

New Effective Approaches for Matrix Factorization

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ABSTRACT: In the mathematical discipline of linear algebra a matrix decomposition or matrix factorization is a factorization of a matrix into a product of matrices. There are many different matrix decompositions; each finds use among a particular class of problems. Matrix decomposition is a fundamental theme in linear algebra, it plays an important role in linear algebra, and we use it in solving systems of linear equation besides other applications. If we take factor analysis as an example of the use of matrix decomposition, we see that factor analysis has many applications in different fields like psychometric, marketing, physical sciences, biological sciences etc. This paper proposed miscellaneous new effective methods equipped with different numerical examples

KEYWORDS: Factorization of a matrix, reduced echelon form, reduced row form, invertible matrix, matrix transpose, upper triangular matrix, lower triangular matrix.

I. INTRODUCTION

Some of the most successful realizations of latent factor models are based on matrix factorization. In its basic form, matrix factorization characterizes both items and users by vectors of factors inferred from item rating patterns. High correspondence between item and user factors leads to a recommendation. These methods have become popular in recent years by combining good scalability with predictive accuracy. In addition, they offer much flexibility for modeling various real-life situations. This paper proposed different approaches for matrix decomposition, and it was organized as follows:

Section 2 gives a method with which we can decompose an square full rank matrix, and then take two numerical examples. Section 3 presented a method for the factorization of square deficient matrix with an example. Section gives a method for factorizing a rectangular matrix which has no full column rank. Section 5 gives an example through which we give a method for factorizing a rectangular full column rank matrix. Section 6 the conclusion of the paper.

II. FACTORIZATION OF AN SQUARE FULL RANK MATRIX

If we have a matrix X given as a product two matrices C,R (i.e $X=CR$), we can consider the columns of X as linear combination of the columns of the matrix C or as linear combination of transpose of the rows of C . So we can form the reduced Echelon form matrix, and then take the transpose matrix of it as a matrix A for the decomposition of a matrix M such that $M = AB$ (1)

Then we search for the matrix B , if we get the reduced echelon row form of the matrix M and then take the non zero rows matrix of it, call this matrix E , we note that E is an square upper triangular matrix and the transpose of it will be an square lower triangular matrix and we note that if we take A in equation(1) equal E^T , since A has full rank E^T also will be of full rank and it will be invertible, so (1) can be written as :

$$M = E^T B \quad (2)$$

Now we can multiply both sides of the equation (2) by the inverse of E^T from the left to get

$$E^{-T} M = B \quad (3)$$

Then we can substitute for B from (3) into (1) to have the following decomposition

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$$M = E^T (E^{-T} M) \tag{4}$$

To make this approach clear we will give the following numerical example
Example

Suppose we have a matrix

$$M = \begin{bmatrix} 4 & 3 & 2 & 1 \\ 1 & 2 & 3 & 4 \\ 4 & 1 & 3 & 2 \\ 1 & 3 & 2 & 4 \end{bmatrix}$$

Now we use Gauss elimination

$$R_2 - \frac{1}{4}R_1 \rightarrow R_2 \quad \begin{bmatrix} 4 & 3 & 2 & 1 \\ 0 & \frac{5}{4} & \frac{2}{5} & \frac{15}{4} \\ 4 & 1 & 3 & 2 \\ 1 & 3 & 2 & 4 \end{bmatrix}$$

$$R_3 - R_1 \rightarrow R_3 \quad \begin{bmatrix} 4 & 3 & 2 & 1 \\ 0 & \frac{5}{4} & \frac{2}{5} & \frac{15}{4} \\ 0 & -2 & 1 & 1 \\ 1 & 3 & 2 & 4 \end{bmatrix}$$

$$R_2 - \frac{1}{4}R_1 \rightarrow R_2 \quad \begin{bmatrix} 4 & 3 & 2 & 1 \\ 0 & \frac{5}{4} & \frac{2}{5} & \frac{15}{4} \\ 0 & -2 & 1 & 1 \\ 1 & 3 & 2 & 4 \end{bmatrix}$$

$$R_4 - \frac{1}{4}R_1 \rightarrow R_4 \quad \begin{bmatrix} 4 & 3 & 2 & 1 \\ 0 & \frac{5}{4} & \frac{2}{5} & \frac{15}{4} \\ 0 & -2 & 1 & 1 \\ 0 & \frac{9}{4} & \frac{3}{2} & \frac{15}{4} \end{bmatrix}$$

$$R_3 + \frac{8}{5}R_2 \rightarrow R_3 \quad \begin{bmatrix} 4 & 3 & 2 & 1 \\ 0 & \frac{5}{4} & \frac{2}{5} & \frac{15}{4} \\ 0 & 0 & \frac{5}{7} & \frac{15}{7} \\ 0 & \frac{9}{4} & \frac{3}{2} & \frac{15}{4} \end{bmatrix}$$

$$R_4 - \frac{9}{5}R_2 \rightarrow R_4 \quad \begin{bmatrix} 4 & 3 & 2 & 1 \\ 0 & \frac{5}{4} & \frac{2}{5} & \frac{15}{4} \\ 0 & 0 & \frac{5}{7} & \frac{15}{7} \\ 0 & 0 & -3 & -3 \end{bmatrix}$$

$$R_4 + \frac{3}{5}R_3 \rightarrow R_4 \quad \begin{bmatrix} 4 & 3 & 2 & 1 \\ 0 & \frac{5}{4} & \frac{2}{5} & \frac{15}{4} \\ 0 & 0 & \frac{5}{7} & \frac{15}{7} \\ 0 & 0 & 0 & \frac{6}{5} \end{bmatrix}$$

$$\frac{1}{4}R_1, \frac{4}{5}R_2, \frac{1}{5}R_3, \frac{5}{6}R_4 \rightarrow \begin{bmatrix} 1 & \frac{3}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & 1 & \frac{2}{5} & \frac{3}{4} \\ 0 & 0 & 1 & \frac{7}{5} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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So let

$$A = E = \begin{bmatrix} 1 & \frac{3}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & 1 & 2 & \frac{3}{7} \\ 0 & 0 & 1 & \frac{5}{7} \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow E^T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{3}{4} & 1 & 0 & 0 \\ \frac{1}{2} & 2 & 1 & 0 \\ \frac{1}{4} & 3 & \frac{7}{5} & 1 \end{bmatrix}$$

$$E^T B = M \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{3}{4} & 1 & 0 & 0 \\ \frac{1}{2} & 2 & 1 & 0 \\ \frac{1}{4} & 3 & \frac{7}{5} & 1 \end{bmatrix} B = \begin{bmatrix} 4 & 3 & 2 & 1 \\ 1 & 2 & 3 & 4 \\ 4 & 1 & 3 & 2 \\ 1 & 3 & 2 & 4 \end{bmatrix}$$

$$\Rightarrow (E^{-T} E^T) B = E^{-T} M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{3}{4} & 1 & 0 & 0 \\ \frac{1}{2} & 2 & 1 & 0 \\ \frac{1}{4} & 3 & \frac{7}{5} & 1 \\ 4 & 3 & 2 & 1 \\ 1 & 2 & 3 & 4 \\ 4 & 1 & 3 & 2 \\ 1 & 3 & 2 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 4 & 3 & 2 & 1 \\ 1 & 2 & 3 & 4 \\ 4 & 1 & 3 & 2 \\ 1 & 3 & 2 & 4 \end{bmatrix}$$

$$\Rightarrow B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{3}{4} & 1 & 0 & 0 \\ \frac{1}{2} & -2 & 1 & 0 \\ \frac{3}{5} & -\frac{1}{5} & -\frac{7}{5} & 1 \end{bmatrix} \begin{bmatrix} 4 & 3 & 2 & 1 \\ 1 & 2 & 3 & 4 \\ 4 & 1 & 3 & 2 \\ 1 & 3 & 2 & 4 \end{bmatrix}$$

$$\Rightarrow B = \begin{bmatrix} 4 & 3 & 2 & 1 \\ -2 & -\frac{1}{4} & \frac{3}{2} & \frac{13}{4} \\ 6 & -0 & -1 & -5 \\ -\frac{12}{5} & 3 & -\frac{8}{5} & 1 \end{bmatrix}$$

Now substituting for the two matrices (The transpose of E and the matrix B) in equation (2) we get

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$$M = \begin{bmatrix} 4 & 3 & 2 & 1 \\ 1 & 2 & 3 & 4 \\ 4 & 1 & 3 & 2 \\ 1 & 3 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{3}{4} & 1 & 0 & 0 \\ \frac{1}{4} & 2 & 1 & 0 \\ \frac{1}{4} & 3 & \frac{7}{5} & 1 \end{bmatrix} \begin{bmatrix} 4 & 3 & 2 & 1 \\ -2 & -\frac{1}{4} & \frac{3}{2} & \frac{13}{4} \\ 6 & 0 & -1 & -5 \\ -\frac{12}{5} & 3 & -\frac{8}{5} & 1 \end{bmatrix}$$

We can take another numerical example by letting

$$M = \begin{bmatrix} 1 & 2 & 3 & 3 & 2 & 1 & 1 \\ 2 & 1 & 4 & 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 0 & 1 & 1 & 4 \\ 1 & 1 & 1 & 2 & 3 & 2 & 1 \\ 3 & 2 & 1 & 1 & 2 & 4 & 1 \\ 2 & 0 & 1 & 3 & 1 & 2 & 4 \\ 3 & 0 & 3 & 0 & 2 & 0 & 1 \end{bmatrix}$$

Then we can use Gauss elimination to reduce M to the equivalence matrix F ,such that

$$F = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & -3 & 0 & 0 & 0 & 0 & 0 \\ 3 & -2 & -4/3 & 0 & 0 & 0 & 0 \\ 3 & -3 & 0 & -3 & 0 & 0 & 0 \\ 2 & -2 & 5/3 & -1 & -20/3 & 0 & 0 \\ 1 & -1 & 4/3 & 0 & -3 & 49/80 & 0 \\ 1 & -2 & 2/3 & 2 & -14/3 & 267/40 & 1244/49 \end{bmatrix}$$

Now we note that F is not in the reduced row echelon form,

but we can decompose M by writing it

$$F^T B = M \tag{5}$$

Then we can find B as follows:

$$F^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & -3 & 0 & 0 & 0 & 0 & 0 \\ 3 & -2 & -4/3 & 0 & 0 & 0 & 0 \\ 3 & -3 & 0 & -3 & 0 & 0 & 0 \\ 2 & -2 & 5/3 & -1 & -20/3 & 0 & 0 \\ 1 & -1 & 4/3 & 0 & -3 & 49/80 & 0 \\ 1 & -2 & 2/3 & 2 & -14/3 & 267/40 & 1244/49 \end{bmatrix}$$

and

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$$F^{-T} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2/3 & -1/3 & 0 & 0 & 0 & 0 & 0 \\ 5/4 & 1/2 & -3/4 & 0 & 0 & 0 & 0 \\ 1/3 & 1/3 & 0 & -1/3 & 0 & 0 & 0 \\ 29/80 & 7/40 & -3/16 & 1/20 & -3/20 & 0 & 0 \\ -73/49 & -38/49 & 5/7 & 12/49 & -36/49 & 80/49 & 0 \\ 513/1244 & 53/311-63/311 & & -9/311 & 103/622 & -267/622 & 49/1244 \end{bmatrix}$$

Then by multiplying both sides of equation (5) by F^{-T} from left we get

$$B = F^{-T}M$$

$$= \begin{bmatrix} 1 & 2 & 3 & 3 & 2 & 1 & 1 \\ 0 & 1 & 2/3 & 1 & 2/3 & 1/3 & 2/3 \\ 9/4 & 3/4 & 17/4 & 21/4 & 11/4 & 1 & -7/4 \\ 2/3 & 2/3 & 2 & 4/3 & 1/3 & 0 & 0 \\ 5/16 & 7/80 & 21/16 & 25/16 & 59/80 & -3/20 & -39/80 \\ -85/49 & -139/49 & -5 & -15/7 & -143/49 & -36/49 & 363/49 \\ 299/622 & 429/622 & 833/622 & 707/1244 & 533/622 & 157/1244 & -603/311 \end{bmatrix}$$

We can verify this decomposition by showing that the product $F^T B$ is equal to M.

III. FACTORIZATION OF AN SQUARE MATRIX WHICH HAS NO FULL RANK (DEFICIENT MATRIX)

In this case we can use the basis of the column space of the square matrix M as a matrix A, for writing the matrix $M=AB$, then look for the matrix B .

Suppose M is an m square matrix of rank n such that $n < m$, then by using Gauss elimination we can find the matrix of the basis of the column space of the matrix M , call it C , so C will be an $n \times m$ matrix , then we have

$$n \times m \quad CB = M \tag{5}$$

Now multiplying both sides of (5) by the transpose of C or any other matrix to get

$$(C^T C)B = C^T M$$

$$m \times n \quad \Rightarrow DB = C^T M \tag{6}$$

Where $D = (C^T C)$, then multiplying both sides of (6) from left with D inverse we get

$$B = (D^{-1}C^T)M \tag{6}$$

Then substituting for B from (6) into (5) we have

$$M = C(D^{-1}C^T M) \tag{7}$$

As an example ,let us take

$$M = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 2 & 1 \\ 3 & 4 & 1 & 1 \\ 2 & 5 & 2 & 3 \end{bmatrix}$$

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By finding the reduced row form of M , we deduce that the basis of the column space of matrix are the three first column so for the factorization as in equation (5) we take

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 4 & 1 \\ 2 & 5 & 2 \end{bmatrix}, \quad C^T = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 4 & 5 \\ 3 & 2 & 1 & 2 \end{bmatrix}$$

$$D = C^T C = \begin{bmatrix} 14 & 24 & 10 \\ 24 & 46 & 22 \\ 10 & 22 & 18 \end{bmatrix} \Rightarrow D^{-1} = \begin{bmatrix} 43/51 & -53/102 & 1/6 \\ -53/102 & 19/51 & -1/6 \\ 1/6 & -1/6 & 1/6 \end{bmatrix}$$

Then we have

$$C^T M = \begin{bmatrix} 14 & 24 & 10 & 10 \\ 24 & 46 & 22 & 22 \\ 10 & 22 & 18 & 12 \end{bmatrix}$$

$$B = D^{-1} C^T M = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

From (7)

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 4 & 1 \\ 2 & 5 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

IV. FACTORIZATION OF A RECTANGULAR MATRIX WHICH HAS NO FULL COLUMN RANK

Here we can follow the same procedure which we use in section (2) for the deficient matrix.

Let

$$M = \begin{bmatrix} 2 & 3 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 3 & 2 & 2 \\ 2 & 4 & 2 & 1 \\ 0 & 3 & 3 & 2 \\ 1 & 5 & 4 & 3 \end{bmatrix}$$

Also with finding the reduced row form of M , we deduce that this matrix has no full column rank, and the basis of the column space of it are the first, the second and the fourth column, so for the factorization as in equation (5) we take

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$$C = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 1 \\ 1 & 3 & 2 \\ 2 & 4 & 1 \\ 0 & 3 & 2 \\ 1 & 5 & 3 \end{bmatrix}, C^T = \begin{bmatrix} 2 & 1 & 1 & 2 & 0 & 1 \\ 3 & 2 & 3 & 4 & 3 & 5 \\ 1 & 1 & 2 & 1 & 2 & 3 \end{bmatrix}$$

$$D = C^T C = \begin{bmatrix} 11 & 24 & 10 \\ 24 & 72 & 36 \\ 10 & 36 & 20 \end{bmatrix} \Rightarrow D^{-1} = \begin{bmatrix} 1 & -5/6 & 1 \\ -5/6 & 5/6 & -13/12 \\ 1 & -13/12 & 3/2 \end{bmatrix}$$

$$C^T M = \begin{bmatrix} 11 & 24 & 13 & 10 \\ 24 & 72 & 48 & 36 \\ 10 & 36 & 26 & 20 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & -5/6 & 1 \\ -5/6 & 5/6 & -13/12 \\ 1 & -13/12 & 3/2 \end{bmatrix} \begin{bmatrix} 11 & 24 & 13 & 10 \\ 24 & 72 & 48 & 36 \\ 10 & 36 & 26 & 20 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So the decomposition is

$$M = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 1 \\ 1 & 3 & 2 \\ 2 & 4 & 1 \\ 0 & 3 & 2 \\ 1 & 5 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Verification

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 1 \\ 1 & 3 & 2 \\ 2 & 4 & 1 \\ 0 & 3 & 2 \\ 1 & 5 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 3 & 2 & 2 \\ 2 & 4 & 2 & 1 \\ 0 & 3 & 3 & 2 \\ 1 & 5 & 4 & 3 \end{bmatrix}$$

V. FACTORIZATION OF A RECTANGULAR MATRIX WHICH FULL COLUMN RANK

Now let us take a full column matrix, and the decomposition is $M=AB$, for example take

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$$M = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 3 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

We can use the previous method which we used in section (4), but we follow another different method for the decomposition of this matrix M, since the reduced row form of M is

$$E = \begin{bmatrix} 1 & 3 & 2 \\ 0 & -1 & -1 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

This matrix can be reduced to

$$R = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R^T = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} \Rightarrow R^{-T} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

Then by using block matrix we can take a matrix

$$A = \begin{bmatrix} R^T \\ C \end{bmatrix}$$

Such that $AB=M$, C has 4 rows and 3 columns, B has 3 rows and 3 columns, so we can write this multiplication in block form as

$$AB = \begin{bmatrix} R^T \\ C \end{bmatrix} [B] = \begin{bmatrix} R^T B \\ CB \end{bmatrix} = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 3 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\Rightarrow R^T B = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} \quad (8)$$

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$$CB = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 3 \end{bmatrix} \quad (9)$$

Then multiplying both sides of (8) by R^{-1} from left we get

$$B = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 2 \\ -2 & -7 & -5 \\ 1 & 1 & 3 \end{bmatrix}$$

Since we know B from (9) we can deduce C by multiplying both sides of it from right by B^{-1} to get

$$C = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} 16/3 & 7/3 & 1/3 \\ -1/3 & -1/3 & -1/3 \\ -5/3 & -2/3 & 1/3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 3 & 0 \\ 19/3 & 7/3 & 1/3 \\ 0 & 0 & 1 \\ -11/3 & -5/3 & 1/3 \end{bmatrix}$$

Then

$$A = \begin{bmatrix} R^T \\ C \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 1 & 1 \\ 8 & 3 & 0 \\ 19/3 & 7/3 & 1/3 \\ 0 & 0 & 1 \\ -11/3 & -5/3 & 1/3 \end{bmatrix}$$

So we have

$$M = AB = \begin{bmatrix} R^T \\ C \end{bmatrix} B = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 1 & 1 \\ 8 & 3 & 0 \\ 19/3 & 7/3 & 1/3 \\ 0 & 0 & 1 \\ -11/3 & -5/3 & 1/3 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

VI. CONCLUSION

This paper presented new, easy, effective and different approaches for matrix decomposition or matrix factorization. So we hope this approaches will give good contributions for solving many problems which we face in linear algebra, physics, computer program or , statistical fields or any other different applications.

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