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# Optimal Control of Output Signal of a Chaotic Oscillator Using Genetic Algorithm Based Discrete Time PI Controller

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**ABSTRACT:** In this study, state output variable of Lorenz chaotic oscillator which has nonlinear equations was controlled using reference based negative feedback discrete time PI controller. The coefficients belonging to controller parameters were chosen in the most suitable way by using Genetic optimization algorithm. So, input reference value change was provided independently without regarding linearization point in the control process. The study was implemented in a co-simulation environment in Matlab-Simulink. PI controller parameters optimization was implemented by converting discrete time negative feedback closed loop control system to hardware codes with PI parameters, utilizing genetic algorithms taking place in the Matlab-Optimtool GUI. The simulation results obtained by using controller showed that the suggested method is favorable for variable reference input control of nonlinear chaotic systems.

KEYWORDS: Lorenz chaotic oscillator, Discrete time, Dynamic behavior, Genetics, Optimization, PID control.

#### I. INTRODUCTION

Chaotic systems are deterministic signals which indicates non-linear behaviors. It is quite difficult to follow the behaviors. Initial conditions could significantly affect dynamic behaviors [1]. Dynamic structures of chaos systems are shown with mathematical equations related to situational variables. Chaotic systems with non-linear equations, controlling output of situational variables provides significant adventages to analyze these variables. In addition, it will improve areas of usage of chaotic systems.

In literature, many studies have been performed in a lot of approaches in order to control chaos systems. A lot of time variant and time invariant control strategies have been developed for synchronization of chaos [2]. The dynamic programming technique has been suggested for optimal control of chaotic continuous-time [3]. In another study, Zhang-j. et al. have applied situation feedback technique for optimal control [4]. Park has designed non-linear controller for synchronization of chaotic systems [5]. In another study, Salarieh have developed an adaptive non-linear control approach in order to avoid chaotic shear [6]. Pai has used sliding mode control for stabilizing uncertain timedelay chaotic systems with input non linearity [7]. Köse has suggested passive and sliding mode control for controlling of continuous time non-linear Shimizu-Morioka chaotic system [8]. In another important study, PID controller is implemented in order to investigate the robustness and vibration suppression performance of the closed-loop system [9]. Determining controller coefficient optimally is a necessary in order to performances of aforementioned control methods can be raised at its highest level. Accordingly, a lot of studies have been done for optimization of the controllers in the literature. Kumar et al., the gains coefficients of the PI/PID/2-DOF PID controller for automation generation control are optimized employing a Teaching Learning Based Optimization (TBLO) algorithm [10]. Li et al. develop control algorithms using the model predictive control (MPC) methodology, which incorporates wind forecasts fort he next few hours when determining wind scheduling. The MPC algorithm is developed by solving a non-linear optimization problem to minimize operation costs to the wind power producer [11]. Karer et al., the presented approach enables a simple, computationally tractable and efficient constrained optimization solution for tuning the parameters of the PID



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controller [12]. Köse et al. in order to control the voltage of the Western System Coordinating Council (WSCC) system, the sliding mode parameters have been obtained by using algorithm (GA) optimization technique [13].

In this study, for optimization genetic algorithm have been chosen and then used. Genetic algorithm, for the first time has been invented by John Holland at Michigan University. Holland has used theory of evolution for optimization in genetic algorithms. Genetic algorithms create a solution set consisted of different solution instead of creating one solution for the problems. Thus, a lot of points can be regarded at the same time at search space and as a result, possibility of reaching holistic solutions have been raised [14].

Genetic algorithms provide successful solutions for parameter and system identification, control systems, applications of robot, image and sound recognition, engineering designs, planning, application of artificial intelligence, specialized systems, function and combinational optimization problems, web design problems, road finding problems, social and economic planning problems [14].

In this study, the coefficients belonging to controller parameters were chosen in the most suitable way by using genetic optimization algorithm. The simulation results have confirmed this state.

#### II. DESCRIPTIONS OF THE LORENZ CHAOTIC SYSTEMS

In this study, continuous time Lorenz chaotic oscillator [1] has been chosen to be controlled by a controller structured as PI. Here, the chaotic oscillator of Lorenz has non-linear equations where x, y and z are state variables and a, b, c are positive constant parameters, Equ 1.

$$\dot{x} = \mathbf{a}(y - x) 
\dot{y} = \mathbf{c}x - xz - y 
\dot{z} = xy - \mathbf{b}z$$
(1)

Using the Laplace transformation method [15], t-domain Lorenz equations are transformed to S-domain Lorenz equations as seen in Equ-2. For controlling state variables of the chaotic oscillator the outputs (Uxyz) of the controllers have been put into the equations. The reason of the discrete time PI controller usage, controllers' outputs have been given on z-domain form.

$$X(s) = \frac{1}{s} [aY(s) - aX(s) + U_{x}(s)]$$

$$Y(s) = \frac{1}{s} [cX(s) - X(s)Z(s) - Y(s) + U_{y}(s)]$$

$$Z(s) = \frac{1}{s} [X(s)Y(s) - bZ(s) + U_{z}(s)]$$
(2)

In this study, three different PI(z) algorithm with different parameters have been used for the control of three state variables of Lorenz oscillator. Control signals are integrated the oscillator state variable before the integral operators. Lorenz non-linear oscillator block diagram structure with the designed controller is shown in Figure 1.



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Fig.1. Lorenz MATLAB/SIMULINK chaos control model

s-domain equivalent of PI(t) control algorithms has been given the Equation 3.

$$U_x(s) = e_x(s) \left( K_p + K_i \frac{1}{s} \right)$$
(3)

e(s) function used in Equation 4 is the difference between the input reference value and the output value.  $e_x(s) = R(s) - X(s)$  (4)

Using forward difference method [13], Equation 5, PI(s) has been converted to PI(z) block diagram.

$$s \approx \frac{1}{T_s} \left( \frac{1 - z^{-1}}{z^{-1}} \right)$$
 (5)

Where,  $T_s$  (named with t sample on Simulink), is the sample period [16]. After discrete t operation the PI algorithm states as,

$$U_{x}(z) = e_{z}(z) \left( K_{p} + K_{i} \frac{T_{s} z^{-1}}{(1 - z^{-1})} \right)$$
(6)  
$$e_{x}(z) = R(z) - X^{*}(s)$$
(7)

Using the direct programming method [17] for Equation 6, the Z-domain PI controller transfer functions have been converted to Z-domain PI block diagram based on 1 sample delay, as 1/z, Fig. 2. The task of the key that was used in Fig. 2 is arranged the starting time of the error signal for controller.



Fig.2.Improved critical key



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#### **III. GENETIC ALGORITHM**

The basic principles of GA were first proposed by Holland. The technique was inspired by the mechanism of natural selection, a biological process in which stronger indivisual is likely to be the winners in a competing environment. GA uses a direct analogy of such natural evolution to do global optimization in order to solve highly complex problems. It presumes that the potential solution of a problem is an indivisual and can be represented by a set of parameters. These parameters are regarded as genes of a chromosome and can be structured by a string of concatenated values. The form of variables representation is defined by the encoding scheme. The variables can be represented by binary, real numbers or other forms, depending on the application data. Its range, the search space, is usually defined by the problem. An illustrative flowchart of the GA algorithm implementation is shown in the Fig. 3. In the beginning an initial chromosome is randomly generated. The chromosomes are candidate solutions to the problem. Then, the fitness values of all chromosomes are evaluated by calculating the objective function in decoded form. So, based on the fitness of each indivisual, a group of the best chromosomes is selected through the selection process. The Genetic operators, crossover and mutation, are applied to this surviving population in order to improve the next generation solution. The process continues until the population converges to the global maximum or another stopping criterion is reached.



Fig.3.Flow-chart for Lorenz algorithm structure



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The most crucial step in applying GA is to choose the objective functions that are used to evaluate the fitness of each chromosome. In this study, the integral squared error in the shown Eq. (8) is chosen as the objective function.

$$I_{ise}\left(k\right) = \sum_{k=0}^{n} e^{2}\left(k\right) \tag{8}$$

In shown Fig. 3, the algorithms of the flow-chart has been runned in the Matlab-Optimtool GUI. In this method, in order to optimize parameters of the system, the system is transformed to the discrete time program code and then the transformed system is integrated in the GUI. Firstly, the hybrid time oscillator and controller equations are transformed to discrete time equations to perform this transformation process and then, the expressions given in the Equation 9 are converted to the software codes by using the direct programming technique.

$$e_{x}(z) = \operatorname{Ref}(z) - X^{*}(s)$$

$$X(z) = T_{s} \frac{z^{-1}}{1 - z^{-1}} \left[ aY(z) - aX(z) + e_{x}(z) \left[ K_{p} + T_{s} \frac{z^{-1}}{1 - z^{-1}} K_{i} \right] \right]$$

$$Y(z) = T_{s} \frac{z^{-1}}{1 - z^{-1}} \left[ cX(z) - X(z)Z(z) - Y(z) \right]$$

$$Z(z) = T_{s} \frac{z^{-1}}{1 - z^{-1}} \left[ X(z)Y(z) - bZ(z) \right]$$
(9)

#### **IV. OPTIMIZATION RESULTS**

The important genetic algorithm parameters are given in Table 1.

| Population Size    | 500                  |             |
|--------------------|----------------------|-------------|
| Fitness Scaling    | Rank                 |             |
| Bounds             | [0.3:300]            |             |
| Initial range      | [0.3;300]            |             |
| Selection function | Rulette              |             |
| Mutation function  | Gaussian             | Shrink:0.01 |
| Crossover          | Constraint dependent |             |
| Generation         | max 1000             |             |
| Fitness limit      | 1e-5                 |             |
| Function tolerans  | 1e-5                 |             |

| Table 1.  | Genetic | algorithm | narameters |
|-----------|---------|-----------|------------|
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PI(z) control parameters for each state variable in the optimization process is optimized as independently. But, the optimization results for only x variable are shown in Fig. 4, 5 and Fig. 6.



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Fig.4.The fitness change in the optimization process of  $PI_x(z)$ 



Fig.5.The Kp change in the optimization process of  $PI\_x(z)$ 



Fig.6.The Ki change in the optimization process of PI\_x(z)

Optimization results has been reflected in Table 2.



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#### Table 2. Kp, Ki and fitness values

| Table 2. Kp, Ki and fittless values |       |         |         |  |  |
|-------------------------------------|-------|---------|---------|--|--|
|                                     | Кр    | Ki      | Fitness |  |  |
| PI_x(z)                             | 211.2 | 17623.9 | 6.9e-05 |  |  |
| PI_y(z)                             | 183.1 | 10840.3 | 3.2e-05 |  |  |
| PI z(z)                             | 249.4 | 22184.7 | 0.033e0 |  |  |

#### V. SIMULATION RESULTS

The simulation results are shown in Fig. 7, 8 and Fig. 9. The obtained results using the Ux(z) controller variable for only the x variable is shown in Figure 7. In addition, the change results of the y and z variables is presented in the same graph. 30 value for the x variable has been selected as the reference value. In the 25 seconds, the controller is to be active. With the introduction of the controller, the x variable is seen that immediately reaches the selected reference value.



Fig.7.The time series of the Lorenz chaotic system for x, y and z state variables



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Fig.8.The XY phase portrait of the Lorenz chaotic system

In another case study, Ux(z), Uy(z) and Uz(z) as three different controller is used to the control of state variables x, y and z. The system variables of these three controllers variables have been want to move to 3 different reference point. This reference values is x=30, y=40 and z=50. When controllers are activated in 25 seconds, moved to the desired point of the system variables are shown in Figure 9.



Fig.9.The time series of the Lorenz chaotic system using as three different controller for x, y and z state variables



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#### VI. CONCLUSION

In this work, the output variables of the Lorenz chaotic system having non-linear equations are controlled by means of the optimization of the PI controller known as classical algorithms. The implemented parameter optimization process for the controller is performed by using genetic algorithm (GA). The obtained results showed that the performance of the GA is good for this optimization process that calculated the parameters of the controller in the defined interval. Thus, the non-linear output of the system for the variation input reference tracks the value of reference with minimum calculation time and minimum integrated squared error. Also, x, y, z 1-D, 2-D graphics change have been confirmed each other.

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