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# Heuristic for Three Machines in Tandem with Setup and Transportation Times of Jobs 

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#### Abstract

In this paper, we have considered the problem of three machines in tandem including the setup times and transportation times for jobs. Also, we have considered break-down time of machines and weights of jobs according to their importance in the sequence. A heuristic approach has been made for finding an optimal schedule. The procedure has been illustrated with the help of a numerical example.


KEYWORDS: Machines in tandem, setup time, transportation time, break-down time, weight of jobs.

## I. INTRODUCTION

Jackson [10] gave the idea of two production stages while studying a queuing system concerned with an industry in which the production of an item takes place in two distinct but successive stages. Jackson [10] called such stages in tandem (or in series). Johnson [11] and Bellman [3] studied the problem of scheduling of $n$ jobs on two machines arranged in tandem where time required to transport jobs from first machine to the second was assumed to be negligible. Ignall and Schrage [9] proposed an algorithm to optimize makespan for multi-machine and $n$ jobs. As the problem size increases, NP- completeness of flow shop problems necessitates the development of heuristics to get near optimal solutions. Campbell, Dudek and Smith [4] proposed a heuristic algorithm to minimize the makespan. In their work, they split the given $m$ machine problem into a series of equivalent two machine flowshop problem and solving it using Johnson's method. Another heuristic was proposed by Nawaz, Enscore and Ham [15] for minimizing makespan time.
Apart from the makespan objective, other significant objectives like total flow time, tardiness and midle time of machines have been proposed by different authors. Ho and Chang [8] have attempted not only minimizing makespan but also minimizing total flow time and machine idle time. Chou and Lee [6] attempted to solve two machine flowshop bi-criteria scheduling problem with release dates for the jobs, in which the objective function is to minimize the weighted sum of total flow time and makespan. Allahverdi and Aldowaisan [1] reported in their work about a three machines no-wait flow-shop problem with the objective of minimizing total completion time where setup times are considered as separate from processing times and sequence independent. Rajendran and Ziegler [19] have considered the objective of minimizing the total weighted flow times of jobs. Rajendran and Chaudhuri [18] have proposed a heuristic algorithm to minimize the total flow time by approaching the problem in multistage. Maggu and Das [14] used the concept of transportation time in going from one stage to the other. They studied a system in which an infinite number of transport agents were available and no transport agent was required to return to stage 1 from stage 2 . It was assumed that Machine- 1 starts processing the next item immediately after finishing the preceding one. Here, we have considered the problem of three machines in tandem including the setup times and transportation times for jobs. Further, we have considered break-down time of machines and weights of jobs according to their importance in the sequence.

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## II. THREE MACHINES AND N-JOBS FLOW-SHOP SCHEDULING PROBLEM INVOLVING TRANSPORTATION TIME AND SETUP TIME OF JOBS

Let us consider $n$ items $\left(I_{1}, I_{2}, \ldots \ldots, I_{n}\right)$ being processed through three machines ( $A, B$ and $C$ ) in the order $A B C$ with agents who transport an item processed at machine $A$ to the machine $B$ and then to machine $C$. Let item $i$ to be processed on the three machines ( $A, B$ and $C$ ) requires setup times denoted by $a_{i}, b_{i}$ and $c_{i}$ respectively. Let $t_{i}$ and $g_{i}$ be the transportation times for item $i$ to carry it from machine $A$ to $B$ and $B$ to $C$ respectively. The problem is to find an optimal schedule of items to minimize the total production time for completing all the items.

Theorem1 An optimal sequence is obtained by sequencing the item $i-1, i, i+1$ such that:

$$
\begin{aligned}
& \min \left\{a_{i}+A_{i}+t_{i}+b_{i}+B_{i}+g_{i}, t_{i+1}+b_{i+1}+B_{i+1}+g_{i+1}+c_{i+1}+C_{i+1}\right\} \\
& \quad<\min \left\{a_{i+1}+A_{i+1}+t_{i+1}+b_{i+1}+B_{i+1}+g_{i+1}, t_{i}+b_{i}+B_{i}+g_{i}+c_{i}+C_{i}\right\}
\end{aligned}
$$

Proof To prove this theorem, we first prove the following lemma:
Lemma1 If $\min \left\{a_{i}+A_{i}+t_{i}\right\} \geq \max \left\{b_{i}+B_{i}+t_{i}\right\}$, then $C A_{p}+t_{p} \geq C B_{p-1}$.
Proof Consider the statement $P(q)$ for an arbitrary number $q$ defined as:

$$
P(q): C A_{q+1}+t_{q+1} \geq C B_{q}(q=1,2, \ldots)
$$

For any arbitrary natural number $q$

$$
\begin{gathered}
C A_{1}=a_{1}+A_{1} \\
C B_{1}=a_{1}+A_{1}+t_{1}+b_{1}+B_{1} \\
C A_{2}+t_{2}=a_{1}+A_{1}+a_{2}+A_{2}+t_{2}
\end{gathered}
$$

Since $\min \left\{a_{i}+A_{i}+t_{i}\right\} \geq \max \left\{b_{i}+B_{i}+t_{i}\right\}$

$$
\Rightarrow C A_{2}+t_{2}>C B_{1}
$$

Hence, $P(q)$ be true for $q=1$.
Let $P(q)$ be true for $q=m$, i.e.

$$
C A_{m+1}+t_{m+1} \geq C B_{m} .
$$

Now,

$$
C B_{m+1}=\max \left\{C A_{m+1}+t_{m+1}, C B_{m}\right\}+b_{m+1}+B_{m+1}
$$

$=C A_{m+1}+\left(t_{m+1}+b_{m+1}+B_{m+1}\right)$
But $C A_{m+2}+t_{m+2}=C A_{m+1}+a_{m+2}+A_{m+2}+t_{m+2}$ and $a_{m+2}+A_{m+2}+t_{m+2} \geq b_{m+1}+B_{m+1}+t_{m+1}$
Hence, $C A_{m+2}+t_{m+2} \geq C B_{m+1}$.
Therefore, $P(q)$ is true for $q=m+1$.
We now proceed to the proof of the theorem.
Let $S$ and $S^{\prime}$ denote the sequences of items given by:

$$
\begin{aligned}
& S=\left(I_{1}, I_{2}, \ldots \ldots, I_{i-1}, I_{i}, I_{i+1} . I_{i+2}, \ldots \ldots, I_{n}\right) \\
& S^{\prime}=\left(I_{1}^{\prime}, I_{2}^{\prime}, \ldots ., I_{i-1}^{\prime}, I_{i+1}^{\prime}, I_{i}^{\prime} \cdot I_{i+2}^{\prime}, \ldots ., I_{n}^{\prime}\right)
\end{aligned}
$$

Let $\left(X_{p}, X_{p}^{\prime}\right)$ and $\left(C X_{p}, C^{\prime} X_{p}\right)$ be respectively the processing time and completion time of any item $p$ on machine $X(=A$ or $B)$ for the sequences $\left(S, S^{\prime}\right)$.Let $\left(t_{p}, t_{p}^{\prime}\right)$ and $\left(g_{p}, g_{p}^{\prime}\right)$ denotes the transportation times of item $p$ to transport it from machine $A$ to machine $B$ and from machine $B$ to machine $C$ respectively for the sequences $\left(S, S^{\prime}\right) . \operatorname{Let}\left(a_{p}, a_{p}^{\prime}\right)$, $\left(b_{p}, b_{p}^{\prime}\right)$ and $\left(c_{p}, c_{p}^{\prime}\right)$ be respectively the setup times of an item $p$ on machines $A, B$ and $C$ for the sequences $\left(S, S^{\prime}\right)$.

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The completion time of $p^{t h}$ item on machines $B$ and $C$ is given by

$$
\begin{align*}
C B_{p} & =\max \left\{C A_{p}+t_{p}, C B_{p-1}\right\}+b_{p}+B_{p} \\
& =C A_{p}+t_{p}+b_{p}+B_{p} \\
C C_{p} & =\max \left\{C B_{p}+g_{p}, C C_{p-1}\right\}+c_{p}+C_{p} \\
& =\max \left\{C A_{p}+t_{p}+b_{p}+B_{p}+g_{p}, C C_{p-1}\right\}+c_{p}+C_{p} \tag{1}
\end{align*}
$$

Now, we will choose the sequence $S$ if

$$
\begin{equation*}
C C_{n}<C^{\prime} C_{n} \tag{2}
\end{equation*}
$$

i.e., if
$\max \left(C A_{n}+t_{n}+b_{n}+B_{n}+g_{n}, C C_{n-1}\right)+c_{n}+C_{n}<\max \left(C^{\prime} A_{n}+t_{n}^{\prime}+b_{n}^{\prime}+B_{n}^{\prime}+g_{n}^{\prime}, C^{\prime} C_{n-1}\right)+c_{n}^{\prime}+C_{n}^{\prime}$
Now $C A_{n}+t_{n}+b_{n}+B_{n}+g_{n}=C^{\prime} A_{n}+t_{n}^{\prime}+b_{n}^{\prime}+B_{n}^{\prime}+g_{n}^{\prime}, c_{n}=c_{n}^{\prime}$ and $C_{n}=C_{n}^{\prime}$, so the result (2) will be true if:

$$
\begin{equation*}
C C_{n-1}<C^{\prime} C_{n-1} \tag{3}
\end{equation*}
$$

Proceeding in this way we get that inequality (2) is true if:

$$
\begin{equation*}
C C_{p}<C^{\prime} C_{p}(p=i+1, i+2, \ldots \ldots, n \text { and } i=1,2, \ldots . n-1) \tag{4}
\end{equation*}
$$

We now calculate the values of $C C_{i+1}$ and $C^{\prime} C_{i+1}$

$$
\begin{align*}
C C_{i+1}= & \max \left\{C B_{i+1}+g_{i+1}, C C_{i}\right\}+c_{i+1}+C_{i+1} \\
= & \max \left\{C A_{i+1}+t_{i+1}+b_{i+1}+B_{i+1}+g_{i+1}, C C_{i}\right\}+c_{i+1}+C_{i+1} \\
C C_{i+1}= & \max \left\{C A_{i+1}+t_{i+1}+b_{i+1}+B_{i+1}+g_{i+1}, \max \left\{C B_{i}+g_{i}, C C_{i-1}\right\}+c_{i}+C_{i}\right\}+c_{i+1}+C_{i+1} \\
= & \max \left\{C A_{i+1}+t_{i+1}+b_{i+1}+B_{i+1}+g_{i+1}, C B_{i}+g_{i}+c_{i}+C_{i}, C C_{i-1}+c_{i}+C_{i}\right\}+c_{i+1}+C_{i+1} \\
= & \max \left\{C A_{i+1}+t_{i+1}+b_{i+1}+B_{i+1}+g_{i+1}, \max \left\{C A_{i}+t_{i}, C B_{i-1}\right\}+b_{i}+B_{i}+g_{i}+c_{i}+C_{i},\right. \\
& \left.C C_{i-1}+c_{i}+C_{i}\right\}+c_{i+1}+C_{i+1} \\
= & \max \left\{C A_{i+1}+t_{i+1}+b_{i+1}+B_{i+1}+g_{i+1}, C A_{i}+t_{i}+b_{i}+B_{i}+g_{i}+c_{i}+C_{i},\right. \\
& \left.C C_{i-1}+c_{i}+C_{i}\right\}+c_{i+1}+C_{i+1} \\
C C_{i+1}= & \max \left\{C A_{i-1}+a_{i}+A_{i}+a_{i+1}+A_{i+1}+t_{i+1}+b_{i+1}+B_{i+1}+g_{i+1}+c_{i+1}+C_{i+1},\right. \\
& C A_{i-1}+a_{i}+A_{i}+t_{i}+g_{i}+b_{i}+B_{i}+c_{i}+C_{i}+c_{i+1}+C_{i+1},  \tag{5}\\
& \left.C C_{i-1}+c_{i}+C_{i}+c_{i+1}+C_{i+1}\right\}
\end{align*}
$$

Similarly,

$$
\begin{gather*}
C^{\prime} C_{i+1}=\max \left\{C^{\prime} A_{i-1}+a_{i}^{\prime}+A_{i}^{\prime}+a_{i+1}^{\prime}+A_{i+1}^{\prime}+t_{i+1}^{\prime}+b_{i+1}^{\prime}+B_{i+1}^{\prime}+g_{i+1}^{\prime}+c_{i+1}^{\prime}+C_{i+1}^{\prime},\right. \\
C^{\prime} A_{i-1}+a_{i}^{\prime}+A_{i}^{\prime}+t_{i}^{\prime}+g_{i}^{\prime}+b_{i}^{\prime}+B_{i}^{\prime}+c_{i}^{\prime}+C_{i}^{\prime}+c_{i+1}^{\prime}+C_{i+1}^{\prime},  \tag{6}\\
\left.C^{\prime} C_{i-1}+c_{i}^{\prime}+C_{i}^{\prime}+c_{i+1}^{\prime}+C_{i+1}^{\prime}\right\}
\end{gather*}
$$

Comparing the sequences $S$ and $S^{\prime}$, it is obvious that

$$
\begin{gather*}
C A_{i-1}=C^{\prime} A_{i-1}, C C_{i-1}=C^{\prime} C_{i-1} \\
X_{i}=X_{i+1}^{\prime}, X_{i+1}=X_{i}^{\prime}(X=a, b, A, B \text { or } C)  \tag{7}\\
t_{i}=t_{i+1}^{\prime}, t_{i+1}=t_{i}^{\prime}, g_{i}=g_{i+1}^{\prime}, g_{i+1}=g_{i}^{\prime}
\end{gather*}
$$

Writing (4) for $p=i+1$ and using (7), we get

$$
\begin{gather*}
\max \left\{C A_{i-1}+a_{i}+A_{i}+a_{i+1}+A_{i+1}+t_{i+1}+b_{i+1}+B_{i+1}+g_{i+1}+c_{i+1}+C_{i+1}, C A_{i-1}+a_{i}+A_{i}+\right. \\
\left.t_{i}+g_{i}+b_{i}+B_{i}+c_{i}+C_{i}+c_{i+1}+C_{i+1}, C C_{i-1}+c_{i}+C_{i}+c_{i+1}+C_{i+1}\right\} \\
<\max \left\{C A_{i-1}+a_{i+1}+A_{i+1}+a_{i}+A_{i}+t_{i}+b_{i}+B_{i}+g_{i}+c_{i}+C_{i}, C A_{i-1}+a_{i+1}+A_{i+1}+\right.  \tag{8}\\
\left.t_{i+1}+g_{i+1}+b_{i+1}+B_{i+1}+c_{i+1}+C_{i+1}+c_{i}+C_{i}, C C_{i-1}+c_{i+1}+C_{i+1}+c_{i}+C_{i}\right\}
\end{gather*}
$$

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Subtracting last term from both sides and further subtracting $C A_{i-1}+a_{i}+A_{i}+a_{i+1}+A_{i+1}+t_{i}+t_{i+1}+$ $g_{i}+g_{i+1}+b_{i}+B_{i}+b_{i+1}+B_{i+1}+c_{i}+C_{i}+c_{i+1}+C_{i+1}$ from each side, we get
$\max \left\{-t_{i}-g_{i}-b_{i}-B_{i}-c_{i}-C_{i},-a_{i+1}-A_{i+1}-t_{i+1}-g_{i+1}-b_{i+1}-B_{i+1}\right\}$
$<\max \left\{-t_{i+1}-g_{i+1}-b_{i+1}-B_{i+1}-c_{i+1}-C_{i+1},-a_{i}-A_{i}-t_{i}-g_{i}-b_{i}-B_{i}\right\}$
$\min \left\{a_{i}+A_{i}+t_{i}+g_{i}+b_{i}+B_{i}, t_{i+1}+g_{i+1}+b_{i+1}+B_{i+1}+c_{i+1}+C_{i+1}\right\}$
$<\min \left\{a_{i+1}+A_{i+1}+t_{i+1}+g_{i+1}+b_{i+1}+B_{i+1}, t_{i}+g_{i}+b_{i}+B_{i}+c_{i}+C_{i}\right\}$
The utility of above theorem can be summarized into following steps to give us decomposition algorithm, that is, numerical method to obtain an optimal schedule minimizing total elapsed time for a 3-machine, n-job sequencing problem where setup and transportation times are taken into account. Our problem can be represented in tableau form as follows:

| Item $i$ | $a_{i}$ | Machine $A\left(A_{i}\right)$ | $t_{i}$ | $b_{i}$ | Machine $B\left(B_{i}\right)$ | $g_{i}$ | $c_{i}$ | Machine $C\left(C_{i}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $a_{1}$ | $A_{1}$ | $t_{1}$ | $b_{1}$ | $B_{1}$ | $g_{1}$ | $c_{1}$ | $C_{1}$ |
| 2 | $a_{2}$ | $A_{2}$ | $t_{2}$ | $b_{2}$ | $B_{2}$ | $g_{2}$ | $c_{2}$ | $C_{2}$ |
| $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\bullet$ | $\cdot$ | $\bullet$ |
| $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\bullet$ | $\bullet$ | $\cdot$ | $\bullet$ |
| $n$ | $a_{n}$ | $A_{n}$ | $t_{n}$ | $b_{n}$ | $B_{n}$ | $g_{n}$ | $c_{n}$ | $C_{n}$ |

Where $A_{i}, B_{i}, C_{i}$ are the service times on $A, B$ and $C$ respectively. $a_{i}, b_{i}, c_{i}$ are the setup times on $A, B$ and $C$ respectively. $t_{i}, g_{i}$ are the transportation times of item $i$ from machine $A$ to $B$ and $B$ to $C$ respectively satisfying the structural relationship:
i) $\quad \min \left(a_{i}+A_{i}+t_{i}\right) \geq \max \left(b_{i}+B_{i}+t_{i}\right)$
ii) $\min \left(c_{i}+C_{i}+g_{i}\right) \geq \max \left(b_{i}+B_{i}+g_{i}\right)$

ALGORITHM1 The result of theorem-1 gives the following procedure for an optimal sequence:
Step1: Assume that there are two fictitious machines G and H in place of $A$ and $B$ respectively. The service times for these fictitious machines are given by $G_{i}$ and $H_{i}$ where

$$
G_{i}=a_{i}+A_{i}+t_{i}+b_{i}+B_{i}+g_{i}, \quad H_{i}=t_{i}+b_{i}+B_{i}+g_{i}+c_{i}+C_{i}
$$

Step2: Applying Johnson's (1954) rule to the fictitious machine times $G \& H$ constructed in step 1, we obtain the optimal sequence.

## III. FLOW-SHOP SCHEDULING ALSO INVOLVING JOB WEIGHTS AND BREAK-DOWN INTERVALS OF MACHINES

Let job $i$ be assigned with the weight $w_{i}$ according to its relative importance for performance in the given sequence. The performance measure studied is weighted mean flow time defined by:

$$
\bar{F}_{w}=\frac{\sum_{i=1}^{n} w_{i} f_{i}}{\sum_{i=1}^{n} w_{i}}, \text { where } f_{i} \text { is the flow time of } i^{\text {th }} \text { job. }
$$

Let the break-down interval $(a, b)$ is already known to us, i.e., deterministic nature andthe break-down interval length is ( $b-a$ ), which is known. Then our aim is to find out an optimal sequence of jobs so as to minimize the total elapsed time.

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The given problem can be represented in tableau form as follows:

| Item $i$ | $a_{i}$ | Machine $A\left(A_{i}\right)$ | $t_{i}$ | $b_{i}$ | Machine $B\left(B_{i}\right)$ | $g_{i}$ | $c_{i}$ | Machine $C\left(C_{i}\right)$ | $w_{i}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $a_{1}$ | $A_{1}$ | $t_{1}$ | $b_{1}$ | $B_{1}$ | $g_{1}$ | $c_{1}$ | $C_{1}$ | $w_{1}$ |
| 2 | $a_{2}$ | $A_{2}$ | $t_{2}$ | $b_{2}$ | $B_{2}$ | $g_{2}$ | $c_{2}$ | $C_{2}$ | $w_{2}$ |
| $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ |
| $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ |
| $n$ | $a_{n}$ | $A_{n}$ | $t_{n}$ | $b_{n}$ | $B_{n}$ | $g_{n}$ | $c_{n}$ | $C_{n}$ | $w_{n}$ |

Algorithm 2 The steps are as follows:
Step1: Modify the problem into two machines flow-shop problem using fictitious machines ( $G$ and $H$ ) as in algorithm1.The modified problem in the tabular form is:

| Item $i$ | Machine $G$ <br> $G_{i}=a_{i}+A_{i}+t_{i}+b_{i}+B_{i}+g_{i}$ | Machine $H$ <br> $H_{i}=t_{i}+b_{i}+B_{i}+g_{i}+c_{i}+C_{i}$ | Weight $w_{i}$ |
| :---: | :---: | :---: | :---: |
| 1 | $G_{1}$ | $H_{1}$ | $w_{1}$ |
| 2 | $G_{2}$ | $H_{2}$ | $w_{2}$ |
| $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ |
| $\cdot$ | - | $H_{n}$ | $\cdot$ |
| $n$ | $G_{n}$ | $w_{n}$ |  |

Step2: Find $\min \left(G_{i}, H_{i}\right)$
i) If $\min \left(G_{i}, H_{i}\right)=G_{i}$ then define $G_{i}^{\prime}=G_{i}-w_{i}$ and $H_{i}^{\prime}=H_{i}$.
ii) If $\min \left(G_{i}, H_{i}\right)=H_{i}$ then define $G_{i}^{\prime}=G_{i}$ and $H_{i}^{\prime}=H_{i}+w_{i}$.

Step3: Define a new reduced problem in the tabular form as:

| $\operatorname{Item} i$ | $G_{i}^{\prime}$ | $H_{i}^{\prime}$ |
| :---: | :---: | :---: |
| 1 | $G_{1}^{\prime} / w_{i}$ | $H_{1}^{\prime} / w_{1}$ |
| 2 | $G_{2}^{\prime} / w_{2}$ | $H_{2}^{\prime} / w_{2}$ |
| $\cdot$ | $\cdot$ | $\cdot$ |
| $\dot{n}$ | $G_{n}^{\prime} / w_{n}$ | $H_{n}^{\prime} / w_{n}$ |

Step4: Determine the optimal sequence by using Johnson's algorithm for the new reduced problem obtained in step3 and see the effect of break-down interval $(a, b)$ on different jobs.
Step5: Formulate a new problem with processing time $A_{i}^{\prime}, B_{i}^{\prime}$ and $C_{i}^{\prime}$ where
$A_{i}^{\prime}=A_{i}+(b-a), B_{i}^{\prime}=B_{i}+(b-a)$ and $C_{i}^{\prime}=C_{i}+(b-a)$, if $(a, b)$ has an effect on job $i$.
$A_{i}^{\prime}=A_{i}, B_{i}^{\prime}=B_{i}, C_{i}^{\prime}=C_{i}$, if $(a, b)$ has no effect on job $i$.
Step6: Now repeat the procedure to get the optimal sequence.
This sequence is an optimal for the original problem. By this sequence, we can determine the total elapsed time and weighted mean-flow time.

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EXAMPLE Let a machine tandem queuing problem be given in the following tableau form:

| Item $i$ | $a_{i}$ | Machine $A\left(A_{i}\right)$ | $t_{i}$ | $b_{i}$ | Machine $B\left(B_{i}\right)$ | $g_{i}$ | $c_{i}$ | Machine $C\left(C_{i}\right)$ | $w_{i}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 4 | 6 | 2 | 3 | 2 | 3 | 5 | 3 |
| 2 | 3 | 6 | 4 | 1 | 4 | 6 | 4 | 7 | 5 |
| 3 | 2 | 4 | 7 | 2 | 3 | 2 | 3 | 8 | 4 |
| 4 | 4 | 9 | 3 | 3 | 6 | 8 | 2 | 10 | 2 |

SOLUTION Now $\min \left(a_{i}+A_{i}+t_{i}\right)=12$

$$
\max \left(b_{i}+B_{i}+t_{i}\right)=12
$$

Hence, Structural condition (i) is satisfied. Now, using the step 1, the reduced problem is:

| Item $i$ | Machine $G$ <br> $G_{i}=a_{i}+A_{i}+t_{i}+b_{i}+B_{i}+g_{i}$ | Machine $H$ <br> $\quad 19$ | $t_{i}+b_{i}+B_{i}+g_{i}+c_{i}+C_{i}$ |
| :---: | :---: | :---: | :---: | | Weight |
| :---: |
| $w_{i}$ |
| 1 |

Using steps 2 to 4 and applying Johnson's rule, the optimal sequence is (3, 1, 4, 2). Now the effects of break-down interval $(18,25)$ on sequence $(3,1,4,2)$ is read as follows:

| Item $i$ | $a_{i}$ | $A$ <br> In-out | $t_{i}$ | $b_{i}$ | $B$ <br> In-out | $g_{i}$ | $c_{i}$ | $C$ <br> In-out | $w_{i}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 2 | $2-6$ | 7 | 2 | $15-18$ | 2 | 3 | $\mathbf{2 3 - 3 1}$ | 4 |
| 1 | 2 | $8-12$ | 6 | 2 | $\mathbf{2 0 - 2 3}$ | 2 | 3 | $34-39$ | 3 |
| 4 | 4 | $\mathbf{1 6 - 2 5}$ | 3 | 3 | $31-37$ | 8 | 2 | $47-57$ | 2 |
| 2 | 3 | $28-34$ | 4 | 1 | $39-43$ | 6 | 4 | $61-68$ | 5 |

Hence, with effect of break-down interval the original problem gets modified to a new problem (as per step 5).

| Item $i$ | $a_{i}$ | Machine $A\left(A_{i}^{\prime}\right)$ | $t_{i}$ | $b_{i}$ | Machine $B\left(B_{i}^{\prime}\right)$ | $g_{i}$ | $c_{i}$ | Machine $C\left(C_{i}^{\prime}\right)$ | $w_{i}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 4 | 6 | 2 | 10 | 2 | 3 | 5 | 3 |
| 2 | 3 | 6 | 4 | 1 | 4 | 6 | 4 | 7 | 5 |
| 3 | 2 | 4 | 7 | 2 | 3 | 2 | 3 | 15 | 4 |
| 4 | 4 | 16 | 3 | 3 | 6 | 8 | 2 | 10 | 2 |

Now, repeating the procedure, we get the sequence $(2,3,1,4)$ which is optimal or near optimal and the final table is:

| Item $i$ | $a_{i}$ | $A$ <br> In-out | $t_{i}$ | $b_{i}$ | $B$ <br> In-out | $g_{i}$ | $c_{i}$ | $C$ <br> In-out | $w_{i}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 3 | $3-9$ | 4 | 1 | $14-18$ | 6 | 4 | $28-35$ | 5 |
| 3 | 2 | $11-15$ | 7 | 2 | $24-27$ | 2 | 3 | $38-53$ | 4 |
| 1 | 2 | $17-21$ | 6 | 2 | $29-39$ | 2 | 3 | $56-61$ | 3 |
| 4 | 4 | $25-41$ | 3 | 3 | $47-53$ | 8 | 2 | $63-73$ | 2 |

Mean weighted flow time is $=\frac{35 \times 5+(53-9) \times 4+(61-15) \times 3+(73-21) \times 2}{5+4+3+2}$

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## $=42.36 \mathrm{hrs}$.

Hence, the total elapsed time is 73 hrs . and mean weighted flow time is 42.36 hrs .

## IV. Conclusions

The paper dealt with an important problem in scheduling theory with the objectives to minimize the makespan and the mean weighted flow time for three machines in tandem including the setup times and transportation times for jobs. The concept of setup time is very important in some cases as in dyeing industries, automobile industries etc. Also, we have considered break-down time of machines and weights of jobs according to their importance in the sequence, this case can also help in considering lunch times for workers or servicing needed for machines. A heuristic approach has been made for finding an optimal schedule. This approach is fast to achieve the objectives.

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