

Row-Wise Representation of Arbitrary Rhotrix

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ABSTRACT : Mathematical arrays that are in some way between two-dimensional vectors and 2×2 dimensional matrices were suggested by Atanssor and Shannon [4]. Ajibade [1] introduced the concept of rhotrix owing to some ideals he conceived of matrix – tertions and matrix – noitrets [2]. He identified a rhotrix as an object which is in some ways between a 2×2 dimensional matrix and 3×3 dimensional matrix.

Some Preliminary Concepts:

The axes of a rhotrix:

A vertical axis of a rhotrix is an array of entries running from the top to the bottom of the rhotrix while a horizontal axis of a rhotrix is an array of entries running from the left to the right hand side of the rhotrix. This is the same as the diagonals in Ajibade [1]. Every rhotrix has a major vertical axis and a major horizontal axis. For example, Consider the rhotrix

$$R_5 = \left\langle \begin{array}{cccc} & & a & \\ & f & d & b \\ k & i & g & e \\ & l & j & h \\ & & m & \\ & & & c \end{array} \right\rangle$$

The vertical axes of R_5 include:

$$\begin{array}{cc} a \\ f & d \\ k, i & g \\ j & j \\ m \end{array}$$

while the horizontal axis include $m \quad f \quad d \quad b$ and $k \quad i \quad g \quad e \quad c$; the major axes being

the last of the two lists.

The dimension of a rhotrix:

The dimension of a rhotrix is the number of entries in a major axis of the rhotrix. The dimension of a rhotrix is sometimes attached as a subscript to the variable used to indicate the rhotrix. For instance a rhotrix of the fifth dimension may be denoted as R_5 .

The rows and columns of a rhotrix:

The rows and columns of a rhotrix we follow the definitions of a row and a column of a 3 – dimensional rhotrix [5]. A row of a rhotrix is an array of entries running from the top – left to the right – bottom side of the rhotrix. A column of a rhotrix is an array of entries running from the top – right to the left – bottom side of the rhotrix.

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For example, consider the 5 – dimensional rhotrix R_5 in Definition 1 above. The first row is $\begin{matrix} a & b & c \\ d & e & f \\ g & h & i \\ j & k & l \\ m & n & o \end{matrix}$, the second row is $\begin{matrix} a & b & c \\ d & e & f \\ g & h & i \\ j & k & l \\ m & n & o \end{matrix}$ and the third row is $\begin{matrix} a & b & c \\ d & e & f \\ g & h & i \\ j & k & l \\ m & n & o \end{matrix}$, on the other hand the first column is $\begin{matrix} a & b & c \\ d & e & f \\ g & h & i \\ j & k & l \\ m & n & o \end{matrix}$, the second column is $\begin{matrix} a & b & c \\ d & e & f \\ g & h & i \\ j & k & l \\ m & n & o \end{matrix}$ while the fifth column is $\begin{matrix} a & b & c \\ d & e & f \\ g & h & i \\ j & k & l \\ m & n & o \end{matrix}$.

Heart – oriented multiplication of rhotrices:

Heart – oriented multiplication of rhotrices Ajibade [1] defined rhotrix multiplication ‘ \circ ’ as follows,
For any two rhotrices

$$\left\langle \begin{matrix} a \\ b & h(R) \\ e \end{matrix} \right\rangle d \quad \text{and} \quad \left\langle \begin{matrix} f \\ g & h(Q) \\ k \end{matrix} \right\rangle j$$

$$R \circ Q = \left\langle \begin{matrix} ah(Q) + fh(R) \\ bh(Q) + gh(R) & h(R)h(Q) \\ eh(Q) + kh(R) \end{matrix} \right\rangle dh(Q) + jh(R)$$

This form of rhotrix multiplication is called heart – oriented multiplication of rhotrices. Note that $h(R)$ and $h(Q)$ are the hearts of R and Q respectively.

Row – column multiplication of rhotrices:

Row – column multiplication of rhotrices sani [5, 6] defined row – column multiplication of 3-dimensional rhotrices as follows:

Let

$$A = \left\langle \begin{matrix} a \\ d & c \\ e \end{matrix} \right\rangle b \quad \text{and} \quad B = \left\langle \begin{matrix} f \\ i & h \\ j \end{matrix} \right\rangle g$$

be two rhotrice of the third dimension . the row – column product ‘ \circ ’ of A and B is expressed as

$$A \circ B = \left\langle \begin{matrix} af + bi \\ df + ei & ch \\ dg + ej \end{matrix} \right\rangle ag + bj$$

Row – column multiplication formula:

In the following theorem, we two rhotrices of the same dimension and represented in row – wise style of arbitrary rhotrices to devlope a formula for their row – column multiplication.

Theorem:

The row – column product $\langle c_{ij} \rangle$ of two rhotrices $\langle a_{ij} \rangle$ and $\langle b_{ij} \rangle$ of dimension d in row – wise form is established by the equation

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$$c_{ij} = \sum_{k=1}^{\frac{1}{2}[d-(-1)^i]} a_{i,k} b_{2k} - \frac{1}{2}[(-1)^{i+1} + 1].j$$

where $i = 1, 2, \dots, d$ and $j = 1, 2, \dots, \frac{1}{2}[d - (-1)^i]$.

Proof:

Let d be an odd positive integer and let A and B denote the d dimensional rhotrices in row – wise form of arbitrary rhotrix representation as follows

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1,n-1} & a_{n1} \\ a_{21} & a_{22} & \dots & a_{2,n-1} & a_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{d-2,1} & a_{d-2,2} & \dots & a_{d-2,n-1} & a_{d-2,n} \\ a_{d-1,1} & a_{d-1,2} & \dots & a_{d-1,n-1} & a_{dn} \\ a_{d,1} & a_{d,2} & \dots & a_{d,n-1} & a_{dn} \end{pmatrix}$$

$$B = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1,n-1} & b_{n1} \\ b_{21} & b_{22} & \dots & b_{2,n-1} & b_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ b_{d-2,1} & b_{d-2,2} & \dots & b_{d-2,n-1} & b_{d-2,n} \\ b_{d-1,1} & b_{d-1,2} & \dots & b_{d-1,n-1} & b_{dn} \\ b_{d,1} & b_{d,2} & \dots & b_{d,n-1} & b_{dn} \end{pmatrix}$$

By definition of row – column rhotrix multiplication [6]

$$A \circ B = C = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1,n-1} & c_{n1} \\ c_{21} & c_{22} & \dots & c_{2,n-1} & c_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ c_{d-2,1} & c_{d-2,2} & \dots & c_{d-2,n-1} & c_{d-2,n} \\ c_{d-1,1} & c_{d-1,2} & \dots & c_{d-1,n-1} & c_{dn} \\ c_{d,1} & c_{d,2} & \dots & c_{d,n-1} & c_{dn} \end{pmatrix}$$

By comparing entries we have the following entries for the principal matrix of C

$$\begin{aligned} c_{11} &= a_{11} b_{11} + a_{12} b_{31} + \dots + a_{1,n-1} b_{d-2,1} + a_{n1} b_{d,1} \\ c_{12} &= a_{11} b_{12} + a_{12} b_{32} + \dots + a_{1,n-2} b_{d-2,2} + a_{n1} b_{d,2} \\ c_{31} &= a_{31} b_{11} + a_{32} b_{31} + \dots + a_{3,n-1} b_{d-2,1} + a_{3n} b_{d,1} \end{aligned}$$

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$$c_{32} = a_{31}b_{12} + a_{32}b_{32} + \dots + a_{3,n-1}b_{d-2,2} + a_{3n}b_{d,2}$$

.....

$$c_{d-2,n-1} = a_{d-2,1}b_{1,n-1} + a_{d-2,2}b_{3,n-1} + \dots + a_{d-2,n-1}b_{d-2,n-1} + a_{d-2,n}b_{d,n-1}$$

$$c_{d-2,n} = a_{d-2,1}b_{1,n} + a_{d-2,2}b_{3,n} + \dots + a_{d-2,n-1}b_{d-2,n} + a_{d-2,n}b_{d,n}$$

$$c_{d,n-1} = a_{d,1}b_{1,n-1} + a_{d,2}b_{3,n-1} + \dots + a_{d,n-1}b_{d-2,n-1} + a_{d,n}b_{d,n-1}$$

$$c_{d,n} = a_{d,1}b_{1,n} + a_{d,2}b_{3,n} + \dots + a_{d,n-1}b_{d-2,n} + a_{d,n}b_{d,n}$$

Entries for the complementary matrix of C

$$c_{21} = a_{21}b_{11} + a_{22}b_{31} + \dots + a_{2,n-1}b_{d-2,1}$$

$$c_{22} = a_{21}b_{12} + a_{22}b_{32} + \dots + a_{2,n-1}b_{d-2,2}$$

$$c_{41} = a_{41}b_{11} + a_{42}b_{31} + \dots + a_{4,n-1}b_{d-2,1}$$

.....

$$c_{d-1,n-1} = a_{d-1,1}b_{1,n-1} + a_{d-1,2}b_{3,n-1} + \dots + a_{d-1,n-1}b_{d-2,n-1}$$

Note that commas have been introduced between indices in the summations to avoid ambiguity. In more compact forms, we have,

Entries for the principal matrix of C

$$c_{11} = \sum_{k=1}^n a_{1,k}b_{2k-1,1}$$

$$c_{12} = \sum_{k=1}^n a_{1,k}b_{2k-1,2}$$

$$c_{31} = \sum_{k=1}^n a_{3,k}b_{2k-1,1}$$

$$c_{32} = \sum_{k=1}^n a_{3,k}b_{2k-1,2}$$

$$c_{d-2,n-1} = \sum_{k=1}^n a_{d-2,k}b_{2k-1,n-1}$$

$$c_{d-2,n} = \sum_{k=1}^n a_{d-2,k}b_{2k-1,n}$$

$$c_{d,n-1} = \sum_{k=1}^n a_{d,k}b_{2k-1,n-1}$$

$$c_{d,n} = \sum_{k=1}^n a_{d,k}b_{2k-1,n}$$

Entries for the complementary matrix of C :

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$$c_{21} = \sum_{k=1}^{n-1} a_{2,k} b_{2k,1}$$

$$c_{22} = \sum_{k=1}^{n-1} a_{2,k} b_{2k,2}$$

$$c_{41} = \sum_{k=1}^{n-1} a_{4,k} b_{2k,1}$$

.....

$$c_{d-1,n-1} = \sum_{k=1}^{n-1} a_{d-1,k} b_{2k,n-1}$$

More densely,

$$c_{ij} = \sum_{k=1}^n a_{i,k} b_{2k-1,j} \quad \dots \dots (5)$$

(entries for the principal matrix of C)

$$c_{ij} = \sum_{k=1}^{n-1} a_{i,k} b_{2k,j} \quad \dots \dots (6)$$

(entries for the complementary matrix of C)

The difference between the two summations above are the number of summands n and the first index $2k - 1$ of b in (5) and the number of summands $n - 1$ and the first index $2k$ of b in (6). Now, i is odd and the first index for b is $2k - 1$, for the entries in the principal matrix of C , while i is even and the first index for b is $2k - 0$ (i.e. $2k$) for the entries in the complementary matrix of C .

Consider the function

$$f(x) = \begin{cases} 1, & \text{if } x \text{ is odd} \\ 0, & \text{if } x \text{ is even} \end{cases}, \text{ defined by } f(x) = \frac{1}{2} [(-1)^{x+1} + 1], x = 1, 2, \dots$$

Replacing 1 and 0 with $f(i)$ in the first index of b in (5) and (6) respectively, we have

$$c_{ij} = \sum_{k=1}^n a_{i,k} b_{2k - \frac{1}{2}[(-1)^{i+1} + 1], j} \quad \dots \dots (7)$$

(entries for the principal matrix of C)

$$c_{ij} = \sum_{k=1}^{n-1} a_{i,k} b_{2k - \frac{1}{2}[(-1)^{i+1} + 1], j} \quad \dots \dots (8)$$

(entries for the complementary matrix of C)

Again, consider the function

$$g(x) = \begin{cases} n, & \text{if } x \text{ is odd} \\ n - 1, & \text{if } x \text{ is even} \end{cases}, \text{ defined by } g(x) = \frac{1}{2} [2n - 1 - (-1)^x], \\ x = 1, 2, \dots$$

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We replace with $g(i)$ the only difference in (7) and (8), which are the total number of summands n and $n - 1$, respectively, and obtain the single equation

$$c_{ij} = \sum_{k=1}^{\frac{1}{2}[2n-1-(-1)^i]} a_{i,k} b_{2k - \frac{1}{2}[(-1)^{i+1}+1],j}$$

where $i = 1, 2, \dots, d$ and $j = 1, 2, \dots, \frac{1}{2}[2n - 1 - (-1)^i]$.

But n is the order of the principal matrix of C . Removing the entries of this matrix from C leaves us with the complementary matrix of C with order $n - 1$. By the second property in section 5, $2n - 1 = n + (n - 1) = d$,

This gives the result

$$c_{ij} = \sum_{k=1}^{\frac{1}{2}[d-(-1)^i]} a_{i,k} b_{2k - \frac{1}{2}[(-1)^{i+1}+1],j}$$

where $i = 1, 2, \dots, d$ and $j = 1, 2, \dots, \frac{1}{2}[d - (-1)^i]$.

CONCLUSION

In this dissertation, we present some methods of representing arbitrary rhotrices, of particular interest is the row – wise method with which we have established some formulae for rhotrix multiplication. The concepts of subrhotrix and submatrix of a rhotrix promises to pave way for a new dimension in describing various algebraic properties of rhotries.

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