

# **Influence of Velocity Slip on the Peristaltic Pumping of a Jeffrey Fluid in a Non Uniform Annulus**

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**ABSTRACT:** Influence of velocity slip on the peristaltic transport of a Jeffrey fluid in a non uniform annulus is investigated. A Sinusoidal wave with is travelling on the outer tube wall and the inner tube is rigid, uniform and moving with a constant velocity. Expressions for velocity, volume flow rate and pressure rise are obtained. The effect of various parameters on the pumping characteristics is discussed with the help of graphs.

**KEYWORDS:** Peristaltic pumping, Jeffrey fluid, non uniform annulus , permeable wall.

## **I. INTRODUCTION**

Peristaltic pumping is a form of material transport induced by a progressive wave of area contraction or expansion along the length of a distensible tube. Peristaltic motion in a channel/tube is now known as an important type of flow occurring in several engineering and physiological processes. In living systems it is a distinctive pattern of smooth muscle contractions that propel the contents of the tube, as food stuffs through oesophagus and alimentary canal, in transport of urine from kidney to bladder, the movement of chyme in the gastro-intestinal tract, vasomotion of the small blood vessels and in many other glandular ducts. Engineers have used the peristaltic mechanism in developing pumps which have several industrial applications.

The problem of mechanism of peristaltic transport has attracted the attention of many researchers since the experimental investigation of Latham [1]. Subsequently a number of analytical, numerical and experimental studies of peristaltic flow of different fluids have been reported under different conditions with reference to physiological and mechanical situations. Ravi Kumar et.al [2] studied the unsteady peristaltic pumping in a finite length tube with permeable wall.

Most of the researchers assumed that blood and the most of the physiological fluids behave like non-Newtonian fluids. They considered various non-Newtonian fluids like Viscoelastic fluid ( Bohme and Friedrich [3] ), power law fluid ( Ravi Kumar et al. [4] ), Casson fluid ( Krishna Kumari et al. [5] ), micro polar fluid ( Srinivasacharya et.al [6] ),

# International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 1, Januray 2016

Herschel-Bulkley fluid (Ramesh Babu [7] al.) in their theoretical investigations.

The Jeffrey model is relatively simpler linear model using time derivatives instead of converted derivatives for example the Oldroyd – B model does, it represents a rheology different from the Newtonian. Krishna Kumari et al.[8 ] studied the peristaltic transport of a Jeffrey fluid in a porous tube by considering Beavers – Joseph conditions and Saffman conditions. Peristaltic pumping of a Jeffrey fluid under the effect of a magnetic field in an inclined channel is studied by Krishna Kumari et al. [9].

Gupta and Seshadri [10] discussed the peristaltic flow through non uniform channels and tubes with particular reference to the flow of spermatic fluid in vas deferens considering the inertia terms to be small in comparison to the viscous terms. Chaturani and Ranganatha [11] discussed a mathematical model for solute transfer in ultra filtering glomerular capillaries, where capillaries are taken as permeable tubes. Further it is reported that the blood flow in small vessels occurs due to peristalsis. Slip effect on the flow of a fractional second grade fluid through a cylindrical tube is analyzed by Rathod.V et al.[12].

With the above discussion in mind, we propose to study the peristaltic transport of Jeffrey fluid through the gap between coaxial tubes, where the outer tube is non-uniform and has a sinusoidal wave traveling down its wall and the inner one is a rigid and uniform tube and moving with a constant velocity V. This investigation may have application in many clinical applications such as the endoscope problem.

## Jeffrey model

The constitutive equations for an incompressible Jeffrey fluid are

$$\bar{T} = -\bar{p}\bar{I} + \bar{S}$$

$$\bar{S} = \frac{\mu}{1+\lambda_1} \left( \frac{\partial \bar{\gamma}}{\partial t} + \lambda_2 \frac{\partial^2 \bar{\gamma}}{\partial t^2} \right)$$

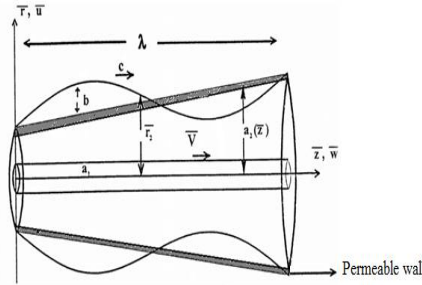
where  $\bar{T}$  and  $\bar{S}$  are Cauchy stress tensor and extra stress tensor,  $\bar{P}$  is the pressure,  $\bar{I}$  is the identity tensor,  $\lambda_1$  is the ratio of the relaxation to retardation times,  $\lambda_2$  is the retardation time and  $\bar{\gamma}$  is the shear rate.

## II. MATHEMATICAL FORMULATION OF THE PROBLEM

Let  $a_1$  be the radius of the inner tube and  $a_2(\bar{z})$  be the radius of the outer tube at any co-axial distance  $\bar{z}$  from the

inlet.(Fig.1) ; The geometry of the wall surfaces are  $\bar{r}_1 = a_1$  (1)

$$\bar{r}_2 = a_2(\bar{z}) + b \sin \left[ \frac{2\pi}{\lambda} (\bar{z} - ct) \right] \quad (2)$$



**Fig. 1. Physical Model**

where  $a_2(\bar{z}) = a_3 + k\bar{z}$  ( $k = 0$  corresponds to uniform tube) (3)

Here  $a_3$  is the radius of the outer tube at the inlet,  $k$  ( $\ll 1$ ) is a constant whose magnitude depends on the length of the outer tube,  $b$  is the wave amplitude,  $\lambda$  is the wave length and  $\bar{t}$  is the time. We choose a cylindrical co-ordinate system  $(\bar{r}, \bar{z})$  where the  $\bar{z}$ -axis lies along the centre line of the inner and outer tubes and  $\bar{r}$  is the distance measured radially.

The equations of motion of the flow in the annulus between the inner and the outer tubes are

$$\frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} (\bar{r} \bar{u}) + \frac{\partial \bar{w}}{\partial \bar{z}} = 0 \tag{4}$$

$$\rho \left( \frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{u}}{\partial \bar{r}} + \bar{w} \frac{\partial \bar{u}}{\partial \bar{z}} \right) = - \frac{\partial \bar{p}}{\partial \bar{r}} + \frac{\mu}{1 + \lambda_1} \left\{ \frac{\partial}{\partial \bar{r}} \left( \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} (\bar{r} \bar{u}) \right) + \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} \right\} \tag{5}$$

$$\rho \left( \frac{\partial \bar{w}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{w}}{\partial \bar{r}} + \bar{w} \frac{\partial \bar{w}}{\partial \bar{z}} \right) = - \frac{\partial \bar{p}}{\partial \bar{z}} + \frac{\mu}{1 + \lambda_1} \left\{ \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left( \bar{r} \frac{\partial \bar{w}}{\partial \bar{r}} \right) + \frac{\partial^2 \bar{w}}{\partial \bar{z}^2} \right\} \tag{6}$$

where  $\bar{u}$ ,  $\bar{w}$  are velocity components in  $\bar{r}$  and  $\bar{z}$  directions respectively,  $\bar{p}$  is the pressure,  $\mu$  is the viscosity coefficient and  $\lambda_1$  is the Jeffrey parameter.

The boundary conditions are

$$\bar{u} = 0, \quad \bar{w} = V \quad \text{at} \quad \bar{r} = \bar{r}_1 \tag{7}$$

$$\bar{u} = \frac{\partial \bar{r}_2}{\partial \bar{t}}, \quad \bar{w} = - \frac{\sqrt{k}}{\alpha} \frac{\partial \bar{w}}{\partial \bar{r}} \quad \text{at} \quad \bar{r} = \bar{r}_2 \tag{8}$$

we introduce the following non-dimensional quantities

$$z = \frac{\bar{z}}{\lambda}, \quad r = \frac{\bar{r}}{a_3}, \quad w = \frac{\bar{w}}{c}, \quad u = \frac{\lambda \bar{u}}{a_3 c}, \quad p = \frac{a_3^2}{\lambda \mu c} \bar{p}, \quad t = \frac{\bar{t} c}{\lambda}, \quad \text{Re} = \frac{\rho c a_3}{\mu}, \quad \delta = \frac{a_3}{\lambda}, \quad Da = \frac{k}{a_3^2}, \quad v = \frac{\bar{v}}{c}, \quad r_1 = \frac{\bar{r}_1}{a_3},$$

# International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 1, Januray 2016

$r_2 = \frac{\bar{r}_2}{a_3} = 1 + \frac{\lambda z k}{a_3} + \phi \text{Sin}[2\pi(z-t)]$ , where  $\phi = \frac{b}{a_3} < 1$  (amplitude ratio),  $\text{Re}$  is the Reynolds number  $\delta$  is the

wave number ratio and  $V$  is the constant velocity of the inner tube.

By using the non-dimensional quantities, the equations (4) to (6) become,

$$\frac{1}{r} \frac{\partial}{\partial r} \left( \frac{ru}{1+\lambda_1} \right) + \frac{\partial w}{\partial z} = 0 \quad (9)$$

$$\frac{\text{Re} \delta^3}{1+\lambda_1} \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} \right) = \frac{-\partial p}{\partial r} + \delta^2 \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{ru}{1+\lambda_1} \right) \right) + \delta^4 \frac{\partial^2 u}{\partial z^2} \quad (10)$$

$$\frac{\text{Re} \delta}{1+\lambda_1} \left( \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right) = \frac{-\partial p}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial w}{\partial r} \right) + \delta^2 \frac{\partial^2 w}{\partial z^2} \quad (11)$$

Using the long wave length, low Reynolds number assumptions and dropping terms of  $\delta$  and higher order, the Eqn. (10) to (11) become,

$$\frac{\partial p}{\partial r} = 0 \quad (12)$$

$$\frac{\partial p}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{r}{1+\lambda_1} \frac{\partial w}{\partial r} \right) \quad (13)$$

The dimensionless boundary conditions are

$$w = V \quad \text{at} \quad r = r_1 \quad (14)$$

$$w = \frac{-\sqrt{Da}}{\alpha} \frac{\partial w}{\partial r} \quad \text{at} \quad r = r_2(z,t) = 1 + \frac{\lambda z k}{a_3} + \phi \cdot \text{Sin}[2\pi(z-t)] \quad (15)$$

where  $\alpha$  is the slip parameter and  $Da$  is the Darcy number.

### III. SOLUTION OF THE PROBLEM

Solving the Eqns. (12) and (13) together with boundary conditions (14) and (15), we get the velocity field as

$$w = \frac{\partial p}{\partial z} \left( \frac{r^2 - r_1^2}{4} \right) (1+\lambda_1) + V - \frac{r_2 \alpha (1+\lambda_1)}{r_2 \alpha \log \left( \frac{r_2}{r_1} \right) + \sqrt{Da}} \left\{ \frac{\partial p}{\partial z} \left( \frac{r_2^2 - r_1^2}{4} \right) + \frac{V}{1+\lambda_1} + \frac{\sqrt{Da}}{\alpha} \left( \frac{r_2}{4} \right) \right\} \log \left( \frac{r}{r_1} \right) \quad (16)$$

The instantaneous volume flow rate  $Q(z,t)$  is given by

$$Q = \int_{r_1}^{r_2} 2\pi r \left[ \frac{\partial p}{\partial z} \left( \frac{r^2 - r_1^2}{4} \right) (1 + \lambda_1) + V - \frac{r_2 \alpha (1 + \lambda_1)}{r_2 \alpha \log \left( \frac{r_2}{r_1} \right) + \sqrt{Da}} \left\{ \frac{\partial p}{\partial z} \left( \frac{r_2^2 - r_1^2}{4} \right) + \frac{V}{1 + \lambda_1} + \frac{\sqrt{Da}}{\alpha} \left( \frac{r_2}{4} \right) \right\} \log \left( \frac{r}{r_1} \right) \right] dr \quad (17)$$

The time mean flow over time period  $T = \lambda/c$  is given by

$$\bar{Q}(z, t) = \frac{1}{T} \int_0^T Q(z, t) dt \quad (18)$$

From equation (17), we get the pressure gradient as

$$\frac{\partial p}{\partial z} = \frac{Q \left( r_2 \alpha \log \left( \frac{r_2}{r_1} \right) + \sqrt{Da} \right) + 2\pi r_2 \alpha V \left( \frac{r_2^2}{2} \log \left( \frac{r_2}{r_1} \right) - \left( \frac{r_2^2 - r_1^2}{4} \right) \right)}{\left( \frac{r_2^2 - r_1^2}{4} \right) (1 + \lambda_1) \left\{ \left( \frac{r_2^2 - r_1^2}{4} \right) 2\pi c_1 - 2\pi r_2 \alpha \left( \frac{r_2^2}{2} \log \left( \frac{r_2}{r_1} \right) - \left( \frac{r_2^2 - r_1^2}{4} \right) \right) \right\} - (1 + \lambda_1) \left( \frac{r_2^2}{2} \right) \frac{\sqrt{Da}}{\alpha} c_1} \quad (19)$$

The pressure rise  $\Delta p$  is given by,  $\Delta p = \int_0^\Lambda \frac{\partial p}{\partial z} dz$  where  $\Lambda = L/\lambda$ ,  $L$  is length of the tube,  $\lambda$  is wave length

#### IV. RESULTS AND DISCUSSIONS

The effect of Jeffrey parameter  $\lambda_1$  on the pumping characteristics for uniform and non-uniform cases are discussed through Fig 2. It is noticed that for a give time mean flow rate  $\bar{Q}$ , the pressure rise decreases with increasing  $\lambda_1$  in all the three regions i.e. pumping region ( $\Delta p > 0$ ), co-pumping region ( $\Delta p < 0$ ) and free pumping region ( $\Delta p = 0$ ). It is observed that the pumping is more in uniform than in non uniform annulus for any given  $\bar{Q}$ . Also the pumping is more for a Newtonian fluid and the pumping decreases as the non Newtonian nature of the fluid increases.

The effect of slip parameter  $\alpha$  on the pumping phenomenon is observed from Figs 3 & 4. It is concluded from these Figs that the pressure rise decreases with increasing  $\alpha$  for a fixed  $\bar{Q}$ . Also for a given  $\Delta p$ , the increase in  $\alpha$  decreases the flow rate in the pumping region. The variation of  $z$  affects the pumping phenomenon as the pumping is more for uniform case than non uniform case.

From figures (5) and (6) the effect of Darcy number  $Da$  is noticed. For a given Darcy number  $Da$  the pressure rise decreases for increasing  $\bar{Q}$ . It is observed that an increase in the flow rate decreases the pressure rise. Also for fixed flow rate, the pressure rise increases with increasing  $Da$ . For a given  $Da$ , the pressure rise is more for non uniform case than the uniform case. The effect of non Newtonian behavior is also observed from these figures.

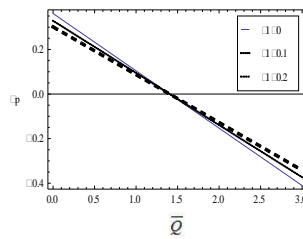
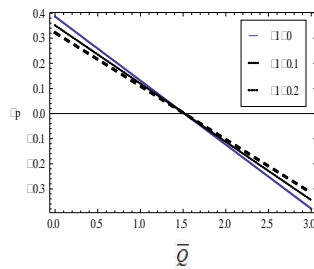
From equation (19), we calculated the pressure rise  $\Delta p$  as a function of  $\bar{Q}$  for different values of  $r_1$  (radius

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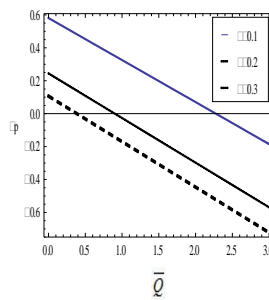
Vol. 5, Issue 1, Januray 2016

of the inner tube) and is shown in Figures (7) and (8) considering uniform and non uniform cases. in the case of uniform annulus. It is observed that for fixed flow rate, the pressure rise  $\Delta p$  increases with increasing  $r_1$ . It is observed that for a given  $\Delta p$ , the increase in  $r_1$  increased the flow rate in all the three pumping regions.

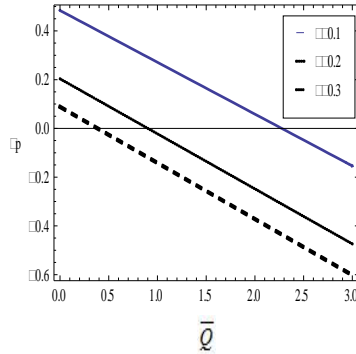


(b)

Fig. 2. Variation of pressure rise  $\Delta p$  with  $\bar{Q}$  for different values of Jeffrey parameter  $\lambda_1$  (a) uniform annulus (b) non uniform annulus.



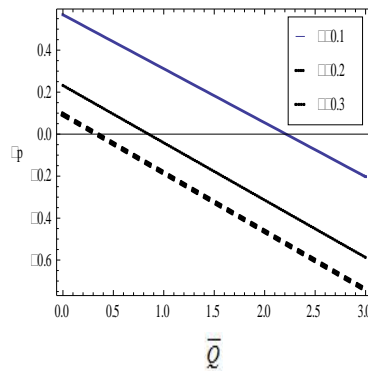
(a)



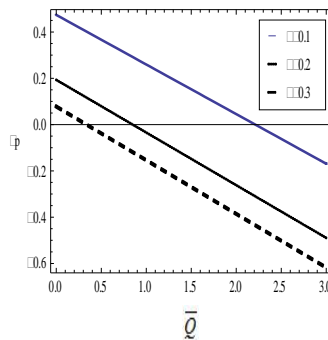
(b)

Fig. 3. Variation of pressure rise  $\Delta p$  with  $\bar{Q}$  for a uniform annulus for different values of slip parameter  $\alpha$  for (a)

Newtonian (b) Non Newtonian fluids



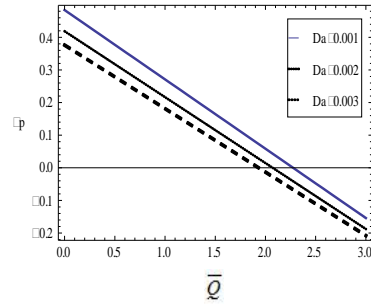
(a)



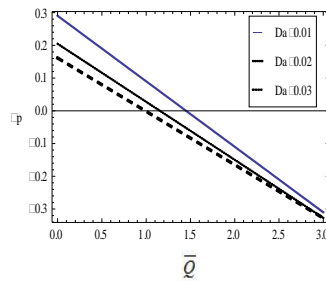
(b)

Fig 4. Variation of pressure rise  $\Delta p$  with  $\bar{Q}$  for a non-uniform annulus for different values of  $\alpha$  with (a) Newtonian

(b) Non Newtonian fluids



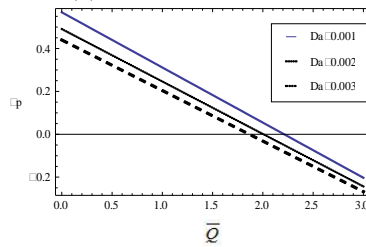
(a)



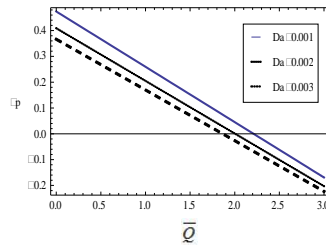
(b)

Fig. 5. Variation of pressure rise  $\Delta p$  with  $\bar{Q}$  for a uniform annulus for different values of Da with (a) Newtonian

(b) Non Newtonian fluids



(a)



(b)

Fig.6. Variation of pressure rise  $\Delta p$  with  $\bar{Q}$  for a non-uniform annulus for different values of Da with with (a)

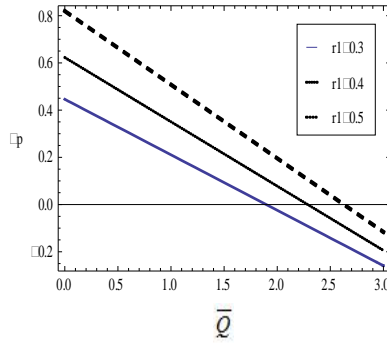
Newtonian (b) Non Newtonian fluids



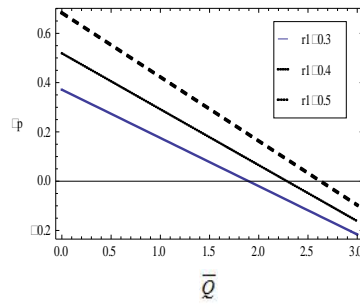
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Vol. 5, Issue 1, Januray 2016

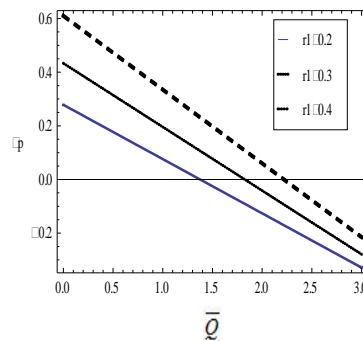


(a)

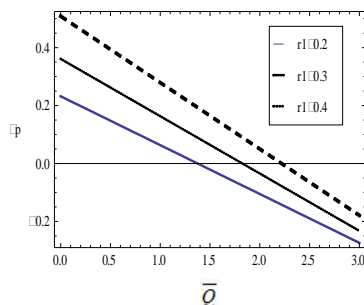


(b)

Fig. 7. Variation of pressure rise  $\Delta p$  with  $\bar{Q}$  for a uniform annulus for different values of  $r_1$  with (a) Newtonian (b) Non Newtonian fluids



(a)



(b)

Fig.8. Variation of pressure rise  $\Delta p$  with  $\bar{Q}$  for a non-uniform annulus for different values of  $r_1$  with (a) Newtonian  
(b) Non Newtonian fluids

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