

# A Review of Speed Estimation Algorithms for Incremental Optical Encoder

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**ABSTRACT:** Incremental Quadrature encoder provides useful feedback information about motor positioning essential in open as well as closed loop control systems. Velocity estimation is analysed from quantized data samples obtained from encoder which needs precision in measurement of both time as well as position samples. As such feedback signals suffered from constraints like encoder resolution, quadrature phase offsets, distortion in waveforms, backlash errors inherent in motors accelerated or decelerated motions which cause deviation in actual velocity. In this review paper various techniques for velocity estimation algorithms discussed for electric drives which reduces maximum relative error arrives in measurement and also improves accuracy through interpolation methods that estimates intermediate position samples.

**KEYWORDS:** Incremental Encoder, Velocity Estimation, RBF

## I. INTRODUCTION

Micro motors found a wide variety of applications in the fields like robotics where number of motors works in co-ordination, synchronized form in CNC machines where precise cutting of job-work is done, and optical laboratories required for accurately moving lenses to desired positions. So controlling speed of such electric drives is essential part of servo control systems. For getting information about motors a low cost sensing system integrated within motor called "Incremental encoder", which gives feedback signals when motor rotates. Interpretation of this signals give position as well as direction information about motors. Angular velocity (or Linear velocity for actuator type motors) measurement are formulated from this position information. Important issues occurs in such estimation are nonlinearity errors like backlash errors, quadrature phase errors, distorted sinusoidal pulses, encoder resolution restricted due to manufacturing technology like grating for optical encoder.

All such issues are considered in paper and different techniques are reviewed in Literature Survey section. Conventional algorithms (1) discussed which tries to make trade-off between noise reductions, reliability, tuning of parameters while Adaptive first order windowing proposed improves velocity estimation without trade-off of parameters. An hybrid algorithm proposed (2) for electrical drives that analytically formulates the observation time and corrects this formulation adaptively for both high speed, low speed and even dynamic accelerated or decelerated motions. A neural based approach (3,4) using radial basis function adaptively corrects all imperfection associated with encoder and adaptively interpolates the position data for both high and low speeds by mapping higher order sinusoids. Finally a conclusion presents innovative approach for choosing new kinds of MEMS based motors.

## II. INCREMENTAL ENCODER

Incremental Encoder serves as positional transducer uses two signals generated from optical arrangements such that they are in quadrature phase apart. It uses optically etched discs with transmissive and reflective radial pattern and light detected by photodetectors and whole arrangement integrated in motor. Sometimes a reference mark signal provided when motor completes a rotation also called zero reference. This quadrature signal enables an up-down pulse counter to be interfaced for position counting where a forward motion counts up while reverse decrements the counter value.

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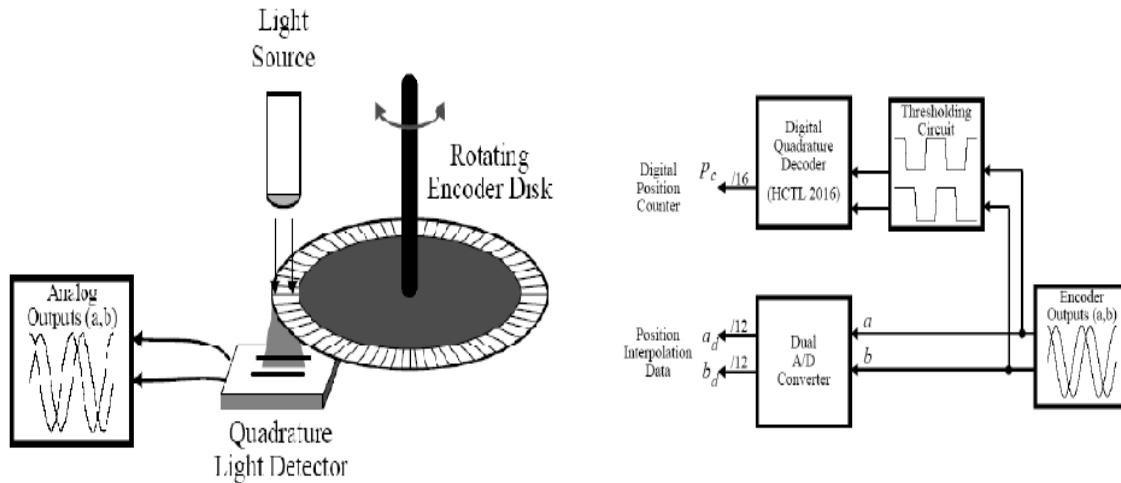


Fig.1 . Principle of Optic Encoder

The figure 1 shows the principle of optic encoder. The principle is applied in project.

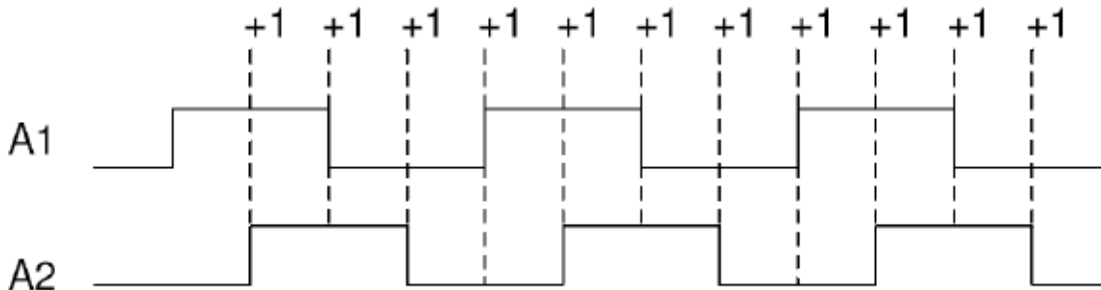


Fig2. Encoder quadrature output pulse trains

### III. VELOCITY ESTIMATION

Position signal  $x(t)$  sampled at  $T$  and  $x(k)$  is true position at time  $k(T)$  and  $y(k)=x(k) + e(k)$  is measurement with error  $e(k)$ . Such that  $-d \leq e(k) \leq d$ , where variances  $V = \text{var}(e(k)) = E(e(k))^2 = \frac{d^2}{3}$  and velocity is given by  $V(k)$  depends upon  $y(i)_{k-n}^k$ , where 'n' window size.

### IV. LITERATURE REVIEW

Different algorithms discussed for velocity estimation:

Farrokh Jannabi-Sharifi, Vincent Haywards, and Chung-Shin J Chen (1) presents method for velocity estimation from quantized position samples. Conventional methods compromise in 1) Noise Reduction 2) Control Delay 3) Estimation accuracy 4) Reliability 5) Computational Load 6) Transient Preservation 7) Difficulty in tuning. In this method 1<sup>st</sup> order adaptive windowing achieves maximum accuracy of estimates and minimizes velocity error tolerances.

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Previous Methods and their features:

a) For finite difference and inverse time method:

Method breaks at higher sampling rate as 'T' becomes very low and even x(k) and x(k-1) difference reduces but (e(k) - e(k-1))/T gets amplified, hence not usable at high time resolution. In inverse time approach, 1/v(k) is estimated but again method fails at high position resolution.

It uses Euler approximation:

$$V = \text{var}(e_k) = E(e_k^2) = \frac{d^2}{3} \quad \text{and} \quad (1)$$

$$\frac{1}{V_k} = \frac{y_k - y_{k-1}}{T} = \frac{x_k - x_{k-1}}{T} + \frac{e_k - e_{k-1}}{T} \quad (2)$$

Method breaks at higher sampling rate as 'T' becomes very low and even x(k) and x(k-1) difference reduces but (e(k) - e(k-1))/T gets amplified, hence not usable at high time resolution. In inverse time approach, 1/v(k) is estimated but again method fails at high position resolution as (x(k) - x(k-1)) even reduces error gets amplified.

$$\frac{1}{V_k} \approx \frac{T}{x_k - x_{k-1}} + \frac{e_k - e_{k-1}}{x_k - x_{k-1}} \quad (3)$$

b) Kalman Filter:

It describes the system by discrete stochastic dynamical equation:

$$X_{k+1} = AX_k + G W_{k+1} \quad (4)$$

$$Y_k = HX_k + e_k \quad (5)$$

$$\text{WHERE } X_k = \begin{bmatrix} X_k \\ \dot{X}_k \\ \ddot{X}_k \end{bmatrix} = \begin{bmatrix} \text{POSITION} \\ \text{VELOCITY} \\ \text{ACCELERATION} \end{bmatrix}; A = \begin{bmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}; H = [1 \quad 0 \quad 0];$$

G is identity matrix and W is gaussian noise in measurement.

Demerits: 1) Kalman Filter needs retuning of process noise covariance 2) Convergence of KF is not always guaranteed 3) Requires 'n<sup>3</sup>' operation not suitable for real time application.

c) Fixed Filter Method:

Assume noisy position signal can be separated into spectral components: 1) a low frequency component from which a velocity estimate can be reliably derived and noisy component filter out, through Butterworth filter, evaluates the weighted sum of filtered and raw velocity estimates from finite differences given by,

$$\bar{v}_k = \sum_{j=1}^n a_j \bar{v}_{k-j} + \sum_{j=0}^n b_j \bar{v}_{k-j} \quad (6)$$

where a & b filter coefficients. As order of 'n' increases filter becomes ideal low pass.

Demerits: 1) Attenuation 2) Phase Lag 3) Cut-off precision

Adaptive Windowing:

a) 1<sup>st</sup> order adaptive windowing:

Euler approximation applied to two position sample is more precise if they are far apart. The larger the window length the smaller the variance and also reduces sampling rate. In order to trade-off precision against reliability window size adaptively adjusted. Window size should be short when velocity is high and large when velocity is low. Noise reduction and precision puts lower bound on window size.

$$d = \|e_k\| \text{ and } \sigma_{vk}^2 = \frac{2d^2}{3n^2T^2}, \frac{y_k - y_{k-n-2d}}{nT} \text{ to } \frac{y_k - y_{k-n+2d}}{nT} \quad (7)$$

thus largest 'n' minimizes variance of velocity error. Hence this method is good over previous methods.

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S.D’Arco- LPiegari-R, Rizzo (2) proposes an “Optimized algorithm for velocity Estimation method for electrical drives” by Hybrid algorithm for digital motor control. Ensures a reduced relative error since in dynamic conditions the error in the measurement of angular velocity with an encoder strictly concern with observation time, a correct method for dimensioning time is discussed.

Fixed time method: The time is fixed and no. of pulses counted. Here space covered is always lack in that of actual. Hence velocity is lower than real velocity. Also algorithm degrades at lower velocity range.

$$\epsilon_h = \text{RELATIVE ERROR} = 1 - \left[ \frac{w r_e T_m}{2 \pi} \right] \times \frac{2 \pi}{w r_e T_m} \rightarrow \epsilon_h \propto \frac{1}{w}$$

where  $T_m, w, r_e$  denotes fixed time, angular frequency ,no. of pulses per revolution while bracket indicates integer part. Fixed space algorithm for fixed no. of pulses time is calculated. Hence time is measured always lack than actual and hence velocity is always higher than actual. Also algorithm fails at higher velocity.

$$\epsilon_{fs} = -1 + \frac{\frac{2\pi f_s N_M}{w r_e}}{\left[ \frac{2\pi f_s N_M}{w r_e} \right]} \rightarrow \epsilon_h \propto w \quad (8)$$

An analytical approach given to dimensioning of Time, 1<sup>st</sup> select fixed time  $T_m$  then some samples from encoder o/p rejected,hence measurement made in reduced time .

$$w_m = \frac{\left[ r_e \frac{T_m - \gamma}{\pi} \right]_{r_e} f_s}{T_m f_s - \frac{\gamma}{w} f_s - \left[ \frac{T_m w - \gamma - \left[ r_e \frac{T_m - \gamma}{\pi} \right]_{r_e}}{w f_s} \right] - 1} ; \epsilon_h = \frac{w}{f_s (w r_e T_m + 2\pi)} \quad (9)$$

Where ‘ $\gamma$ ’ is angle before the 1<sup>st</sup> edge and estimated velocity. As for constant sampling frequency, encoder resolution i.e  $r_e$ , and fixed  $T_m$ ,evaluated velocity is function of  $\gamma$  and real velocity. To remove effect of  $\gamma$ , maximum relative error is given by,

$$e_h = \frac{w (w r_e T_m + 2\pi)}{f_s} \quad (10)$$

Here is other analytical approached for dynamic conditions, like static error for constant acceleration time, the relative error given by,

$$\epsilon_d = \epsilon_s + \frac{a T_m}{2} \quad (11)$$

Where  $a$  is constant acceleration and  $T_m$  =Fixed Time.

Dynamic error depends on errors in the method as well as on speed variation in given observation time  $T_m$ , during last ‘ $m$ ’ estimations of velocity to define ‘mobile mean value’ and dynamic error given by,

$$\epsilon_d = \frac{n r_e}{f_s [n r_e T_m - 60]} + \frac{a T_m}{2n} \quad (12)$$

where  $T_m$  is adjusted to have minimum relative error.

Kok Kiong Tan proposes “New interpolation method for quadrature encoder”(3) and “ Adaptive online correction and interpolation of quadrature encoder pulses using RBF radial basis function” (4).

Paper considers development of 2 stage RBF neural network. 1<sup>st</sup> stage adaptively corrects imperfection in encoder signals such as mean, phase offsets, amplitude deviation and waveform distortion. 2<sup>nd</sup> stage serves as inferencing machine to adaptively map quadrature encoder signals to higher order sinusoids thus enabling intermediate interpolations. In order to compensate sinusoidal imperfections a method introduced by Heydman using least square fitting for error correction and interpolation requires precision ADC & DSP blocks to increase resolution explicitly.

$$\text{RBF algorithm: } f(\bar{x}) = \sum_{i=1}^N w_i \phi(\bar{x})_i \quad (12)$$

$$\phi(\bar{x})_i = \exp \left[ - \left( \left\| \frac{\bar{x}_i - c_i}{2\sigma_i} \right\|^2 \right) \right] \quad (13)$$

Where,  $\phi(\bar{x})$  is basis function and hidden layers contains mean parameter value  $c_i$ .

In (3) from encoder quadrature pulses

$$u_1 = A1 \cos \alpha ; u_2 = A2 \sin \alpha ; \quad (14)$$

from Hyderman practical signal:  $\bar{u}_1 = u_1 + m_1 ; \bar{u}_2 = \frac{A1 \cos(\alpha - \epsilon)}{G} + m_2 \quad (15)$

from simplification:  $k_1 \bar{u}_1^2 + k_2 \bar{u}_2^2 + k_3 \bar{u}_1 \bar{u}_2 + k_4 \bar{u}_1 + k_5 \bar{u}_2 = 1 \quad (16)$

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From least square fitting above parameters optimized to:

$$\epsilon = \sin^{-1} \left( \frac{k_3}{\sqrt{4k_1k_2}} \right); G = \sqrt{\frac{k_2}{k_1}}; m_1 = \frac{2k_2k_4 - k_3k_5}{k_3^2 - 4k_1k_2}; m_2 = \frac{2k_1k_5 - k_3k_4}{k_3^2 - 4k_1k_2}; \quad (17)$$

$$A1 = \sqrt{\frac{4k_2(1+k_1m_1^2+k_2m_2^2+k_3(m_1m_2))}{4k_1k_2-k_3^2}} \quad (18)$$

In (4) for RBF network which updates its parameter as follows:

For 1<sup>ST</sup> stage:

$$e(k)_j^i = d_j^i - u_j^i; E(k)_j^i = \sum_{i=1}^N \frac{e_{jk}^2}{M}; u_{ji} = \sum_{r=1}^N w_{rj}^i \phi_{rj}^i u(k)_j^i; \quad (19)$$

$$\phi(u_{rj}^k)^i = \exp \left( - \left\| \frac{(u(k)_j - c(k)_j)^2}{2\sigma(k)_j^2} \right\| \right) \quad (20)$$

$$w(k)_j = [w(k)_{1j} \quad \dots \quad w(k)_{Nj}]^T \quad (21)$$

$$\phi(k)_j = [\phi(k)_{1j} \quad \dots \quad \phi(k)_{Nj}]^T \quad (22)$$

The updation rule :

$$w(k)_j = w(k-1)_j + \delta(k)_j \lambda(k)_j \quad (23)$$

$$\delta(k)_j = \frac{\phi(k)_j^i}{\|\phi(k)_j\|} \left[ 1 - \frac{\rho e(k-1)_j^i}{|\lambda(k)_j|} \right] \quad (24)$$

$$\lambda(k)_j^i = d_j^i - w(k-1)_j^T \phi(k)_j^i \quad (25)$$

So for above set tuning of 'δ' is according to Lyapunov stability.

For 2<sup>nd</sup> stage RBF network, interpolation of sinusoidal waveforms:

From trigonometry theories we have,

$$\sin(n\alpha) = n \cos^{n-1}(\alpha) \sin(\alpha) - C_n^3 \cos^{n-3}(\alpha) \sin^3(\alpha) + C_n^5 \cos^{n-5}(\alpha) \sin^5(\alpha) - \dots \quad (26)$$

$$\cos(n\alpha) = \cos^n(\alpha) - C_n^2 \cos^{n-2}(\alpha) \sin^2(\alpha) + C_n^4 \cos^{n-4}(\alpha) \sin^4(\alpha) - \dots \quad (27)$$

Here it requires look up table to serve as inference engine. Hence avoid use of precision ADC'S. The paper concludes that: 1) For encoders with Sinusoidal quadrature signals can provide higher interpolation stages than square quadrature signals. 2) higher order sinusoidal still affected by measurement noise. 3) also zero crossing of 'N' order harmonics should be perfectly measured, which constraints limits on maximum interpolation order.

## IV. CONCLUSION

In this review paper all different velocity estimation algorithms discussed and their merits and demerits are analysed. Optical Encoder are inexpensive tool for motor measurement but this kind of encoder readily suffers from any spurious spikes arriving due to electrical noise and cause false measurement hence requires buffering of pulses. Such algorithms eliminates use of high precision ADC's at the cost of lower resolution. Such speed estimation improves control system of servo motors.

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