

Distributional Generalized Two Dimensional Fractional Sine Transform and Its Operators

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ABSTRACT: The purpose of this paper is two dimensional fractional sine transform is extended in the distributional two dimensional generalized sense. Operators on the testing function E and its dual space E^* are obtained.

KEYWORDS: fractional Fourier transform, fractional Cosine transform, fractional Sine transform.

I.INTRODUCTION

The FrFT is a generalization of the ordinary Fourier transform [1] with an order parameter α and is identical to the ordinary Fourier transform when this order α is equal to $p/2$. Since the ordinary Fourier transform and related techniques are of importance in various different areas like communications, signal processing and control systems, it is natural to expect the FrFT to find many applications in these fields as well. In fact, the FrFT has already found many applications in the areas of signal processing and communications [2-3].

We know that the Cosine and Sine transforms and their discrete versions are useful tools in signal and image processing, such as signal coding [4], watermarking [5] and restoration of de-focused images [6]. In the Ref. [7]

Pei and Yeh extended the Cosine transform to the discrete fractional cosine transform (DFrCT) and the discrete fractional sine transform (DFrST). Both of them possess well the angle additivity property of the DFrFT.

Moreover, the DFrCT and DFrST are used in the digital computation of FrFT for the reducing computational load of the DFrFT.

The success of FrFT in its application has promoted the development of other kinds of fractional transforms like fractional Hartley transform, fractional Hadamard transform, fractional cosine transform and fractional sine transform (FrST). Pei Soo-Chang redefined the fractional cosine transform and fractional sine transform based on fractional Fourier transform in 2001 [8-9]. FrST is the extension of sine transform and it has been widely used in domain of digital signal and image processing.

1.1. Generalized two dimensional fractional Sine transform

Two dimensional fractional Sine transform with parameter α $f(x, y)$ denoted by $F_S^\alpha(x, y)$ perform a linear operation given by the integral transform.

$$F_S^\alpha\{f(x, y)\}(u, v) = \int_0^\infty \int_0^\infty f(x, y)K_\alpha(x, y, u, v) dx dy \dots \dots \dots (1.1)$$

Where the kernel,

$$K_S^\alpha(x, y, u, v) = \sqrt{\frac{1-icota}{2\pi}} e^{\frac{i(x^2+y^2+u^2+v^2)cota}{2}} e^{i(\frac{\pi}{2}-\alpha)} \sin(coseca. ux) . \sin(coseca. vy) \dots \dots \dots (1.2)$$

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1.2. The test function space E

An infinitely differentiable complex valued function ϕ on R^n belongs to $E(R^n)$ if for each compact set $I \subset S_{a,b}$, where,

$$S_{a,b} = \{x, y: x, y \in R^n, |x| \leq a, |y| \leq b, a > 0, b > 0\}, I \in R^n$$

$$\gamma_{E,p,q}(\phi) = \sup_{x,y} |D_{x,y}^{p,q} \phi(x, y)| < \infty \text{ Where, } p, q = 1, 2, 3, \dots$$

Thus $E(R^n)$ will denote the space of all $\phi \in E(R^n)$ with support contained in $S_{a,b}$

Note that the space E is complete and therefore a Frechet space. Moreover, we say that f is a fractional sine transformable, if it is a member of E^* , the dual space of E.

In the present work, two dimensional fractional sine transform is extended in the distributional two dimensional generalized sense. Operators on the testing function E and its dual space E^* as shifting operator, differential operator, ad joint shifting operator, ad joint differential operator etc. are obtained

II. DISTRIBUTIONAL TWO-DIMENSIONAL FRACTIONAL SINE TRANSFORM

The two dimensional distributional fractional Cosine transform of $f(x, y) \in E^*(R^n)$ defined by

$$F_S^\alpha \{f(x, y)\} = F^\alpha(u, v) = \langle f(x, y), K_\alpha(x, y, u, v) \rangle \dots \dots \dots (2.1)$$

$$K_S^\alpha(x, y, u, v) = \sqrt{\frac{1 - i \cot \alpha}{2\pi}} e^{\frac{i(x^2 + y^2 + u^2 + v^2) \cot \alpha}{2}} e^{i(\frac{\pi}{2} - \alpha)} \sin(\operatorname{cosec} \alpha . ux) . \sin(\operatorname{cosec} \alpha . vy) \dots \dots (2.2)$$

Where , RHS of equation (2.1) has a meaning as the application of $f \in E^*$ to $K_\alpha(x, y, u, v) \in E$

III. SHIFTING OPERATOR

If $\phi(x, y) \in E$ and a and b are real numbers then $\phi(x + a, y + b) \in E$, $x + a > 0, y + b > 0$ where $\phi(x, y) = K_S^\alpha(x, y, u, v)$

Proof: consider

$$\gamma_{E,p,q}(\phi(x + a, y + b)) = \sup_{x,y \in I} |D_x^p D_y^q \phi(x + a, y + b)|$$

$$\gamma_{E,p,q}(\phi(x + a, y + b)) = \sup_{x,y \in I} |D_x^p D_y^q K_S^\alpha(x + a, y + b, u, v)|$$

$$\gamma_{E,p,q}(\phi(x + a, y + b)) = \sup_{x,y \in I} |D_x^p D_y^q K_S^\alpha(x + a, u, y + b, v)|$$

$$\gamma_{E,p,q}(\phi(x + a, y + b)) = \sup_{x,y \in I} |D_x^p . D_y^q . K_S^\alpha(x', u, y', v)|$$

$$< \infty$$

For any fixed x' and y' , $u, v \in R^n$ and any fixed p, q, $0 < \alpha \leq \frac{\pi}{2}$

Thus $\phi(x + a, y + b) \in E$, $x + a > 0, y + b > 0$

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IV.THE TRANSLATION (SHIFTING) OPERATOR

$T: \varphi(x, y) \rightarrow \varphi(x + a, y + b)$ Is a topological automorphism on E, for $a > 0, y + b > 0$

V. If $\varphi(x, y) \in E$ and $a > 0, b > 0$ then $\varphi(ax, by) \in E$

Proof: consider

$$\gamma_{E,p,q}(\varphi(ax, by)) = \sup_{x,y \in I} |D_x^p D_y^q \varphi(ax, by)|$$

$$\gamma_{E,p,q}(\varphi(ax, by)) = \sup_{x,y \in I} |D_x^p D_y^q K_s^\alpha(ax, by, u, v)|$$

$$\gamma_{E,p,q}(\varphi(ax, by)) = \sup_{x,y \in I} |D_x^p D_y^q K_s^\alpha(ax, u, by, v)|$$

$$\gamma_{E,p,q}(\varphi(ax, by)) = \sup_{x,y \in I} |D_x^p D_y^q K_s^\alpha(x', u, y', v)|$$

For any fixed x' and $y', u, v \in R^n$ and any fixed $p, q, 0 < \alpha \leq \frac{\pi}{2}$

Thus $\varphi(ax, by) \in E, x + a > 0, y + b > 0$

VI. If $\varphi(x, y) \in E$ and $a > 0, b > 0$ then scaling operator $R: E \rightarrow E$ defined $R\varphi = \psi$

where $\varphi(x, y) = \varphi(ax, by)$ is topological automorphism. Combining result 2 and 3, immediately yields next proposition.

VII. Proposition: if $a, b, c, d \in R$ such that $x + a > 0, y + b > 0$ and $cx > 0, dy > 0$

Then the shifting scaling operator $S: E \rightarrow E$ defined $S(\varphi) = \psi$

Where $\psi(x, y) \rightarrow \varphi(c(x + a), d(y + b))$ is a topological automorphism

VIII.DIFFERENTIAL OPERATOR

The operator $\varphi(x, y) \rightarrow D_{x,y}\varphi(x, y)$ is defined on the space E and transform this space E into itself, where $\varphi(x, y) = K_s^\alpha(x, y, u, v)$

Proof: let $D_{x,y}\varphi(x, y) = \varphi(x, y)$

$$\gamma_{E,p,q}(\varphi(x, y)) = \sup_{x,y \in I} |D_x^p D_y^q \varphi(x, y)|$$

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$$\gamma_{E,p,q}(\varphi(x, y)) = \sup_{x,y \in I} |D_x^p D_y^q D_{x,y} \varphi(x, y)|$$

$$\gamma_{E,p,q}(\varphi(x, y)) = \sup_{x,y \in I} |D_x^{p+1} D_y^{q+1} K_s^\alpha(x, y, u, v)| \dots \dots \dots (1)$$

$$D_x^{p+1} K_s^\alpha(x, y, u, v)$$

$$= \sqrt{\frac{1 - icota}{2\pi}} e^{\frac{i}{2}(u^2+v^2+x^2+y^2)cota} e^{i(\frac{\pi}{2}-\alpha)} \sin(csc\alpha \cdot vy) \sum_{n=0}^{p+1} \sum_{r=0}^k \binom{p}{n} \frac{n!}{(k-2r)! r!} (icota)^{k-r} (2x)^{(k-2r)} (csc\alpha \cdot ux)^{p-n}$$

$$\sin\left(csc\alpha \cdot ux + \frac{(p+1-n)\pi}{2}\right)$$

$$D_{x,y}^{p+1,q+1} K_s^\alpha(x, y, u, v) = \sqrt{\frac{1 - icota}{2\pi}} e^{i(\frac{\pi}{2}-\alpha)} e^{\frac{i}{2}(x^2+y^2+u^2+v^2)cota} \sum_{m=0}^q \sum_{s=0}^l \sum_{n=0}^p \sum_{r=0}^k \binom{p}{n} \binom{q}{m} \frac{n!}{(k-2r)! r!}$$

$$\frac{m!}{(l-2s)! s!} (icota)^{k+l-r-s} (2x)^{(k-2r)} (2y)^{(l-2s)} (csc\alpha \cdot ux)^{p-n} (csc\alpha \cdot vy)^{q-m}$$

$$\sin\left(csc\alpha \cdot ux + \frac{(p-n)\pi}{2}\right) \sin\left(csc\alpha \cdot vy + \frac{(q-m)\pi}{2}\right)$$

Put in equation (1)

$$\gamma_{E,p,q}(\varphi(x, y)) = \sup_{x,y \in I} \left| \sqrt{\frac{1 - icota}{2\pi}} e^{\frac{i}{2}(x^2+y^2+u^2+v^2)cota} e^{i(\frac{\pi}{2}-\alpha)} \sum_{m=0}^q \sum_{s=0}^l \sum_{n=0}^p \sum_{r=0}^k \binom{p}{n} \binom{q}{m} \frac{n!}{(k-2r)! r!} \right.$$

$$\left. \frac{m!}{(l-2s)! s!} (icota)^{k+l-r-s} (2x)^{(k-2r)} (2y)^{(l-2s)} (csc\alpha \cdot ux)^{p-n} (csc\alpha \cdot vy)^{q-m} \right.$$

$$\left. \sin\left(csc\alpha \cdot ux + \frac{(p-n)\pi}{2}\right) \sin\left(csc\alpha \cdot vy + \frac{(q-m)\pi}{2}\right) \right|$$

$$< \infty$$

For any fixed $x, y \in R^n$ and fixed p and q , $0 < \alpha \leq \frac{\pi}{2}$

$\therefore \varphi(x, y) \in E$

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$\therefore D_{x,y}\varphi(x, y) \in E$

IX. ADJOINT SHIFTING OPERATOR

The adjoint shifting operator is a continuous function from E^* to E^* . the operator of (2) is

$f(x, y) \rightarrow f(x - a, y - b)$ Leads to the operation transform formula.

Solution: $F_\alpha^c \{f(x + a, y + b)\} = \langle f(x + a, y + b), K_c^\alpha(x, y, u, v) \rangle$

$F_\alpha^c \{f(x + a, y + b)\} =$

$$\langle f(x + a, y + b), \sqrt{\frac{1 - icota}{2\pi}} e^{\frac{i}{2}(x^2+y^2+u^2+v^2)cota} e^{i(\frac{\pi}{2}-a)} \sin(csc\alpha \cdot ux) \sin(csc\alpha \cdot vy) \rangle$$

$$F_\alpha^c \{f(x + a, y + b)\} = \langle f(x, y), \sqrt{\frac{1 - icota}{2\pi}} e^{\frac{i}{2}((x+a)^2+(y+b)^2+u^2+v^2)cota} e^{i(\frac{\pi}{2}-a)} \sin(csc\alpha \cdot u(x + a)) \sin(csc\alpha \cdot v(y + b)) \dots (1).$$

$$e^{\frac{i}{2}((x+a)^2+(y+b)^2+u^2+v^2)cota} e^{i(\frac{\pi}{2}-a)} \sin(csc\alpha \cdot u(x + a)) \sin(csc\alpha \cdot v(y + b))$$

$$= e^{\frac{i}{2}(x^2+y^2+2xa+2yb+a^2+b^2+u^2+v^2)cota} e^{i(\frac{\pi}{2}-a)} \sin(csc\alpha \cdot (ux + ua)) \sin(csc\alpha \cdot (vy + vb))$$

$$e^{\frac{i}{2}((x+a)^2+(y+b)^2+u^2+v^2)cota} e^{i(\frac{\pi}{2}-a)} \sin(csc\alpha \cdot u(x + a)) \sin(csc\alpha \cdot v(y + b))$$

$$= e^{\frac{i}{2}(x^2+y^2+u^2+v^2)cota} e^{i(\frac{\pi}{2}-a)} e^{\frac{i}{2}(2xa+2yb+a^2+b^2)cota} [\sin(csc\alpha ux) \cos(csc\alpha ua) + \cos(csc\alpha ux) \sin(csc\alpha ua)] [\sin(csc\alpha vy) \cos(csc\alpha vb) + \cos(csc\alpha vy) \sin(csc\alpha vb)]$$

$$e^{\frac{i}{2}((x+a)^2+(y+b)^2+u^2+v^2)cota} e^{i(\frac{\pi}{2}-a)} \sin(csc\alpha \cdot u(x + a)) \sin(csc\alpha \cdot v(y + b))$$

$$= e^{\frac{i}{2}(x^2+y^2+u^2+v^2)cota} e^{i(\frac{\pi}{2}-a)} e^{\frac{i}{2}(2xa+2yb+a^2+b^2)cota}$$

$$\begin{bmatrix} \sin(csc\alpha ux) \cos(csc\alpha ua) \sin(csc\alpha vy) \cos(csc\alpha vb) \\ + \sin(csc\alpha ux) \cos(csc\alpha ua) \cos(csc\alpha vy) \sin(csc\alpha vb) \\ + \cos(csc\alpha ux) \sin(csc\alpha ua) \sin(csc\alpha vy) \cos(csc\alpha vb) \\ + \cos(csc\alpha ux) \sin(csc\alpha ua) \cos(csc\alpha vy) \sin(csc\alpha vb) \end{bmatrix}$$

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Let

$$P = \cos(\text{csc}a u) \quad Q = \cos(\text{csc}a v) \quad R = \sin(\text{csc}a u) \quad S = \sin(\text{csc}a v)$$

$$\begin{aligned} & e^{\frac{i}{2}((x+a)^2+(y+b)^2+u^2+v^2)\text{cota}} e^{i(\frac{\pi}{2}-a)} \sin(\text{csc}a \cdot u(x+a)) \sin(\text{csc}a \cdot v(y+b)) \\ &= e^{\frac{i}{2}(x^2+y^2+u^2+v^2)\text{cota}} e^{\frac{i}{2}(2xa+2yb+a^2+b^2)\text{cota}} e^{i(\frac{\pi}{2}-a)} \end{aligned}$$

$$\left[\begin{array}{l} PQ \sin(\text{csc}a u) \sin(\text{csc}a v) \\ +PS \sin(\text{csc}a u) \cos(\text{csc}a v) \\ +RQ \cos(\text{csc}a u) \sin(\text{csc}a v) \\ +RS \cos(\text{csc}a u) \cos(\text{csc}a v) \end{array} \right]$$

$$\begin{aligned} & e^{\frac{i}{2}((x+a)^2+(y+b)^2+u^2+v^2)\text{cota}} e^{i(\frac{\pi}{2}-a)} \sin(\text{csc}a \cdot u(x+a)) \sin(\text{csc}a \cdot v(y+b)) = \\ & \left[\begin{array}{l} PQe^{\frac{i}{2}(x^2+y^2+u^2+v^2)\text{cota}} e^{\frac{i}{2}(2xa+2yb+a^2+b^2)\text{cota}} e^{i(\frac{\pi}{2}-a)} \sin(\text{csc}a u) \sin(\text{csc}a v) \\ +PSe^{\frac{i}{2}(x^2+y^2+u^2+v^2)\text{cota}} e^{\frac{i}{2}(2xa+2yb+a^2+b^2)\text{cota}} e^{i(\frac{\pi}{2}-a)} \sin(\text{csc}a u) \cos(\text{csc}a v) \\ +QRe^{\frac{i}{2}(x^2+y^2+u^2+v^2)\text{cota}} e^{\frac{i}{2}(2xa+2yb+a^2+b^2)\text{cota}} e^{i(\frac{\pi}{2}-a)} \cos(\text{csc}a u) \sin(\text{csc}a v) \\ +RSe^{\frac{i}{2}(x^2+y^2+u^2+v^2)\text{cota}} e^{\frac{i}{2}(2xa+2yb+a^2+b^2)\text{cota}} e^{i(\frac{\pi}{2}-a)} \cos(\text{csc}a u) \cos(\text{csc}a v) \end{array} \right] \\ & e^{\frac{i}{2}((x+a)^2+(y+b)^2+u^2+v^2)\text{cota}} e^{i(\frac{\pi}{2}-a)} \sin(\text{csc}a \cdot u(x+a)) \sin(\text{csc}a \cdot v(y+b)) = \end{aligned}$$

$$\left[\begin{array}{l} PQe^{\frac{i}{2}(2xa+2yb+a^2+b^2)\text{cota}} 2DK_s^\alpha(x, y, u, v) \\ +PSe^{\frac{i}{2}(2xa+2yb+a^2+b^2)\text{cota}} 1DK_s^\alpha(x, u) * \sqrt{\frac{2\pi}{1-icota}} 1DK_c^\alpha(y, v) \\ +QRe^{\frac{i}{2}(2xa+2yb+a^2+b^2)\text{cota}} 1DK_c^\alpha(x, u) * \sqrt{\frac{2\pi}{1-icota}} 1DK_s^\alpha(y, v) \\ +RSe^{\frac{i}{2}(2xa+2yb+a^2+b^2)\text{cota}} e^{i(a-\frac{\pi}{2})} 2DK_c^\alpha(x, y, u, v) \end{array} \right]$$

$$e^{\frac{i}{2}((x+a)^2+(y+b)^2+u^2+v^2)\text{cota}} e^{i(\frac{\pi}{2}-a)} \sin(\text{csc}a \cdot u(x+a)) \sin(\text{csc}a \cdot v(y+b)) =$$

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$$e^{\frac{i}{2}(2xa+2yb+a^2+b^2)cot\alpha} \left[\begin{array}{l} PQ2DK_s^\alpha(x, y, u, v) \\ + PS \ 1DK_s^\alpha(x, u) * \sqrt{\frac{2\pi}{1-icot\alpha}} 1DK_c^\alpha(y, v) \\ + QR1DK_c^\alpha(x, u) * \sqrt{\frac{2\pi}{1-icot\alpha}} 1DK_s^\alpha(y, v) \\ + RSe^{i(\alpha-\frac{\pi}{2})} 2DK_c^\alpha(x, y, u, v) \end{array} \right]$$

Equation (1) is

$$F_\alpha^S\{f(x+a, y+b)\} = \langle f(x, y), e^{\frac{i}{2}(2xa+2yb+a^2+b^2)cot\alpha} \left[\begin{array}{l} PQ2DK_s^\alpha(x, y, u, v) \\ + PS \ 1DK_s^\alpha(x, u) * \sqrt{\frac{2\pi}{1-icot\alpha}} 1DK_c^\alpha(y, v) \\ + QR1DK_c^\alpha(x, u) * \sqrt{\frac{2\pi}{1-icot\alpha}} 1DK_s^\alpha(y, v) \\ + RSe^{i(\alpha-\frac{\pi}{2})} 2DK_c^\alpha(x, y, u, v) \end{array} \right] \rangle$$

$$F_\alpha^S\{f(x+a, y+b)\} = e^{\frac{i}{2}(2xa+2yb+a^2+b^2)cot\alpha}$$

$$\left\{ \begin{array}{l} \langle f(x, y), PQ2DK_s^\alpha(x, y, u, v) \rangle + \sqrt{\frac{2\pi}{1-icot\alpha}} \langle f(x, y), PS \ 1DK_s^\alpha(x, u) \ 1DK_c^\alpha(y, v) \rangle \\ + \sqrt{\frac{2\pi}{1-icot\alpha}} \langle f(x, y), QR1DK_c^\alpha(x, u) 1DK_s^\alpha(y, v) \rangle + \langle f(x, y), RSe^{i(\alpha-\frac{\pi}{2})} K_c^\alpha(x, y, u, v) \rangle \end{array} \right\}$$

$$F_\alpha^S\{f(x+a, y+b)\} = e^{\frac{i}{2}(2xa+2yb+a^2+b^2)cot\alpha}$$

$$\left\{ \begin{array}{l} PQ\langle f(x, y), 2DK_s^\alpha(x, y, u, v) \rangle + \sqrt{\frac{2\pi}{1-icot\alpha}} PS\langle f(x, y), \ 1DK_s^\alpha(x, u) \ 1DK_c^\alpha(y, v) \rangle \\ + QR \sqrt{\frac{2\pi}{1-icot\alpha}} \langle f(x, y), 1DK_c^\alpha(x, u) 1DK_s^\alpha(y, v) \rangle + RS \langle f(x, y), e^{i(\alpha-\frac{\pi}{2})} K_c^\alpha(x, y, u, v) \rangle \end{array} \right\}$$

$$F_\alpha^S\{f(x+a, y+b)\} = e^{\frac{i}{2}(2xa+2yb+a^2+b^2)cot\alpha}$$

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$$\left\{ \begin{array}{l} \cos(\text{csc}\alpha u)\cos(\text{csc}\alpha v)\langle f(x, y), 2DK_s^\alpha(x, y, u, v) \rangle \\ +\cos(\text{csc}\alpha u)\sin(\text{csc}\alpha v)\sqrt{\frac{2\pi}{1-icota}}\langle f(x, y), 1DK_s^\alpha(x, u) 1DK_c^\alpha(y, v) \rangle \\ +\cos(\text{csc}\alpha v)\sin(\text{csc}\alpha u)\sqrt{\frac{2\pi}{1-icota}}\langle f(x, y), 1DK_c^\alpha(x, u) 1DK_s^\alpha(y, v) \rangle \\ +\sin(\text{csc}\alpha u)\sin(\text{csc}\alpha v)\langle f(x, y), e^{i(\alpha-\frac{\pi}{2})} K_c^\alpha(x, y, u, v) \rangle \end{array} \right\}$$

$$F_\alpha^s\{f(x+a, y+b)\} = e^{\frac{i}{2}(2xa+2yb+a^2+b^2)cota} \cos(\text{csc}\alpha u)\cos(\text{csc}\alpha v)\langle f(x, y), 2DK_s^\alpha(x, y, u, v) \rangle \\ +e^{\frac{i}{2}(2xa+2yb+a^2+b^2)cota} \cos(\text{csc}\alpha u)\sin(\text{csc}\alpha v)\sqrt{\frac{2\pi}{1-icota}}\langle f(x, y), 1DK_s^\alpha(x, u) 1DK_c^\alpha(y, v) \rangle \\ +e^{\frac{i}{2}(2xa+2yb+a^2+b^2)cota} \cos(\text{csc}\alpha v)\sin(\text{csc}\alpha u)\sqrt{\frac{2\pi}{1-icota}}\langle f(x, y), 1DK_c^\alpha(x, u) 1DK_s^\alpha(y, v) \rangle \\ +e^{\frac{i}{2}(2xa+2yb+a^2+b^2)cota} \sin(\text{csc}\alpha u)\sin(\text{csc}\alpha v)\langle f(x, y), e^{i(\alpha-\frac{\pi}{2})} K_c^\alpha(x, y, u, v) \rangle$$

10. THE AD JOINT DIFFERENTIAL OPERATOR

$$F_\alpha^s\{D_{x,y}f(x, y)\} = \langle D_{x,y}, K_s^\alpha(x, y, u, v) \rangle$$

$$F_\alpha^s\{D_{x,y}f(x, y)\} = \langle f(x, y), -D_{x,y}K_s^\alpha(x, y, u, v) \rangle$$

$$F_\alpha^s\{D_{x,y}f(x, y)\} = \langle f(x, y), -D_{x,y} \left(\sqrt{\frac{1-icota}{2\pi}} e^{\frac{i}{2}(x^2+y^2+u^2+v^2)cota} e^{i(\frac{\pi}{2}-\alpha)} \sin(\text{csc}\alpha \cdot ux) \sin(\text{csc}\alpha \cdot vy) \right) \rangle$$

$$F_\alpha^s\{D_{x,y}f(x, y)\} = \left\langle f(x, y), -\sqrt{\frac{1-icota}{2\pi}} D_x \left(e^{\frac{i}{2}(x^2+u^2)cota} \sin(\text{csc}\alpha \cdot ux) \right) \right\rangle \\ D_y \left(e^{\frac{i}{2}(y^2+v^2)cota} \sin(\text{csc}\alpha \cdot vy) \right)$$

$$\text{Let } A = \sqrt{\frac{1-icota}{2\pi}} e^{\frac{i}{2}(v^2+u^2)cota}$$

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$$F_{\alpha}^s\{D_{x,y}f(x,y)\} = \langle f(x,y), -AD_x \left(e^{\frac{i}{2}(x^2)cot\alpha} \sin(csc\alpha.ux) \right) D_y \left(e^{\frac{i}{2}(y^2)cot\alpha} \sin(csc\alpha.vy) \right) \rangle$$

$$F_{\alpha}^s\{D_{x,y}f(x,y)\} = \langle f(x,y) - A \left(e^{\frac{i}{2}(x^2)cot\alpha} \cos(csc\alpha.ux) csc\alpha.ux + \sin(csc\alpha.ux) e^{\frac{i}{2}(x^2)cot\alpha} ixcot\alpha \right) \left(e^{\frac{i}{2}(y^2)cot\alpha} \cos(csc\alpha.vy) csc\alpha.vy + \sin(csc\alpha.vy) e^{\frac{i}{2}(y^2)cot\alpha} iycot\alpha \right) \rangle$$

$$F_{\alpha}^s\{D_{x,y}f(x,y)\} = \langle f(x,y) - Ae^{\frac{i}{2}(x^2+y^2)cot\alpha} (\cos(csc\alpha.ux) csc\alpha.u + \sin(csc\alpha.ux)ixcot\alpha)(\cos(csc\alpha.vy) csc\alpha.v + \sin(csc\alpha.vy) iycot\alpha) \rangle$$

$$F_{\alpha}^s\{D_{x,y}f(x,y)\} = \langle f(x,y) - Ae^{\frac{i}{2}(x^2+y^2)cot\alpha} \left[\begin{array}{l} \cos(csc\alpha.ux) csc\alpha.u \cos(csc\alpha.vy) csc\alpha.v \\ + \cos(csc\alpha.ux) csc\alpha.usin(csc\alpha.vy) iycot\alpha \\ + \sin(csc\alpha.ux)ixcot\alpha \cos(csc\alpha.vy) csc\alpha.v \\ + \sin(csc\alpha.ux)ixcot\alpha \sin(csc\alpha.vy) iycot\alpha \end{array} \right] \rangle$$

$$F_{\alpha}^s\{D_{x,y}f(x,y)\} = \langle f(x,y) - Ae^{\frac{i}{2}(x^2+y^2)cot\alpha} \left[\begin{array}{l} uv (csc\alpha)^2 \cos(csc\alpha.ux) \cos(csc\alpha.vy) \\ + iycot\alpha csc\alpha.u \cos(csc\alpha.ux) \sin(csc\alpha.vy) \\ + ixcot\alpha csc\alpha.v \sin(csc\alpha.ux) \cos(csc\alpha.vy) \\ - xy(cot\alpha)^2 \sin(csc\alpha.ux) \sin(csc\alpha.vy) \end{array} \right] \rangle$$

$$F_{\alpha}^s\{D_{x,y}f(x,y)\} = \langle f(x,y), \left(\begin{array}{l} -uv(csc\alpha)^2 K_c^{\alpha}(x,y,u,v) \\ -iycot\alpha csc\alpha.u e^{-i(\alpha-\frac{\pi}{2})} 1DK_c^{\alpha}(x,u) * \sqrt{\frac{2\pi}{1-icot\alpha}} 1DK_s^{\alpha}(y,v) \\ -ixcot\alpha csc\alpha.v e^{-i(\alpha-\frac{\pi}{2})} \sqrt{\frac{2\pi}{1-icot\alpha}} 1DK_s^{\alpha}(x,u) * 1DK_c^{\alpha}(y,v) \\ + xy(cot\alpha)^2 e^{-i(\alpha-\frac{\pi}{2})} K_s^{\alpha}(x,y,u,v) \end{array} \right) \rangle$$

$$F_{\alpha}^s\{D_{x,y}f(x,y)\} = -uv(csc\alpha)^2 \langle f(x,y), K_c^{\alpha}(x,y,u,v) \rangle$$

$$-e^{-i(\alpha-\frac{\pi}{2})} \sqrt{\frac{2\pi}{1-icot\alpha}} icot\alpha csc\alpha.u \langle f(x,y), 1DK_c^{\alpha}(x,u) \rangle \langle y, 1DK_s^{\alpha}(y,v) \rangle$$

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$$-e^{-i(\alpha-\frac{\pi}{2})} \sqrt{\frac{2\pi}{1-\cot\alpha}} \text{icotacscav}\langle f(x, y), 1DK_s^\alpha(x, u) \rangle \langle x, 1DK_c^\alpha(y, v) \rangle$$

$$+ (\cot\alpha)^2 e^{-i(\alpha-\frac{\pi}{2})} \langle f(x, y)xy, K_s^\alpha(x, y, u, v) \rangle$$

$$F_\alpha^c \{D_{x,y} f(x, y)\} = -uv(\csc\alpha)^2 F_\alpha^c \{f(x, y)\}$$

$$-e^{-i(\alpha-\frac{\pi}{2})} \sqrt{\frac{2\pi}{1-\cot\alpha}} \text{icotacscav} 1_{DF_\alpha^c} \{f(x, y)\}(u) 1_{F_\alpha^s} \{1.y\}(v)$$

$$-e^{-i(\alpha-\frac{\pi}{2})} \sqrt{\frac{2\pi}{1-\cot\alpha}} \text{icotacscav} 1_{DF_\alpha^c} \{x.1\}(v) 1_{F_\alpha^s} \{f(x, y)\}(u) - (\cot\alpha)^2 e^{-i(\alpha-\frac{\pi}{2})} F_\alpha^s \{f(x, y)\}$$

XI.CONCLUSION

In the present work Operators on the testing function E and its dual space E^* for two dimensional fractional sine transform are obtained

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