_{ij} = t, \quad i = 1, 2, \dots, n;$$

$$\sum_{i=1}^n y_{ij} = 1, \quad j = 1, 2, \dots, n;$$

$$\sum_{i=1}^n \sum_{j=1}^n \gamma_{ij} y_{ij} + (b - a)t = 1;$$

$$t \geq 0, \quad y_{ij} \geq 0, i = 1, 2, \dots, n \quad j = 1, 2, \dots, n;$$

where

$$t = \frac{1}{b - a + \sum_{i=1}^n \sum_{j=1}^n \gamma_{ij} x_{ij}} \text{ and } y_{ij} = tx_{ij}.$$

Now, the dual problem corresponding to (LP), (LD) is given below:

$$(LD) \text{ Minimize } f$$

Subject to

$$u_i + v_j + \alpha_{ij} + f\gamma_{ij} \geq 0, \text{ for all } i \text{ and } j$$

$$- \sum_{i=1}^n u_i - \sum_{j=1}^n v_j + (b - a)f \geq b$$

$$u_1, u_2, \dots, u_n, \quad v_1, v_2, \dots, v_n \text{ and } f \text{ are real numbers.}$$

III. THE PARALLEL MOVING METHOD

Now, we derive the following theorem which connects the solutions of the problem (FP) and its deduction the problems (L) and (U) and is used in the parallel moving method.

Theorem :

Let X_0 be optimal solution to the problem (L) and Y_0 be optimal solution to the problem (U). Then, X_0 and Y_0 are feasible solution to the fuzzy assignment problem and

$$\frac{P(X_0)}{Q(X_0)} \leq \text{Max. } f(X) \leq \frac{P(Y_0)}{Q(Y_0)}$$

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and

$$\frac{P(Y_0)}{Q(Y_0)} \leq \text{Max. } f(X) \leq \frac{P(X_0)}{Q(Y_0)}$$

where

$$P(X) = b - \sum_{i=1}^n \sum_{j=1}^n \alpha_{ij} x_{ij}$$

$$Q(X) = b - a + \sum_{i=1}^n \sum_{j=1}^n \gamma_{ij} x_{ij}$$

If X_0 and Y_0 are the same, then X_0 is an optimal solution to the fuzzy assignment problem and

$$\text{Max. } f(X) = \frac{P(X_0)}{Q(X_0)}$$

.Proof:

Now, since X_0 is optimal solution to the problem (L), then X_0 is optimal solution to the problem (N) where

$$(N) \text{ Maximize } P = b - \sum_{i=1}^n \sum_{j=1}^n \alpha_{ij} x_{ij}$$

Subject to (1), (2) and (3) are satisfied.

Now, since Y_0 is optimal solution to the problem (U), then Y_0 is optimal solution to the problem (D) where

$$(D) \text{ Minimize } Q = b - a + \sum_{i=1}^n \sum_{j=1}^n \gamma_{ij} x_{ij}$$

Subject to (1), (2) and (3) are satisfied.

Now,

$$\frac{P(X)}{Q(X)} = \frac{b - \sum_{i=1}^n \sum_{j=1}^n \alpha_{ij} x_{ij}}{b - a + \sum_{i=1}^n \sum_{j=1}^n \gamma_{ij} x_{ij}} = f(X)$$

Let X be a feasible solution to the problem (FP).

Clearly, X is a feasible solution to the problems (N) and (D). Then, we have

$$P(X) \leq P(X_0) \text{ and } Q(X) \geq Q(Y_0)$$

Since $Q(X) > 0$, we have

$$\frac{P(X)}{Q(X)} \leq \frac{P(X_0)}{Q(Y_0)}$$

This implies that

$$\text{Max.} \left(\frac{P(X)}{Q(X)} \right) \leq \frac{P(X_0)}{Q(Y_0)}$$

Therefore,

$$\text{Max. } f(X) \leq \frac{P(X_0)}{Q(Y_0)} \quad \dots \dots (4)$$

Now, since X_0 and Y_0 are feasible solutions to the problem (FP), we have

$$f(X_0) \leq \text{Max. } f(X) \text{ and } f(Y_0) \leq \text{Max. } f(X) \quad \dots \dots (5)$$

Now, from (4) and (5), we have

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$$\frac{P(X_0)}{Q(X_0)} \leq \text{Max. } f(X) \leq \frac{P(Y_0)}{Q(Y_0)}$$

and

$$\frac{P(Y_0)}{Q(Y_0)} \leq \text{Max. } f(X) \leq \frac{P(X_0)}{Q(X_0)} \quad \dots \dots (6)$$

Now, given that X_0 and Y_0 are the same.

From (6), we have

$$\frac{P(X_0)}{Q(X_0)} \leq \text{Max. } f(X) \leq \frac{P(X_0)}{Q(X_0)}$$

and

$$\frac{P(X_0)}{Q(X_0)} \leq \text{Max. } f(X) \leq \frac{P(X_0)}{Q(X_0)}$$

This implies that,

$$\text{Max. } f(X) = \frac{P(X_0)}{Q(X_0)} = f(X_0)$$

Therefore, X_0 is an optimal solution to the problem (FP) and

$$\text{Max. } f(X) = \frac{P(X_0)}{Q(X_0)}$$

Hence the theorem.

IV. CONCLUSION

In this paper, limitation of an existing method [7] for solving fuzzy assignment problems and fuzzy travelling salesman problem is discussed and to overcome this limitation, the new method is proposed. By comparing the results of the proposed methods and existing method, it is shown that it is better to use the proposed method instead of existing method.

In addition, the new proposed method namely, parallel moving method for solving the fuzzy assignment problems are discussed.

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