

Stochastic Models for a Two Grade Manpower System with Wastage as a Geometric Process

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ABSTRACT: In this paper, the problem of time to recruitment is studied for a two-grade manpower system by assuming that the wastage (loss of manpower) which takes place due to policy decisions form a geometric process. Three mathematical models are constructed by considering three different forms of optional and mandatory exponential thresholds for cumulative wastages in each grade. Explicit analytical expressions for variance of time to recruitment is obtained when the inter-policy decision times (i) are correlated and (ii) form a geometric process.

KEYWORDS: Two grade manpower system, geometric process, correlated random variables, Univariate policy of recruitment, Variance of time to recruitment.

I. INTRODUCTION

Attrition is a common phenomenon in many marketing organizations. This leads to reduction in the total strength of marketing personnel and will adversely affect the sales turnover of the organization, if recruitment is not planned. Recruitment on every occasion of depletion is not advisable since every recruitment involves cost. As the loss of manpower is unpredictable, a suitable recruitment policy has to be designed to overcome this loss. Many researchers have studied several problems in manpower planning using different kinds of wastages, inter-decision times and thresholds. In [2], the authors have obtained the mean and variance of time to recruitment are obtained when the inter-decision times form an order statistics and wastage forms a sequence of independent and identically distributed exponential random variables. In [3], the authors have studied the results in [2] when the wastages form a sequence of exchangeable and constantly correlated exponential random variables. In [4], the system characteristics is obtained when the inter-decision times and wastages form an order statistics separately. In [5], the authors have obtained performance measures assuming different conditions on wastages and inter-decision times. Assuming geometric process for wastages and exponential distribution for optional and mandatory thresholds, an attempt has been made in this paper to study the problem of time to recruitment for a two-grade manpower system when the inter-policy decision times (i) are correlated and (ii) form a geometric process using the univariate policy of recruitment based on shock model approach. This paper is organized as follows: In section II, III and IV model-I, II and III are described and analytical expression for performance measures are derived. In section V, the relevant conclusions are presented.

II. MODEL DESCRIPTION AND ANALYSIS OF MODEL –I

Consider an organization taking decisions at random epoch in $(0, \infty)$ and at every decision epoch a random number of persons quit the organization. There is an associated loss of man-hours (wastages) if a person quits. It is assumed that the wastages are linear and cumulative. Let U_i be a continuous random variable denoting inter-decision time between $(i-1)^{\text{th}}$ and i^{th} decision, $i=1,2,3,\dots$ with cumulative distribution function $F(\cdot)$, probability density function $f(\cdot)$ and F_k is

the cumulative distribution function of $\sum_{i=1}^k U_i$. Let X_i be the wastages in the organization due to the i^{th} decision epoch, $i=1,2,3,\dots$ with cumulative distribution function $G(\cdot)$, probability density function $g(\cdot)$. It is assumed that

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wastage forms a geometric process with rate d . Let $Y_1, Y_2(Z_1, Z_2)$ denotes the optional (mandatory) thresholds for the loss of man-hours in grades 1 and 2, with parameters $\theta_1, \theta_2, \alpha_1, \alpha_2$ respectively, where $\theta_1, \theta_2, \alpha_1, \alpha_2$ are positive. It is assumed that $Y_1 < Z_1$ and $Y_2 < Z_2$. In this model, the optional threshold Y for the cumulative wastages in the organization is taken as $Y = \text{Max}(Y_1, Y_2)$ and the mandatory threshold Z for the cumulative wastages in the organization is taken as $Z = \text{Max}(Z_1, Z_2)$. It is assumed that X_i, Y and Z are statistically independent. Let T be the time to recruitment in the organization with cumulative distribution function $L(\cdot)$, probability density function $l(\cdot)$, mean $E(T)$ and variance $V(T)$. Let $L^*(\cdot)$ and $f^*(\cdot)$, be the Laplace transform of $l(\cdot)$ and $f(\cdot)$, respectively. Let $V_k(t)$ be the probability that there are exactly k decision epochs in $(0, t]$. Since the number of decisions made in $(0, t]$ form a renewal process, we note that $V_k(t) = F_k(t) - F_{k+1}(t)$ with $F_0(t) = 1$. Let p be the probability that the organization is not going for recruitment whenever the total loss of man-hours crosses optional threshold Y . The Univariate recruitment policy employed in this paper is as follows: **If the total loss of man-hours exceeds the optional threshold Y , the organization may or may not go for recruitment. But if the total loss of man-hours exceeds the mandatory threshold Z , the recruitment is necessary.**

Main Results

$$P(T > t) = \sum_{k=0}^{\infty} V_k(t) P\left(\sum_{i=1}^k X_i < Y\right) + p \sum_{k=0}^{\infty} V_k(t) P\left(\sum_{i=1}^k X_i \geq Y\right) \times P\left(\sum_{i=1}^k X_i < Z\right) \quad (1)$$

Consider the first term of (1) and on further simplification we get

$$\begin{aligned} \sum_{k=0}^{\infty} V_k(t) P\left(\sum_{i=1}^k X_i < Y\right) &= 1 - \sum_{k=1}^{\infty} F_k(t) [g_{k-1}^*(\theta_1) - g_k^*(\theta_1)] - \sum_{k=1}^{\infty} F_k(t) [g_{k-1}^*(\theta_2) - g_k^*(\theta_2)] \\ &\quad + \sum_{k=1}^{\infty} F_k(t) [g_{k-1}^*(\theta_1 + \theta_2) - g_k^*(\theta_1 + \theta_2)] \end{aligned}$$

$$(i.e) \sum_{k=0}^{\infty} V_k(t) P\left(\sum_{i=1}^k X_i < Y\right) = 1 - \sum_{k=1}^{\infty} F_k(t) S_1 - \sum_{k=1}^{\infty} F_k(t) S_2 + \sum_{k=1}^{\infty} F_k(t) S_3 \quad (2)$$

Similarly we get

$$\begin{aligned} \sum_{k=0}^{\infty} V_k(t) P\left(\sum_{i=1}^k X_i > Y\right) P\left(\sum_{i=1}^k X_i < Z\right) &= P\left[-\sum_{k=1}^{\infty} F_k(t) S_4 - \sum_{k=1}^{\infty} F_k(t) S_5 + \sum_{k=1}^{\infty} F_k(t) S_6 + \sum_{k=1}^{\infty} F_k(t) S_7 + \sum_{k=1}^{\infty} F_k(t) S_8 - \sum_{k=1}^{\infty} F_k(t) S_9 \right. \\ &\quad \left. + \sum_{k=1}^{\infty} F_k(t) S_{10} + \sum_{k=1}^{\infty} F_k(t) S_{11} - \sum_{k=1}^{\infty} F_k(t) S_{12} - \sum_{k=1}^{\infty} F_k(t) S_{13} - \sum_{k=1}^{\infty} F_k(t) S_{14} + \sum_{k=1}^{\infty} F_k(t) S_{15}\right] \end{aligned} \quad (3)$$

where

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$$\begin{aligned}
 S_1 &= [g_{k-1}^*(\theta_1) - g_k^*(\theta_1)], S_2 = [g_{k-1}^*(\theta_2) - g_k^*(\theta_2)], S_3 = [g_{k-1}^*(\theta_1 + \theta_2) - g_k^*(\theta_1 + \theta_2)] \\
 S_4 &= [g_{k-1}^*(\alpha_1) - g_k^*(\alpha_1)], S_5 = [g_{k-1}^*(\alpha_2) - g_k^*(\alpha_2)], S_6 = [g_{k-1}^*(\alpha_1 + \alpha_2) - g_k^*(\alpha_1 + \alpha_2)] \\
 S_7 &= [g_{k-1}^*(\theta_1)g_{k-1}^*(\alpha_1) - g_k^*(\theta_1)g_k^*(\alpha_1)], S_8 = [g_{k-1}^*(\theta_1)g_{k-1}^*(\alpha_2) - g_k^*(\theta_1)g_k^*(\alpha_2)], \\
 S_9 &= [g_{k-1}^*(\theta_1)g_{k-1}^*(\alpha_1 + \alpha_2) - g_k^*(\theta_1)g_k^*(\alpha_1 + \alpha_2)], S_{10} = [g_{k-1}^*(\theta_2)g_{k-1}^*(\alpha_1) - g_k^*(\theta_2)g_k^*(\alpha_1)] \\
 S_{11} &= [g_{k-1}^*(\theta_2)g_{k-1}^*(\alpha_2) - g_k^*(\theta_2)g_k^*(\alpha_2)], S_{12} = [g_{k-1}^*(\theta_2)g_{k-1}^*(\alpha_1 + \alpha_2) - g_k^*(\theta_2)g_k^*(\alpha_1 + \alpha_2)] \\
 S_{13} &= [g_{k-1}^*(\theta_1 + \theta_2)g_{k-1}^*(\alpha_1) - g_k^*(\theta_1 + \theta_2)g_k^*(\alpha_1)], S_{14} = [g_{k-1}^*(\theta_1 + \theta_2)g_{k-1}^*(\alpha_2) - g_k^*(\theta_1 + \theta_2)g_k^*(\alpha_2)] \\
 S_{15} &= [g_{k-1}^*(\theta_1 + \theta_2)g_{k-1}^*(\alpha_1 + \alpha_2) - g_k^*(\theta_1 + \theta_2)g_k^*(\alpha_1 + \alpha_2)]
 \end{aligned} \tag{4}$$

Since $\{ X_i \}$ is a geometric process

$$g_k^*(s) = \prod_{j=1}^k g^* \left(\frac{s}{d^{j-1}} \right), \quad k = 1, 2, 3, \dots \tag{5}$$

Since $L(t) = 1 - P(T > t)$, $l^*(s) = \frac{d}{ds} L(t)$ and $E(T) = \left. \frac{-d}{ds} [l^*(s)] \right|_{s=0}$ and $E(T^2) = \left. \frac{d^2}{ds^2} [l^*(s)] \right|_{s=0}$

$$\begin{aligned}
 L(t) &= \sum_{k=1}^{\infty} F_k(t)S_1 + \sum_{k=1}^{\infty} F_k(t)S_2 - \sum_{k=1}^{\infty} F_k(t)S_3 + p \left[\sum_{k=1}^{\infty} F_k(t)S_4 + \sum_{k=1}^{\infty} F_k(t)S_5 - \sum_{k=1}^{\infty} F_k(t)S_6 - \sum_{k=1}^{\infty} F_k(t)S_7 \right. \\
 &\quad \left. - \sum_{k=1}^{\infty} F_k(t)S_8 + \sum_{k=1}^{\infty} F_k(t)S_9 - \sum_{k=1}^{\infty} F_k(t)S_{10} - \sum_{k=1}^{\infty} F_k(t)S_{11} + \sum_{k=1}^{\infty} F_k(t)S_{12} + \sum_{k=1}^{\infty} F_k(t)S_{13} \right. \\
 &\quad \left. + \sum_{k=1}^{\infty} F_k(t)S_{14} - \sum_{k=1}^{\infty} F_k(t)S_{15} \right]
 \end{aligned} \tag{7}$$

$$\begin{aligned}
 l^*(s) &= \sum_{k=1}^{\infty} f_k^*(s)S_1 + \sum_{k=1}^{\infty} f_k^*(s)S_2 - \sum_{k=1}^{\infty} f_k^*(s)S_3 + p \left[\sum_{k=1}^{\infty} f_k^*(s)S_4 + \sum_{k=1}^{\infty} f_k^*(s)S_5 - \sum_{k=1}^{\infty} f_k^*(s)S_6 - \sum_{k=1}^{\infty} f_k^*(s)S_7 \right. \\
 &\quad \left. - \sum_{k=1}^{\infty} f_k^*(s)S_8 + \sum_{k=1}^{\infty} f_k^*(s)S_9 - \sum_{k=1}^{\infty} f_k^*(s)S_{10} - \sum_{k=1}^{\infty} f_k^*(s)S_{11} + \sum_{k=1}^{\infty} f_k^*(s)S_{12} + \sum_{k=1}^{\infty} f_k^*(s)S_{13} \right. \\
 &\quad \left. + \sum_{k=1}^{\infty} f_k^*(s)S_{14} - \sum_{k=1}^{\infty} f_k^*(s)S_{15} \right]
 \end{aligned} \tag{8}$$

where S_j 's are given by (4).

Case (i): $U_i, i=1,2,3, \dots$ are exchangeable and constantly correlated exponential random variables.

Taking Laplace Stieltjes Transform on both sides of (7) we get

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$$\begin{aligned}
 L^*(s) = & \sum_{k=1}^{\infty} F_k^*(s)S_1 + \sum_{k=1}^{\infty} F_k^*(s)S_2 - \sum_{k=1}^{\infty} F_k^*(s)S_3 + p \left[\sum_{k=1}^{\infty} F_k^*(s)S_4 + \sum_{k=1}^{\infty} F_k^*(s)S_5 - \sum_{k=1}^{\infty} F_k^*(s)S_6 - \sum_{k=1}^{\infty} F_k^*(s)S_7 \right. \\
 & - \sum_{k=1}^{\infty} F_k^*(s)S_8 + \sum_{k=1}^{\infty} F_k^*(s)S_9 - \sum_{k=1}^{\infty} F_k^*(s)S_{10} - \sum_{k=1}^{\infty} F_k^*(s)S_{11} + \sum_{k=1}^{\infty} F_k^*(s)S_{12} + \sum_{k=1}^{\infty} F_k^*(s)S_{13} \\
 & \left. + \sum_{k=1}^{\infty} F_k^*(s)S_{14} - \sum_{k=1}^{\infty} F_k^*(s)S_{15} \right] \tag{9}
 \end{aligned}$$

It is known that

$$F_k^*(s) = \frac{(1-\rho)(bs+1)^{1-k}}{(1-\rho)(1+bs) + k\rho bs} \tag{10}$$

$$\left[\frac{d}{ds} (F_k^*(s)) \right]_{s=0} = -uk \tag{11}$$

$$\left[\frac{d^2}{ds^2} (F_k^*(s)) \right]_{s=0} = u^2 \left[k^2(1+\rho^2) + k(1-\rho^2) \right] \tag{12}$$

Using (9),(10),(11),(12) and (6)

The first two moments are given by

$$\begin{aligned}
 E(T) = & u \left(\sum_{k=1}^{\infty} kS_1 + \sum_{k=1}^{\infty} kS_2 - \sum_{k=1}^{\infty} kS_3 + p \left[\sum_{k=1}^{\infty} kS_4 + \sum_{k=1}^{\infty} kS_5 - \sum_{k=1}^{\infty} kS_6 - \sum_{k=1}^{\infty} kS_7 - \sum_{k=1}^{\infty} kS_8 \right. \right. \\
 & \left. \left. + \sum_{k=1}^{\infty} kS_9 - \sum_{k=1}^{\infty} kS_{10} - \sum_{k=1}^{\infty} kS_{11} + \sum_{k=1}^{\infty} kS_{12} + \sum_{k=1}^{\infty} kS_{13} + \sum_{k=1}^{\infty} kS_{14} - \sum_{k=1}^{\infty} kS_{15} \right] \right) \tag{13}
 \end{aligned}$$

$$\begin{aligned}
 E(T^2) = & u^2 \left(\sum_{k=1}^{\infty} (k^2(1+\rho^2) + k(1-\rho^2))S_1 + \sum_{k=1}^{\infty} (k^2(1+\rho^2) + k(1-\rho^2))S_2 - \sum_{k=1}^{\infty} (k^2(1+\rho^2) + k(1-\rho^2))S_3 \right. \\
 & + p \left[\sum_{k=1}^{\infty} (k^2(1+\rho^2) + k(1-\rho^2))S_4 + \sum_{k=1}^{\infty} (k^2(1+\rho^2) + k(1-\rho^2))S_5 - \sum_{k=1}^{\infty} (k^2(1+\rho^2) + k(1-\rho^2))S_6 \right. \\
 & - \sum_{k=1}^{\infty} (k^2(1+\rho^2) + k(1-\rho^2))S_7 - \sum_{k=1}^{\infty} (k^2(1+\rho^2) + k(1-\rho^2))S_8 + \sum_{k=1}^{\infty} (k^2(1+\rho^2) + k(1-\rho^2))S_9 \\
 & - \sum_{k=1}^{\infty} (k^2(1+\rho^2) + k(1-\rho^2))S_{10} - \sum_{k=1}^{\infty} (k^2(1+\rho^2) + k(1-\rho^2))S_{11} + \sum_{k=1}^{\infty} (k^2(1+\rho^2) + k(1-\rho^2))S_{12} \\
 & \left. \left. + \sum_{k=1}^{\infty} (k^2(1+\rho^2) + k(1-\rho^2))S_{13} + \sum_{k=1}^{\infty} (k^2(1+\rho^2) + k(1-\rho^2))S_{14} - \sum_{k=1}^{\infty} (k^2(1+\rho^2) + k(1-\rho^2))S_{15} \right] \right) \tag{14}
 \end{aligned}$$

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where S_j 's are given by (4) and $u = \frac{b}{1-\rho}$. (13) give the mean time to recruitment .(13) and (14) together with (6) give the variance of time to recruitment.

Case (iii): $U_i, i=1,2,3,\dots$ form a geometric process

Since $\{U_i\}$ is a geometric process it is known that

$$f_k^*(s) = \prod_{r=1}^k f^*\left(\frac{s}{a^{r-1}}\right), \quad k = 1, 2, 3, \dots \tag{15}$$

From (6),(8) and (15)

$$E(T) = E(U_1) \frac{a}{a-1} \left(\sum_{k=1}^{\infty} \frac{a^k - 1}{a^{k-1}} S_1 + \sum_{k=1}^{\infty} \frac{a^k - 1}{a^{k-1}} S_2 - \sum_{k=1}^{\infty} \frac{a^k - 1}{a^{k-1}} S_3 + p \left[\sum_{k=1}^{\infty} \frac{a^k - 1}{a^{k-1}} S_4 + \sum_{k=1}^{\infty} \frac{a^k - 1}{a^{k-1}} S_5 \right. \right. \\ \left. \left. - \sum_{k=1}^{\infty} \frac{a^k - 1}{a^{k-1}} S_6 - \sum_{k=1}^{\infty} \frac{a^k - 1}{a^{k-1}} S_7 - \sum_{k=1}^{\infty} \frac{a^k - 1}{a^{k-1}} S_8 + \sum_{k=1}^{\infty} \frac{a^k - 1}{a^{k-1}} S_9 - \sum_{k=1}^{\infty} \frac{a^k - 1}{a^{k-1}} S_{10} - \sum_{k=1}^{\infty} \frac{a^k - 1}{a^{k-1}} S_{11} \right. \right. \\ \left. \left. + \sum_{k=1}^{\infty} \frac{a^k - 1}{a^{k-1}} S_{12} + \sum_{k=1}^{\infty} \frac{a^k - 1}{a^{k-1}} S_{13} + \sum_{k=1}^{\infty} \frac{a^k - 1}{a^{k-1}} S_{14} - \sum_{k=1}^{\infty} \frac{a^k - 1}{a^{k-1}} S_{15} \right] \right) \tag{16}$$

$$E(T^2) = \left(\sum_{k=1}^{\infty} \left(\frac{a^{2k} - 1}{(a^2 - 1)a^{2(k-1)}} \sigma_{U_1}^2 + E(U_1) \left[\frac{(a^k - 1)^2}{(a-1)^2 a^{2(k-1)}} \right] \right) S_1 + \sum_{k=1}^{\infty} \left(\frac{a^{2k} - 1}{(a^2 - 1)a^{2(k-1)}} \sigma_{U_1}^2 + E(U_1) \left[\frac{(a^k - 1)^2}{(a-1)^2 a^{2(k-1)}} \right] \right) S_2 \right. \\ \left. - \sum_{k=1}^{\infty} \left(\frac{a^{2k} - 1}{(a^2 - 1)a^{2(k-1)}} \sigma_{U_1}^2 + E(U_1) \left[\frac{(a^k - 1)^2}{(a-1)^2 a^{2(k-1)}} \right] \right) S_3 + p \left[\sum_{k=1}^{\infty} \left(\frac{a^{2k} - 1}{(a^2 - 1)a^{2(k-1)}} \sigma_{U_1}^2 + E(U_1) \left[\frac{(a^k - 1)^2}{(a-1)^2 a^{2(k-1)}} \right] \right) S_4 \right. \right. \\ \left. \left. + \sum_{k=1}^{\infty} \left(\frac{a^{2k} - 1}{(a^2 - 1)a^{2(k-1)}} \sigma_{U_1}^2 + E(U_1) \left[\frac{(a^k - 1)^2}{(a-1)^2 a^{2(k-1)}} \right] \right) S_5 - \sum_{k=1}^{\infty} \left(\frac{a^{2k} - 1}{(a^2 - 1)a^{2(k-1)}} \sigma_{U_1}^2 + E(U_1) \left[\frac{(a^k - 1)^2}{(a-1)^2 a^{2(k-1)}} \right] \right) S_6 \right. \right. \\ \left. \left. - \sum_{k=1}^{\infty} \left(\frac{a^{2k} - 1}{(a^2 - 1)a^{2(k-1)}} \sigma_{U_1}^2 + E(U_1) \left[\frac{(a^k - 1)^2}{(a-1)^2 a^{2(k-1)}} \right] \right) S_7 - \sum_{k=1}^{\infty} \left(\frac{a^{2k} - 1}{(a^2 - 1)a^{2(k-1)}} \sigma_{U_1}^2 + E(U_1) \left[\frac{(a^k - 1)^2}{(a-1)^2 a^{2(k-1)}} \right] \right) S_8 \right. \right. \\ \left. \left. + \sum_{k=1}^{\infty} \left(\frac{a^{2k} - 1}{(a^2 - 1)a^{2(k-1)}} \sigma_{U_1}^2 + E(U_1) \left[\frac{(a^k - 1)^2}{(a-1)^2 a^{2(k-1)}} \right] \right) S_9 - \sum_{k=1}^{\infty} \left(\frac{a^{2k} - 1}{(a^2 - 1)a^{2(k-1)}} \sigma_{U_1}^2 + E(U_1) \left[\frac{(a^k - 1)^2}{(a-1)^2 a^{2(k-1)}} \right] \right) S_{10} \right. \right. \\ \left. \left. - \sum_{k=1}^{\infty} \left(\frac{a^{2k} - 1}{(a^2 - 1)a^{2(k-1)}} \sigma_{U_1}^2 + E(U_1) \left[\frac{(a^k - 1)^2}{(a-1)^2 a^{2(k-1)}} \right] \right) S_{11} + \sum_{k=1}^{\infty} \left(\frac{a^{2k} - 1}{(a^2 - 1)a^{2(k-1)}} \sigma_{U_1}^2 + E(U_1) \left[\frac{(a^k - 1)^2}{(a-1)^2 a^{2(k-1)}} \right] \right) S_{12} \right. \right. \\ \left. \left. + \sum_{k=1}^{\infty} \left(\frac{a^{2k} - 1}{(a^2 - 1)a^{2(k-1)}} \sigma_{U_1}^2 + E(U_1) \left[\frac{(a^k - 1)^2}{(a-1)^2 a^{2(k-1)}} \right] \right) S_{13} + \sum_{k=1}^{\infty} \left(\frac{a^{2k} - 1}{(a^2 - 1)a^{2(k-1)}} \sigma_{U_1}^2 + E(U_1) \left[\frac{(a^k - 1)^2}{(a-1)^2 a^{2(k-1)}} \right] \right) S_{14} \right. \right. \\ \left. \left. - \sum_{k=1}^{\infty} \left(\frac{a^{2k} - 1}{(a^2 - 1)a^{2(k-1)}} \sigma_{U_1}^2 + E(U_1) \left[\frac{(a^k - 1)^2}{(a-1)^2 a^{2(k-1)}} \right] \right) S_{15} \right] \right) \tag{17}$$

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Where $E(U_1) = -f^{*'}(0)$ and $\sigma_{U_1}^2 = f^{*''}(0) - \left(f^{*'}(0)\right)^2$ and S_j 's are given by (4). (16) give the mean time to recruitment. (16) and (17) together with (6) give the variance of time to recruitment.

III. MODEL DESCRIPTION AND ANALYSIS FOR MODEL- II

For this model, the optional and mandatory thresholds for the number of exits in the organization are given by $Y = \min(Y_1, Y_2)$ and $Z = \min(Z_1, Z_2)$. All other assumptions and notations are as in model-I.

Case (i): $U_i, i=1,2,3,\dots$ are exchangeable and constantly correlated exponential random variables.

The first two moments are given by

$$E(T) = u \left[\sum_{k=1}^{\infty} k S_3 + p \left[\sum_{k=1}^{\infty} k S_6 - \sum_{k=1}^{\infty} k S_{15} \right] \right] \quad (18)$$

$$E(T^2) = u^2 \left[\sum_{k=1}^{\infty} (k^2(1+\rho^2) + k(1-\rho^2)) S_3 + p \left[\sum_{k=1}^{\infty} (k^2(1+\rho^2) + k(1-\rho^2)) S_6 - \sum_{k=1}^{\infty} (k^2(1+\rho^2) + k(1-\rho^2)) S_{15} \right] \right] \quad (19)$$

where S_j 's are given by (4). (18) give the mean time to recruitment. (18) and (19) together with (6) give the variance of time to recruitment.

Case (ii): $U_i, i=1,2,3,\dots$ form a geometric process

The first two moments are given by

$$E(T) = E(U_1) \frac{a}{a-1} \left[\sum_{k=1}^{\infty} \frac{a^k - 1}{a^{k-1}} S_3 + p \left[\sum_{k=1}^{\infty} \frac{a^k - 1}{a^{k-1}} S_6 - \sum_{k=1}^{\infty} \frac{a^k - 1}{a^{k-1}} S_{15} \right] \right] \quad (20)$$

$$E(T^2) = \left[\sum_{k=1}^{\infty} \left(\frac{a^{2k} - 1}{(a^2 - 1)a^{2(k-1)}} \sigma_{U_1}^2 + E(U_1) \left[\frac{(a^k - 1)^2}{(a-1)^2 a^{2(k-1)}} \right] \right) S_3 + p \left[\sum_{k=1}^{\infty} \left(\frac{a^{2k} - 1}{(a^2 - 1)a^{2(k-1)}} \sigma_{U_1}^2 + E(U_1) \left[\frac{(a^k - 1)^2}{(a-1)^2 a^{2(k-1)}} \right] \right) S_6 - \sum_{k=1}^{\infty} \left(\frac{a^{2k} - 1}{(a^2 - 1)a^{2(k-1)}} \sigma_{U_1}^2 + E(U_1) \left[\frac{(a^k - 1)^2}{(a-1)^2 a^{2(k-1)}} \right] \right) S_{15} \right] \quad (21)$$

where $E(U_1) = -f^{*'}(0)$ and $\sigma_{U_1}^2 = f^{*''}(0) - \left(f^{*'}(0)\right)^2$. S_j 's are given by (4). (20) give the mean time to recruitment. (20) and (21) together with (6) give the variance of time to recruitment.

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IV. MODEL DESCRIPTION AND ANALYSIS FOR MODEL- III

For this model, the optional and mandatory thresholds for the loss of man-hours in the organization are taken as $Y = Y_1 + Y_2$ and $Z = Z_1 + Z_2$. All the other assumptions and notations are as in model-I.

Case (i): $U_i, i=1,2,3,\dots$ are exchangeable and constantly correlated exponential random variables.

The first two moments are given by

$$E(T) = u \left[\sum_{k=1}^{\infty} k A_2 S_2 - \sum_{k=1}^{\infty} k A_1 S_1 + p \left[\sum_{k=1}^{\infty} k A_5 S_5 - \sum_{k=1}^{\infty} k A_4 S_4 - \sum_{k=1}^{\infty} k A_1 A_4 S_7 + \sum_{k=1}^{\infty} k A_1 A_5 S_8 + \sum_{k=1}^{\infty} k A_2 A_4 S_{10} - \sum_{k=1}^{\infty} k A_2 A_5 S_{11} \right] \right] \quad (22)$$

$$E(T^2) = u^2 \left[\sum_{k=1}^{\infty} (k^2 (1 + \rho^2) + k(1 - \rho^2)) A_2 S_2 - \sum_{k=1}^{\infty} (k^2 (1 + \rho^2) + k(1 - \rho^2)) A_1 S_1 + p \left[\sum_{k=1}^{\infty} (k^2 (1 + \rho^2) + k(1 - \rho^2)) A_5 S_5 - \sum_{k=1}^{\infty} (k^2 (1 + \rho^2) + k(1 - \rho^2)) A_4 S_4 - \sum_{k=1}^{\infty} (k^2 (1 + \rho^2) + k(1 - \rho^2)) A_1 A_4 S_7 + \sum_{k=1}^{\infty} (k^2 (1 + \rho^2) + k(1 - \rho^2)) A_1 A_5 S_8 + \sum_{k=1}^{\infty} (k^2 (1 + \rho^2) + k(1 - \rho^2)) A_2 A_4 S_{10} - \sum_{k=1}^{\infty} (k^2 (1 + \rho^2) + k(1 - \rho^2)) A_2 A_5 S_{11} \right] \right] \quad (23)$$

where S_j 's are given by (4). (22) give the mean time to recruitment. (22) and (23) together with (6) give the variance of time to recruitment.

Case (ii): $U_i, i=1,2,3,\dots$ form a geometric process

The first two moments are given by

$$E(T) = E(U_1) \frac{a}{a-1} \left[\sum_{k=1}^{\infty} \frac{a^k - 1}{a^{k-1}} A_2 S_2 - \sum_{k=1}^{\infty} \frac{a^k - 1}{a^{k-1}} A_1 S_1 + p \left[\sum_{k=1}^{\infty} \frac{a^k - 1}{a^{k-1}} A_5 S_5 - \sum_{k=1}^{\infty} \frac{a^k - 1}{a^{k-1}} A_4 S_4 - \sum_{k=1}^{\infty} \frac{a^k - 1}{a^{k-1}} A_1 A_4 S_7 + \sum_{k=1}^{\infty} \frac{a^k - 1}{a^{k-1}} A_1 A_5 S_8 + \sum_{k=1}^{\infty} \frac{a^k - 1}{a^{k-1}} A_2 A_4 S_{10} - \sum_{k=1}^{\infty} \frac{a^k - 1}{a^{k-1}} A_2 A_5 S_{11} \right] \right] \quad (24)$$

$$E(T^2) = \left[\sum_{k=1}^{\infty} \left(\frac{a^{2k} - 1}{(a^2 - 1)a^{2(k-1)}} \sigma_{U_1}^2 + E(U_1) \left[\frac{(a^k - 1)^2}{(a-1)^2 a^{2(k-1)}} \right] \right) A_2 S_2 - \sum_{k=1}^{\infty} \left(\frac{a^{2k} - 1}{(a^2 - 1)a^{2(k-1)}} \sigma_{U_1}^2 + E(U_1) \left[\frac{(a^k - 1)^2}{(a-1)^2 a^{2(k-1)}} \right] \right) A_1 S_1 + p \left[\sum_{k=1}^{\infty} \left(\frac{a^{2k} - 1}{(a^2 - 1)a^{2(k-1)}} \sigma_{U_1}^2 + E(U_1) \left[\frac{(a^k - 1)^2}{(a-1)^2 a^{2(k-1)}} \right] \right) A_5 S_5 - \sum_{k=1}^{\infty} \left(\frac{a^{2k} - 1}{(a^2 - 1)a^{2(k-1)}} \sigma_{U_1}^2 + E(U_1) \left[\frac{(a^k - 1)^2}{(a-1)^2 a^{2(k-1)}} \right] \right) A_4 S_4 - \sum_{k=1}^{\infty} \left(\frac{a^{2k} - 1}{(a^2 - 1)a^{2(k-1)}} \sigma_{U_1}^2 + E(U_1) \left[\frac{(a^k - 1)^2}{(a-1)^2 a^{2(k-1)}} \right] \right) A_1 A_4 S_7 + \sum_{k=1}^{\infty} \left(\frac{a^{2k} - 1}{(a^2 - 1)a^{2(k-1)}} \sigma_{U_1}^2 + E(U_1) \left[\frac{(a^k - 1)^2}{(a-1)^2 a^{2(k-1)}} \right] \right) A_1 A_5 S_8 + \sum_{k=1}^{\infty} \left(\frac{a^{2k} - 1}{(a^2 - 1)a^{2(k-1)}} \sigma_{U_1}^2 + E(U_1) \left[\frac{(a^k - 1)^2}{(a-1)^2 a^{2(k-1)}} \right] \right) A_2 A_4 S_{10} - \sum_{k=1}^{\infty} \left(\frac{a^{2k} - 1}{(a^2 - 1)a^{2(k-1)}} \sigma_{U_1}^2 + E(U_1) \left[\frac{(a^k - 1)^2}{(a-1)^2 a^{2(k-1)}} \right] \right) A_2 A_5 S_{11} \right] \quad (25)$$

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where

$$A_1 = \frac{\theta_2}{\theta_1 - \theta_2}, A_2 = \frac{\theta_1}{\theta_1 - \theta_2}, A_4 = \frac{\alpha_2}{\alpha_1 - \alpha_2}, A_5 = \frac{\alpha_1}{\alpha_1 - \alpha_2},$$

$E(U) = -f^{*'}(0)$ and $\sigma_U^2 = f^{*''}(0) - \left(f^{*'}(0)\right)^2$. and S_j 's are given by (4). (24) give the mean time to recruitment. (24) and (25) together with (6) give the variance of time to recruitment.

Note: when $a=1$ in case (ii), one can deduce the analogous results for $V(T)$ when U_i 's are i.i.d exponential random variables for all the models.

V. FINDINGS

From the above results the following observations are presented which agree with reality,

- i. When ρ increases and keeping all the other parameter fixed, variance of time to recruitment increase for all the models.
- ii. As $a > 1$ ($a < 1$), U_i 's, $i=1,2,3,\dots$ form a decreasing (increasing) sequence and hence the inter-decision times will decrease (increase). Consequently, the variance of the time to recruitment increase (decrease).

VI. CONCLUSION

In reality, no organization will prefer to go for frequent recruitment as it involves cost. In this context, Model III is more suitable than Models I and II, as in Model III, the average time to recruitment is greater than that for the other two Models.

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