

Special Pythagorean Triangles and 6-Digit Harshad Numbers

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ABSTRACT: Pythagorean triangles, each with a leg represented by a 6-Digit Harshad number are obtained. A few interesting results are given.

KEYWORDS: Pythagorean triangles, 6-Digit Harshad numbers

I. INTRODUCTION

The fascinating branch of mathematics is the theory of numbers where in Pythagorean triangles have been a matter of interest to various mathematicians and to the lovers of mathematics, because it is a treasure house in which the search for many hidden connection is a treasure hunt. For a rich variety of fascinating problems one may refer [1-17]. A careful observer of patterns may note that there is a one to one correspondence between the polygonal numbers and the number of sides of the polygon. Apart from the other patterns we have some more fascinating patterns of numbers namely Jarasandha numbers, Nasty numbers and Dhurva numbers. These numbers have been presented in [18-21].

In [22-24], special Pythagorean triangles connected with polygonal numbers and Nasty numbers are obtained. In [25], special Pythagorean triangles in connection with Hardy Ramanujan number 1729 are exhibited. Recently in [26], special Pythagorean triangles connected with Dhurva numbers. Thus the main objective of this paper is to find out the special Pythagorean triangles in connection with Harshad numbers.

In this communication, we have presented Pythagorean triangles each leg represented by 6-Digit Harshad numbers (155655, 131052) respectively. Also a few interesting relations are obtained.

II. BASIC DEFINITIONS

1. DEFINITION – PYTHAGOREAN EQUATION

The ternary quadratic Diophantine equation given by $x^2 + y^2 = z^2$ is known as Pythagorean equation where x, y, z are natural numbers. The above equations are also referred to as Pythagorean triangle and denote it by $T(x, y, z)$.

Also, in Pythagorean triangle $T(x, y, z)$: $x^2 + y^2 = z^2$, x and y are called its legs and z its hypotenuse.

2. DEFINITION - PRIMITIVE

Most cited solution of the Pythagorean equation is

$x = m^2 - n^2, y = 2mn, z = m^2 + n^2$ where $m > n > 0$. This solution is called primitive, if m, n are of opposite parity and $\gcd(m, n) = 1$.

3. DEFINITION- HARSHAD NUMBER

It is an integer that is divisible by the sum of its digits.

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III. METHOD OF ANALYSIS

In this section, we exhibit Pythagorean triangles; each with a leg represented by the six digit Harshad number 155655 and denote this number by N.

To start with, it is noted that the leg y cannot be represented by N as y is even and N is odd. Also z cannot be written as sum of two squares. Since a positive integer P can be written as a sum of two integer squares if the canonical prime factorization $P = p_1^{r_1} p_2^{r_2} \dots p_r^{r_r}$ (where p_i are distinct primes) satisfies the condition if $p_i \equiv 3 \pmod{4}$ then r_i is even. A prime $p \equiv 1 \pmod{4}$ can be written as $p = a^2 + b^2$

Now, consider $x=N$

$$\Rightarrow m^2 - n^2 = 155655$$

Which is a binary quadratic Diophantine equation. Solving the above equation for m, n, we get 7 integer solutions and thus, we have 7 Pythagorean triangles, each having the leg x to be represented by the six digit Harshad number N = 155655 as shown in table -1:

Table - 1

S.No	m	n	x	y	z
1	77828	77827	155655	12114239510	12114239510
2	25944	25941	155655	1346026608	1346026617
3	15568	15563	155655	484569568	484569593
4	8652	8643	155655	149558472	149558553
5	5196	5181	155655	53840952	53841177
6	2896	2869	155655	16617248	16617977
7	644	509	155655	655592	673817

Note: There are 4 primitive and 3 non-primitive triangles are shown in Table -1.

Table -2

S.No	m	n	x	y	z
1	65526	1	4293656675	131052	4293656677
2	32763	2	1073414165	131052	1073414173
3	21842	3	477072955	131052	477072973
4	10921	6	119268205	131052	119268277
5	978	67	951995	131052	960973
6	489	134	221165	131052	257077
7	402	163	135035	131052	188173
8	326	201	65875	131052	146677

In a similar manner, it seen that, there are 8 Pythagorean triangles, each having the leg y represented by the six digits Harshad number M = 131052 as shown in the table -2:

Note: There are 8 non-primitive triangles.

From the above tables it is observed that

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- The expression $\frac{y}{x} = z + \frac{4A}{P} - y^2$ is a perfect square.
 - $3(P - y)$ is a nasty number.
 - $3y - \frac{12A}{P}$ is a nasty number.
- $z - y = (m - n)^2$
- $z + y = (m + n)^2$

Where A and P are the area and perimeter of the Pythagorean triangle.

IV. CONCLUSION

One may search for the connections between Pythagorean triangles and other Harshad number.

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