Study of Flow Forming Process Using a Non-Newtonian Lubricant

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ABSTRACT: During flow forming various soaps are frequently used as lubricant whose characteristics are approximately by a non Newtonian fluid. In this paper the lubrication problems of flow forming has been studied, assuming the lubricant as a non Newtonian one, whose viscosity is function of pressure also. The analysis is initially covered by neglecting elastic deformation of the blank and the roller and then by considering the elastic deformation of both blank and the roller which are generated by the pressure of the fluid. The analysis includes the pressure distribution, shear stress and film thickness. The results so obtained are presented graphically and discussed critically.

NOMENCLATURE

Unless otherwise specified, the following symbols have been used in the paper

c Clearance between the roller and the mandrel

\( D_R \) Diameter of the roller

\( d \) Diameter of the mandrel

\( D \) Reduced diameter

\( D_0 \) Diameter of the under formed blank

\( E_R \) Young’s modulus of the roller material

\( E_B \) Young’s modulus of the blank material

\( E_0 \) Equivalent young’s modulus of the roller and the blank

\( h \) Thickness of the oil film at any point outside the contact zone

\( h_0 \) Thickness of the oil film squeezed into the area of contact between the roller and the blank

\( h_0 = h_0 \left( \frac{x^2}{b} \right) \) Dimensionless thickness of the oil film squeezed in the area of contact between the roller and the blank

\( h_z \) The gap between the roller and the blank

\( N \) Speed of rotation of the mandrel in rpm.

\( n \) Constant (non Newtonian parameter)

\( p \) Pressure

\( p_0 \) Reduced pressure of the lubricant

\( p_m \) Mean pressure at the contact area

\( q \) Load per unit length

\( r_A \) Radius of the point of contact of the roller and mandrel

\( t \) Thickness of the blank

\( t_0 \) Thickness of the undeformed blank

\( U_R \) Peripheral velocity of the roller

\( U_B \) Peripheral velocity of the blank

\( U = U_R + U_B \) Peripheral velocity of the blank and the roller at the contact zone

\( V \) Feed of the roller

\( W_B \) Deflection of the surface of the blank

\( W_R \) Deflection of the surface of the roller
Width of contact in the direction of revolution

Dimensionless value of $X_o$, $X = \left(\frac{X}{X_o}\right)$

Width of elastic contact in the direction of revolution

Width of contact of the roller and the blank

Reduction of thickness per revolution of blank

Semi angle of the conical mandrel

Pressure exponent of viscosity of the lubricant

Absolute viscosity of the lubricant at atmospheric pressure

Poisson’s ratio of the roller material

Poisson’s ratio of the blank material

Radius of round off of the roller

Radius of round off of the mandrel

I. INTRODUCTION

Flow forming is a process for forming sheet metal into the components having axisymmetric like cones, hemispheres or other circular shapes by a combination of rotation and deformation [1-3]. In this process a roller moves along the surface of a mandrel, while the mandrel rotates about its axis with a blank when the working forces are small, blank can be spun without the use of any lubricant, provided that the feed of the roller and the rotation of the blank are kept within the limits of the small critical values, so as to keep the blank surface from being torn off and resulting flakes of metal from sticking the roller. It is economical then other forming processes and can be used for larger production at lower cost [4]. Blank size upto 2 m can be deformed successfully. As the tube diameter reduces, the surface roughness and force increases [5]. In this process the wrinkle failure, the time and the material wastages is significantly reduced whereas product quality, production efficiency and processes repeatability has improved [6]. This accounts for the use of some sort of lubricants e.g. various types of greases, metallic soaps (sodium and lithium soaps etc) tallow etc. in flow forming [1]. Thus study of the lubrication is of importance in flow forming. In one of the studies of the flow forming [7] the lubricant has been assumed to have Newtonian character. Lubrication studies in deep drawing & cold forging operations have been reported recently [8-11]. It has been shown that the behavior of the metallic soaps (sodium and lithium soaps etc.) and other lubricants, generally used in forming, approximates to that of a power law fluid [12]. The author has not come across any work in the area of Flow forming carried out systematically by considering by the non-Newtonian characteristic of the lubricant. As per the previous knowledge no work has been done, which takes into account the non Newtonian properties of the lubricant. Therefore, the aim of the present paper is to develop and discuss the theories of hydrodynamic lubrication of flow forming process, considering the lubricant as a power law fluid.

II. ANALYSIS OF LUBRICATION IN FLOW FORMING

The analysis of the lubrication process has been carried out into stages, firstly by neglecting the elastic deformation of the roller and the blank & then by considering elastic deformation.
Figure 1 shows the flow forming process in which rotating disc of sheet metal is fixed on a conical mandrel, is deformed into a cone shaped cup with the roller. The clearance $c$ between the roller and the mandrel is kept to be $t_0\sin\alpha$ if the roller is assumed to be in contact in a way shown in figure 2 with the blank near the top of the roller (this assumption means two dimensional contacts of two discs). The width of contact in the direction of revolution is given by the equation

$$X_0 = \sqrt{D\Delta y}$$

Figure 2 showing the Schematic Diagram of Flow Forming Lubrication with the oil film squeezed thickness into the area of contact between the roller and the blank

Where $D$, the reduced diameter is given by

$$D = \frac{d_R d}{d_R + d}$$

In the above relation, the thickness of the blank $t$ is assumed to be small as compared to the diameter of the mandrel, $d$, so $\left(\frac{d}{2} + t\right) = \frac{d}{2}$

Further the lubricant is assumed to be a Non Newtonian one, which is approximated by power law fluid for which
\[ \tau = \left( \mu \frac{du}{dy} \right)^n \] (3)

And viscosity \( \mu \) at a pressure \( p \) is given by the relation
\[ \mu = \mu_0 \exp(\gamma p) \] (4)

The effect of temperature on the properties of the lubricant has been neglected in the analysis.

The gap \( h_s \) between the roller and the blank is given by
\[ h_s = \frac{x^2}{D} (X^2 - 1) \] (5)

If the thickness of the oil film squeezed into the area of contact between the roller and the blank is equal to \( h_s \), the oil gap \( h \) at any point outside the contact zone is given by
\[ h = h_0 + h_s \] (6)

### III. PRESSURE DISTRIBUTION

The Reynolds’s equation for the power law fluid is [15]
\[ \frac{dp}{dX} = \frac{\mu_0 \nu_{0} \exp(\gamma p)(h - h_0)^n}{h^{2n+1}} \] (7)

Where \( U = U_R + U_B \)

Now, \( h_0 = \frac{h}{U} \) (8)

Substituting the value of \( h \) from equation (8) into the equation (7) and integrating from \( X = 1 \) to \( X = \infty \) we get
\[ p_0 = \frac{\mu_0 \nu_{0} \exp(\gamma p)(h - h_0)^n}{h^{2n+1}} \int_{1}^{\infty} \frac{(x^2 - 1)^n}{U} \frac{dx}{(h + x^2 - 1)^{2n+1}} \] (9)

Where
\[ p_0 = \int_{p_0}^{P} e^{-\gamma p} dp = \frac{1}{\gamma} (e^{-\gamma p} - 1) \] (10)

From equation (10), \( \log_e (1 - \gamma p_0) = -\gamma p \) (11)

Therefore it is necessary that \( p_0 = \frac{1}{\gamma} \) (12)

**FILM THICKNESS**

If the peripheral velocity of the blank and the roller at the contact zone is assumed to be same, \( U = \pi ND_R \). Thus the thickness of the oil film squeezed into the area of contact between the roller and the blank is given by
\[ h_0 = 3\pi \mu_0 \nu_D D_R \left( \frac{D}{2D} \right) \] (13)

### IV. SHEAR STRESS

The shear stress \( \tau \) for lubricant following power law is given by
\[ \tau = \left( \mu \frac{du}{dy} \right)^n \] (14)

The value of lubricant film thickness is assumed to be very small, the pressure gradient in the regions is small and hence
\[ \frac{du}{dy} = \frac{U}{h} \] (15)

Substituting the value of \( (du/dy) \) from equation (15) into (14), we get
\[ \tau = \left( \mu \frac{U}{h} \right)^n \] (16)

Equation (16) gives the shear stress values for the lubricated flow forming process.
V. ANALYSIS CONSIDERING ELASTIC DEFORMATION

Now, if the elastic deformation of the roller and the blank by the pressure generated with a fluid whose viscosity is itself changed with pressure is considered. Equation (16) is modified by the elasto hydrodynamic equation which has been derived by Ertel and Grubin [2]. In this case, it is assumed that the shape of the surface outside the Hertzian zone is the same whether a lubricant is there or not. The width of contact $X_{op}$ is given by

$$X_{op} = \sqrt{D(\Delta y + W_R + W_B)}$$

According to the theory of contact by Hertz, elastic contact width in the direction of revolution, $X_{oe}$ is given by

$$X_{oe} = \frac{4P_m D^2}{E_0}$$

Where $P_m$ is the mean pressure at the area of contact and $E_0$ is given by

$$\frac{1}{E_0} = \frac{1-V_B^2}{E_B} + \frac{1-V_R^2}{E_R}$$

Using the equation (18) and (19) equation (17) becomes

$$X_{op} = \sqrt{D \left( \Delta y + 16P_m^2 \frac{D^2}{E_0} \right)}$$

When the deformation of the roller and the blank by the pressure generated by the lubricant is also considered, value of $h_s$ is given by

$$h_s = \frac{x^2}{D} + W_R + W_B - \Delta y$$

According to the theory of contact by hertz, the pressure distribution produced by the contact of two discs is semielliptical. Taking this fact into account, equation (21) can be written as

$$h_s = 2q \frac{X_{oe}}{E_0} \frac{X_{oe}}{X_{oe}^2 - 1} - \log \left( \frac{X_{oe}}{X_{oe}^2 + \frac{X_{oe}^2}{X_{oe}^2 - 1}} \right) - \Delta y$$

Where $q$ is the load per unit length and is given by

$$q = 2X_{oe} \rho_m$$

Now the expression within the square brackets in equation (22) can be approximated in the range $1 \leq \frac{X_{oe}}{X_{oe}} \leq 1.8$ by $2.114 \left( \frac{x}{X_{oe}} - 1 \right)^{1.55}$ to a very high accuracy [8]. Thus equation (22) may be written as

$$h_s = 2q \frac{X_{oe}}{E_0} \left[ 2.114 \left( \frac{x}{X_{oe}} - 1 \right)^{1.55} \right] - \Delta y$$

VI. PRESSURE DISTRIBUTION

In this case also, the Reynolds’s equation (7) has the same form, but here $h_s = h - h_0$ is given by equation (23) in place of the value given by equation (5) in the previous case, where the elastic deformations of the roller and the blank were neglected. Thus in this case, Reynolds’s equation (7), takes the following form,

$$\frac{dp}{dx} = \left[ \frac{(n+1)(2n+1)}{n} \right]^n \mu_0 e^{y \left( \frac{q}{E_0} \right)} \left[ 4.228 \left( \frac{x}{X_{oe}} - 1 \right)^{1.55} - \Delta y \right]^{n} / \left[ 3\pi \mu_0 \gamma D_R \frac{D}{A_y} + \frac{4.228q}{E_0} \left( \frac{x}{X_{oe}} - 1 \right)^{1.55} - \Delta y \right]^{2n+1}$$
Integrating equation
\[ p_0 = \left[ \frac{(n+1)(2n+1)}{n} \right] \mu_0 \int_0^\infty \left( \frac{3}{E_0} \right) 4.228X^{1.55} - \Delta y \right]^n dX'/3\pi N\mu_0\gamma D_R \sqrt{\frac{D}{\Delta y} + \frac{4.228}{E_0} (x' - 1)^{1.55} - \Delta y} \]

(24)

Where \( X' = \frac{X \times 10^{-4}}{X_{oc}} \) and \( p_0 \) is given by equation (10)

The integral on the R. H. S. of equation (25) can be evaluated for different values of \( n \).

VII. CALCUATION OF SHEAR STRESS

The shear stress for a lubricant following power law is given as in the previous case, by
\[ \tau = \left( \mu \frac{du}{dy} \right)^n = \left( \mu \frac{V}{h} \right)^n \]

(26)

But in this case, value of \( h_x \) is given by equation (21), as compared to the previous case, where \( h_x \) was given by equation (5) and \( h \) is given by equation (6).

VIII. DISCUSSION AND CONCLUSIONS

In the present paper, an analysis of the lubrication of flow forming has been presented, considering the lubricant as a power law fluid. The value of non-Newtonian parameter \( n \) will vary within reasonable limits and this variation will have considerable effects on the hydrodynamic pressure and the shear stress distribution. The values of various parameters associated with a typical case of flow forming are given below which are based on the process of deformation and the properties of the lubricant.

\[ D (diameter of mandrel at the section under consideration) = 70 \text{ mm.} \]

\[ D_R = 120 \text{ mm, } 2\alpha = 45^0, \mu_0 = 1.02 \times 10^{-4} \text{ kgsec} \text{ mm}^{-1}, \gamma = 7.12 \times 10^{-2} \text{ kg} \text{ mm}^{-2}, P_R = 8 \text{ mm, } n = 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, N = 55, 220, 880 \text{ RPM.} \]

Figure 3

In figure 3 the values of the thickness of oil film squeezed between the roller and blank do not varies throughout the contact length. Whereas in figure 4 the value of oil film thickness gradually decreases along the length.
In figure 5 reduced pressure $p_0$ of the lubricant vs. $h_0$ the dimensionless thickness of oil film squeezed in the area of contact between the roller and the blank have been plotted for different values of $n$ and $N$ for $0.01 \leq h_0 \leq 0.2$. It has been found that as the value of $n$ increases from 1.0 to 1.6 the value of $p_0$ goes on rising. However the value of $p_0$ are negative for $n=1$ to 1.6 at $N=55$ whereas $n=1$ the value of $p_0$ is also negative at $N=880$ but increases and remain positive for higher value of $n$ and remain constant throughout film thickness ($h_0$). In order to avoid the zone of negative pressure, the operation should be carried out at high rpm by using lubricant of high viscosity.
In figure 6 value of $\tau$ the shear stress in the lubricant film, have been plotted for various values of $n$ and $N$. From this figure, it can be seen that as the value of $n$ increases from 1 to 1.6 the value of shear stress gets reduced for all the values of $x$. Also for a value of $n$ the shear stress $\tau$ goes on increasing as the value of $N$ increases from 55 to 880 RPM. For the point $x = x_0$ the value of $x$, corresponding to the point of contact of the roller and the blank), it is seen that for a given value of $n$, the shear stress $\tau$ is the same for all the values of $N$. However, as $n$ increases from 1 to 1.6 value of $\tau$ at $x = x_0$ goes on reducing and is smallest for $n=1.6$. From the above it is clear that the shear stress in the lubricant film goes on reducing as $n$ increases form $n = 1$ to 1.6, although the pressure $p_0$, developed in the oil film, is larger in magnitude for a larger value of $n$, so a more viscous lubricant (having a larger value of $n$, the non Newtonian parameter) is more suitable for flow forming, although the pressures developed are higher in this case.

![Figure 6: Shear stress $\tau$ versus $x$](image1)

![Figure 7: $\log_{10} p_0$ versus $h_0$](image2)
In figure 7 the reduce pressure $p_0$ of the lubricant vs. $h_0$ the dimensionless thickness of oil film squeezed in the area of contact between the roller and blank have been plotted for the different value of $n$ & $N$ for $0.01 \leq h_0 \leq 0.2$ by considering elastic deformation. It has been found that as the value of $n$ increases $p_0$ goes on increasing and also gradually increases as the film thickness increases. As the process of flow forming are generally carried out at high deformation pressure with lubricant of high value of Newtonian parameter, it is better to find out the value of pressure and shear stress distribution by considering elastic deformation taking place during the process.

![Graph showing the comparison of pressure $p_0$ vs. dimensionless thickness $h_0$ with and without elastic deformation for different values of $n$ and $N$.]

Figure 8 shows the superimposition of the reduce pressure $p_0$ of the lubricant vs. $h_0$ the dimensionless thickness of oil film squeezed in the area of contact between the roller and blank with and without elastic deformation have been plotted for the different value. It has been observed that at high speed ($N \cong 650$ rpm), it is advisable that the process is to be carried out without elastic deformation in order to avoid the deformation of the component.

The solution can be extended by considering the effect of temperature on viscosity of lubricant, if needed. It is evident that it is better to use high viscosity lubricant for flow forming process to have better accuracy and precision.

**APPENDIX I**

The integral in equation (9) can be split up into two integrals in the following way

$$\int_1^{\infty} \frac{(x^2-1)^n \, dx}{(x^2-1+h_0)} = \int_1^{\infty} \frac{dx}{(x^2-1)^{n+1}} - (2n + 1) h_0 \int_1^{\infty} \frac{dx}{(x^2-1)^{n+2}} = \frac{\Gamma(\frac{2n+1}{2}) \Gamma(\frac{1-n}{2})}{\Gamma(\frac{1}{2})} - (2n + 1) h_0 \frac{\Gamma(\frac{2n+1}{2}) \Gamma(\frac{1-n}{2})}{2 \Gamma(\frac{1}{2})}$$

The above integral can be evaluated for various values on $n$, as the values of gamma function are available in mathematical handbooks [14].

When $n=1$, the integral in (9) has the value $\left(\frac{1}{4h_0}\right)$ according to [2] in the range $0.01 \leq h_0 \leq 0.2$.

**APPENDIX II**

The thickness reduction per revolution of the blank $\Delta y$, is obtained from figure 9. If the contour profile of the blank $AA$ is deformed as $BB$ after one revolution, the reduction in thickness is given by the sectioned area. Then the mean value of the reduction per one revolution of blank $\Delta y$ is given by
\[
\Delta y = \frac{\int_{0}^{3v/2} \sqrt{\left[ p_R^2 - (x-V)^2 \right] - \left[ p_R^2 - x^2 \right]} \, dx}{v} 
\]

\( (1C) \)

**Figure 9:** Geometry for calculation of \( \Delta y \)

\[
\Delta y = \frac{p_R}{4} \left[ 3 \left( 1 - \left( \frac{v}{2p_R} \right)^2 \right) - 3 \left( 1 - \left( \frac{3v}{2p_R} \right)^2 \right) \right] + \frac{1}{7} \left( \frac{3}{p_R} \right) - \left( 3 \sin^{-1} \left( \frac{3v}{2p_R} \right) - \sin^{-1} \left( \frac{3v}{2p_R} \right) \right) \]

\( (2C) \)

\( \Delta y \) can be calculated for various values of feed of roller, \( v \) and radius of round of, \( p_R \) of the roller.

**APPENDIX III**

In the present paper the effect of pressure on the viscosity of the lubricant has been taken into account. The viscosity \( \mu \) at pressure \( p \) is given by the relation

\[
\mu = \mu_0 e^{\gamma p}
\]

Where \( \mu_0 \) the viscosity at atmospheric pressure and \( \gamma \) is the pressure exponent of viscosity of lubricant. Graphical relationship between “Reduced Pressure” of the lubricant \( p \) and pressure \( p_0 \) is given in figure 9.
Figure 10: reduced pressure $p_0$ versus pressure $p$

REFERENCES