

Vertex Prime Labeling of Union of Complete Bipartite Graphs

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ABSTRACT: A graph $G(V, E)$ is said to have a vertex prime labeling if its edges can be labeled with distinct integers from $\{1, 2, 3, \dots, |E|\}$ such that for each vertex of degree at least 2, the greatest common divisor of the labels on its incident edges is 1. A graph that admits a vertex prime labeling is called a vertex prime graph.

In this paper, we prove that $mK_{5,5}$, $mK_{6,6}$, $mK_{7,7}$ and $mK_{8,8}$ are vertex prime graphs, where m is any positive integer.

KEYWORDS: labeling of graphs, vertex prime labeling of graphs.

Subject Classification Code (2000): 05C(Primary)

I. INTRODUCTION

Let $G(V, E)$ be a graph. For notations and terminology, we follow [1]. G is called a *vertex prime graph* if there is a bijection $f : E \rightarrow \{1, 2, 3, \dots, |E|\}$ such that for any vertex v , $\gcd \{f(uv) \mid uv \in E\} = 1$. The bijection f is called a *vertex prime labeling* of G .

For example, vertex prime labelings of some known graphs are illustrated in Figure 1.

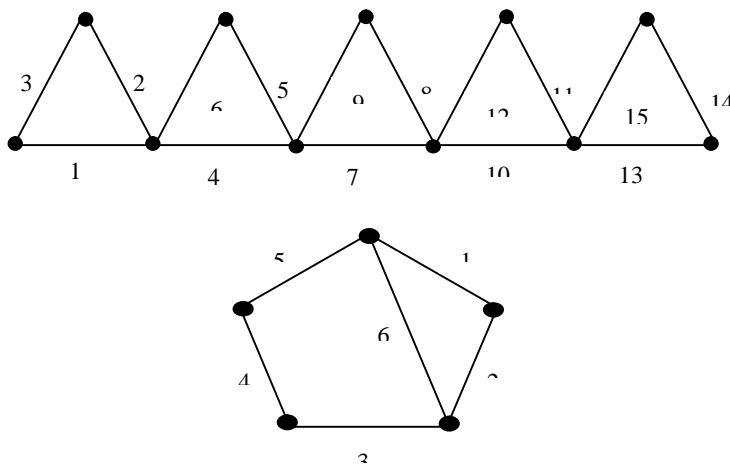


Figure 1

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The concept of vertex prime graphs has been introduced by T. Deretsky, S.M. Lee and J. Mitchem [3] in 1991. They proved that the forests; any connected graph; $C_{2k} \cup C_n$; $C_{2k} \cup C_{2n} \cup C_{2k+1}$; $C_{2m} \cup C_{2n} \cup C_{2t} \cup C_k$; and $5C_{2m}$ are vertex prime. They have further proved that a graph with exactly 2 components, one of which is not an odd cycle has a vertex prime labeling and a 2 – regular graph with at least two odd cycles does not have a vertex prime labeling. They have conjectured that a 2 – regular graph has a vertex prime labeling if and only if it does not have two odd cycles. Let $G = \bigcup_{i=1}^t C_{2n_i}$ and $N = \sum_{i=1}^t n_i$. In [2], I. Borosh, D. Hensley and A. Hobbs proved that there is a

positive constant n_0 such that the conjecture of Deretsky et al., is true for the following cases:

- i) G is the disjoint union of at most seven cycles, or
- ii) G is a union of cycles all of the same even length $2n$ if $n \leq 1,50,000$
or if $n \geq n_0$, or
- iii) $n_i \geq (\log N)^{4 \log \log \log N}$ for all $i = 1, 2, 3, \dots, t$, or
- iv) Each C_{2n_i} is repeated at most n_i times.

In [4], Selvam Avadayappan and R. Sinthu proved that $mK_{2,3}$ is vertex prime. In [5], Selvam Avadayappan and R. Sinthu proved that $mK_{3,3}$ and $mK_{4,4}$ are vertex prime.

In this paper, we prove the vertex primeness of the union of m disjoint copies of the complete bipartite graphs $K_{5,5}$, $K_{6,6}$, $K_{7,7}$ and $K_{8,8}$.

II. RELATED WORK

Mean Graphs, Super Mean Graphs, Irregular Graphs, Domination Parameters in Graphs are the related works.

III. PRESENTATION OF THE MAIN CONTRIBUTION OF THE PAPER / SCOPE OF RESEARCH

We prove that $mK_{5,5}$, $mK_{6,6}$, $mK_{7,7}$ and $mK_{8,8}$ are vertex prime graphs through the definition of vertex prime graphs. We also work on the general case of this theorem.

IV. EXPERIMENTAL RESULTS

We proved that $mK_{5,5}$, $mK_{6,6}$, $mK_{7,7}$ and $mK_{8,8}$ are vertex prime graphs.

Theorem 1 For any positive integer m , $mK_{5,5}$ is a vertex prime graph.

Proof

Let $V(mK_{5,5}) = \{u_1^1, u_2^1, u_3^1, u_4^1, u_5^1; u_1^2, u_2^2, u_3^2, u_4^2, u_5^2; \dots; u_1^m, u_2^m, u_3^m, u_4^m, u_5^m; v_1^1, v_2^1, v_3^1, v_4^1, v_5^1; v_1^2, v_2^2, v_3^2, v_4^2, v_5^2; \dots; v_1^m, v_2^m, v_3^m, v_4^m, v_5^m\}$

and

$E(mK_{5,5}) = \{u_i^r v_j^r : 1 \leq i \leq 5, 1 \leq j \leq 5; 1 \leq r \leq m\}$

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Define $f : E(mK_{5,5}) \rightarrow \{1, 2, 3, \dots, 25m\}$ as follows:

$$\begin{aligned} f(u_1^r v_j^r) &= 25(r-1) + j, 1 \leq j \leq 5, 1 \leq r \leq m; \\ f(u_2^r v_j^r) &= 25r - (j + 14), 1 \leq j \leq 5, 1 \leq r \leq m; \\ f(u_3^r v_j^r) &= 25r - (j + 9), 1 \leq j \leq 5, 1 \leq r \leq m; \\ f(u_4^r v_j^r) &= 25r - (j + 4), 1 \leq j \leq 5, 1 \leq r \leq m; \\ f(u_5^r v_j^r) &= 25r - (j - 1), 1 \leq j \leq 5, 1 \leq r \leq m. \end{aligned}$$

Consider the vertex u_1^r .

$$\begin{aligned} \gcd(\{f(u_1^r v_j^r), 1 \leq j \leq 5, 1 \leq r \leq m\}) \\ = \gcd(25(r-1) + j, 1 \leq j \leq 5, 1 \leq r \leq m) \\ = 1 \end{aligned}$$

Similarly one can check for the remaining vertices $u_2^r, u_3^r, u_4^r, u_5^r, v_1^r, v_2^r, v_3^r, v_4^r$ and v_5^r .

Clearly f is a vertex prime labeling of $mK_{5,5}$.

Hence $mK_{5,5}$ is a vertex prime graph.

For example, the edge labels of a vertex prime labeling of $4K_{5,5}$ is shown in Figure 2. ■

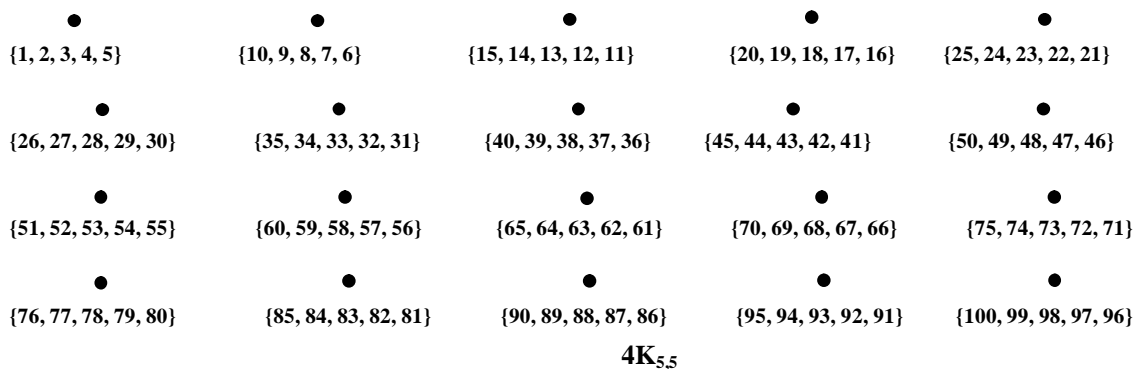


Figure 2

Here first set represents the edge labels of the edges incident with the first vertex of the first copy of $K_{5,5}$ and so on.

Theorem 2 For any positive integer m , $mK_{6,6}$ is a vertex prime graph.

Proof

Let $V(mK_{6,6}) = \{u_1^1, u_2^1, u_3^1, u_4^1, u_5^1, u_6^1; u_1^2, u_2^2, u_3^2, u_4^2, u_5^2, u_6^2; \dots; u_1^m, u_2^m, u_3^m, u_4^m, u_5^m, u_6^m; v_1^1, v_2^1, v_3^1, v_4^1, v_5^1, v_6^1; v_1^2, v_2^2, v_3^2, v_4^2, v_5^2, v_6^2; \dots; v_1^m, v_2^m, v_3^m, v_4^m, v_5^m, v_6^m\}$

and

$$E(mK_{6,6}) = \{u_i^r v_j^r : 1 \leq i \leq 6, 1 \leq j \leq 6; 1 \leq r \leq m\}$$

Define $f : E(mK_{6,6}) \rightarrow \{1, 2, 3, \dots, 36m\}$ as follows:

$$f(u_i^r v_j^r) = 36(r-1) + j, 1 \leq j \leq 6, 1 \leq r \leq m;$$

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$$f(u_2^r v_j^r) = 36r - (j + 23), 1 \leq j \leq 6, 1 \leq r \leq m;$$

$$f(u_3^r v_j^r) = 36r - (j + 17), 1 \leq j \leq 6, 1 \leq r \leq m;$$

$$f(u_4^r v_j^r) = 36r - (j + 11), 1 \leq j \leq 6, 1 \leq r \leq m;$$

$$f(u_5^r v_j^r) = 36r - (j + 5), 1 \leq j \leq 6, 1 \leq r \leq m;$$

$$f(u_6^r v_j^r) = 36r - (j - 1), 1 \leq j \leq 6, 1 \leq r \leq m.$$

Consider the vertex u_3^r

$$\begin{aligned} \gcd((f(u_3^r v_j^r), 1 \leq j \leq 6, 1 \leq r \leq m)) \\ = \gcd(36r - (j + 17), 1 \leq j \leq 6, 1 \leq r \leq m) \\ = 1 \end{aligned}$$

Similarly one can check for the remaining vertices $u_1^r, u_2^r, u_4^r, u_5^r, u_6^r, v_1^r, v_2^r, v_3^r, v_4^r, v_5^r$ and v_6^r .

Clearly f is a vertex prime labeling of $mK_{6,6}$.

Hence $mK_{6,6}$ is a vertex prime graph.

For example, the edge labels of a vertex prime labeling of $4K_{6,6}$ is shown in Figure 3. ■

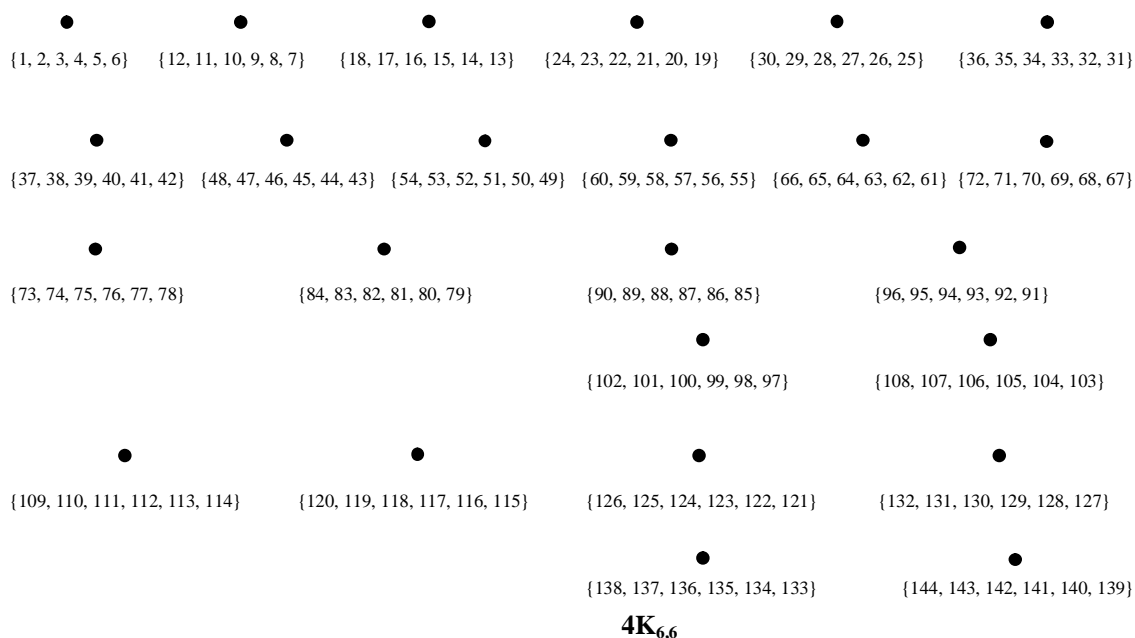


Figure 3

Here first set represents the edge labels of the edges incident with the first vertex of the first copy of $K_{6,6}$ and so on.

Theorem 3 For any positive integer m , $mK_{7,7}$ is a vertex prime graph.

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Proof

Let $V(mK_{7,7}) = \{u_1^1, u_2^1, u_3^1, u_4^1, u_5^1, u_6^1, u_7^1; u_1^2, u_2^2, u_3^2, u_4^2, u_5^2, u_6^2, u_7^2; \dots; u_1^m, u_2^m, u_3^m, u_4^m, u_5^m, u_6^m, u_7^m; v_1^1, v_2^1, v_3^1, v_4^1, v_5^1, v_6^1, v_7^1; v_1^2, v_2^2, v_3^2, v_4^2, v_5^2, v_6^2, v_7^2; \dots; v_1^m, v_2^m, v_3^m, v_4^m, v_5^m, v_6^m, v_7^m\}$

and

$$E(mK_{7,7}) = \{u_i^r v_j^r : 1 \leq i \leq 7, 1 \leq j \leq 7; 1 \leq r \leq m\}$$

Define $f : E(mK_{7,7}) \rightarrow \{1, 2, 3, \dots, 49m\}$ as follows:

$$f(u_1^r v_j^r) = 49(r-1) + j, 1 \leq j \leq 7, 1 \leq r \leq m;$$

$$f(u_2^r v_j^r) = 49r - (j + 34), 1 \leq j \leq 7, 1 \leq r \leq m;$$

$$f(u_3^r v_j^r) = 49r - (j + 27), 1 \leq j \leq 7, 1 \leq r \leq m;$$

$$f(u_4^r v_j^r) = 49r - (j + 20), 1 \leq j \leq 7, 1 \leq r \leq m;$$

$$f(u_5^r v_j^r) = 49r - (j + 13), 1 \leq j \leq 7, 1 \leq r \leq m;$$

$$f(u_6^r v_j^r) = 49r - (j + 6), 1 \leq j \leq 7, 1 \leq r \leq m;$$

$$f(u_7^r v_j^r) = 49r - (j - 1), 1 \leq j \leq 7, 1 \leq r \leq m.$$

Consider the vertex u_2^r

$$\gcd(\{f(u_2^r v_j^r), 1 \leq j \leq 7, 1 \leq r \leq m\})$$

$$= \gcd(49r - (j + 34), 1 \leq j \leq 7, 1 \leq r \leq m)$$

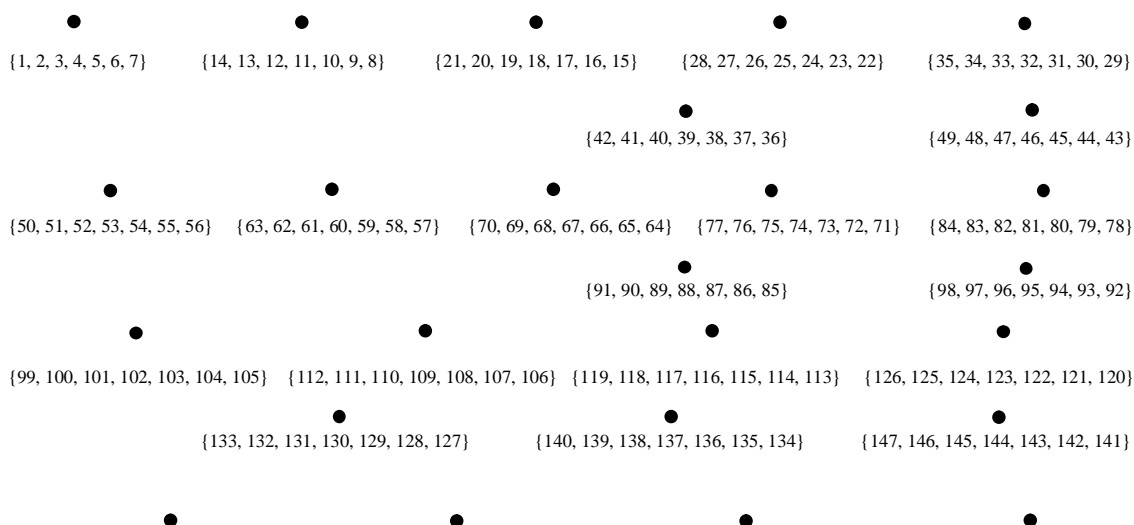
$$= 1$$

Similarly one can check for the remaining vertices $u_1^r, u_3^r, u_4^r, u_5^r, u_6^r, u_7^r, v_1^r, v_2^r, v_3^r, v_4^r, v_5^r, v_6^r$ and v_7^r .

Clearly f is a vertex prime labeling of $mK_{7,7}$.

Hence $mK_{7,7}$ is a vertex prime graph.

For example, the edge labels of a vertex prime labeling of $4K_{7,7}$ is shown in Figure 4. ■



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{148, 149, 150, 151, 152, 153, 154} {161, 160, 159, 158, 157, 156, 155} {168, 167, 166, 165, 164, 163, 162} {175, 174, 173, 172, 171, 170, 169}

● ● ●

{182, 181, 180, 179, 178, 177, 176} {189, 188, 187, 186, 185, 184, 183} {196, 195, 194, 193, 192, 191, 190}

4K_{7,7}

Figure 4

Here first set represents the edge labels of the edges incident with the first vertex of the first copy of $K_{7,7}$ and so on.

Theorem 4 For any positive integer m , $mK_{8,8}$ is a vertex prime graph.

Proof

Let $V(mK_{8,8}) = \{u_1^1, u_2^1, u_3^1, u_4^1, u_5^1, u_6^1, u_7^1, u_8^1; u_1^2, u_2^2, u_3^2, u_4^2, u_5^2, u_6^2, u_7^2, u_8^2; \dots; u_1^m, u_2^m, u_3^m, u_4^m, u_5^m, u_6^m, u_7^m, u_8^m; v_1^1, v_2^1, v_3^1, v_4^1, v_5^1, v_6^1, v_7^1, v_8^1; v_1^2, v_2^2, v_3^2, v_4^2, v_5^2, v_6^2, v_7^2, v_8^2; \dots; v_1^m, v_2^m, v_3^m, v_4^m, v_5^m, v_6^m, v_7^m, v_8^m\}$

and

$$E(mK_{8,8}) = \{u_i^r v_j^r : 1 \leq i \leq 8, 1 \leq j \leq 8; 1 \leq r \leq m\}$$

Define $f : E(mK_{8,8}) \rightarrow \{1, 2, 3, \dots, 64m\}$ as follows:

- $f(u_1^r v_j^r) = 64(r-1) + j, 1 \leq j \leq 8, 1 \leq r \leq m;$
- $f(u_2^r v_j^r) = 64r - (j + 47), 1 \leq j \leq 8, 1 \leq r \leq m;$
- $f(u_3^r v_j^r) = 64r - (j + 39), 1 \leq j \leq 8, 1 \leq r \leq m;$
- $f(u_4^r v_j^r) = 64r - (j + 31), 1 \leq j \leq 8, 1 \leq r \leq m;$
- $f(u_5^r v_j^r) = 64r - (j + 23), 1 \leq j \leq 8, 1 \leq r \leq m;$
- $f(u_6^r v_j^r) = 64r - (j + 15), 1 \leq j \leq 8, 1 \leq r \leq m;$
- $f(u_7^r v_j^r) = 64r - (j + 7), 1 \leq j \leq 8, 1 \leq r \leq m;$
- $f(u_8^r v_j^r) = 64r - (j - 1), 1 \leq j \leq 8, 1 \leq r \leq m.$

Consider the vertex u_2^r

$$\begin{aligned} &\gcd((f(u_2^r v_j^r), 1 \leq j \leq 8, 1 \leq r \leq m)) \\ &= \gcd(64r - (j + 47), 1 \leq j \leq 8, 1 \leq r \leq m) \\ &= 1 \end{aligned}$$

Similarly one can check for the remaining vertices $u_1^r, u_3^r, u_4^r, u_5^r, u_6^r, u_7^r, u_8^r, v_1^r, v_2^r, v_3^r, v_4^r, v_5^r, v_6^r, v_7^r$ and v_8^r .

Clearly f is a vertex prime labeling of $mK_{8,8}$.

Hence $mK_{8,8}$ is a vertex prime graph.

For example, the edge labels of a vertex prime labeling of $4K_{8,8}$ is shown in Figure 5. ■

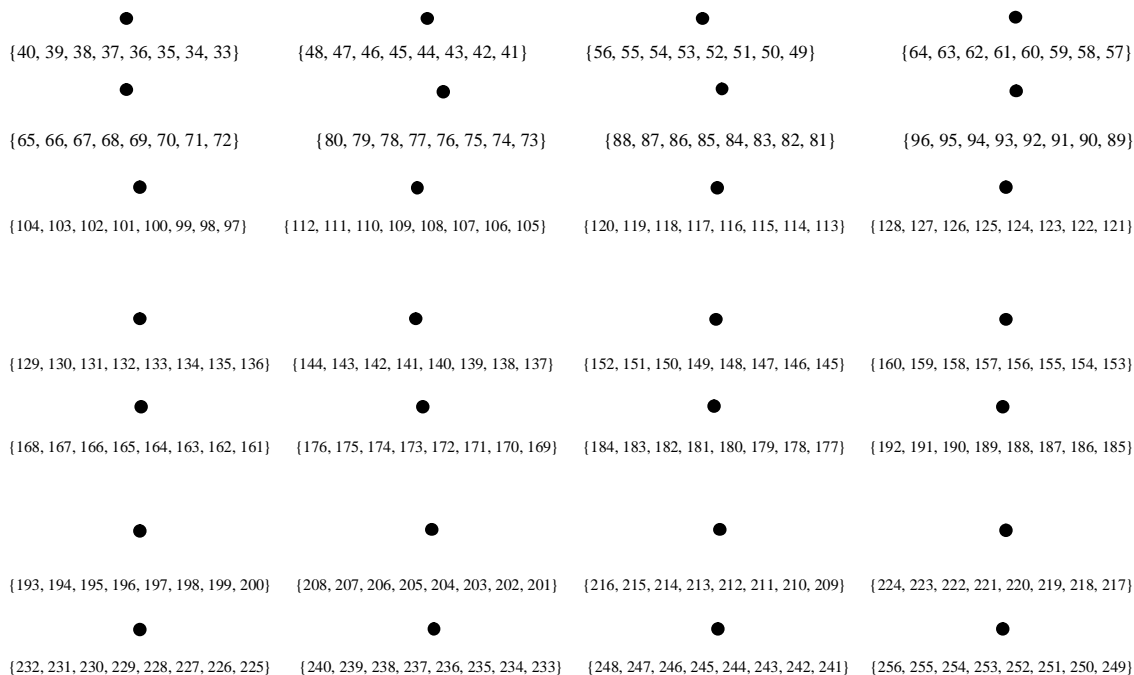
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{1, 2, 3, 4, 5, 6, 7, 8} {16, 15, 14, 13, 12, 11, 10, 9} {24, 23, 22, 21, 20, 19, 18, 17} {32, 31, 30, 29, 28, 27, 26, 25}

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$4K_{8,8}$

Figure 5

Here first set represents the edge labels of the edges incident with the first vertex of the first copy of $K_{8,8}$ and so on.

V. CONCLUSION

In this paper, we present Vertex Prime Labeling if its edges can be labeled with distinct integers. Thus we prove that $mK_{5,5}$, $mK_{6,6}$, $mK_{7,7}$ and $mK_{8,8}$ are vertex prime graphs. Some known graphs and unknown graphs are illustrated in a simple manner.

VI. ACKNOWLEDGEMENT

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