

# On Contra $\pi g\beta$ - Continuous functions in Čech Closure Spaces

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**ABSTRACT:** Every topological space is a closure space, if we define the closure operator of the space as a function that takes any subset to its closure. This paper presents an introduction to Čech closure space. The set of all Čech closure operators on a set is closed under the operations of union. We initiate Contra  $\pi g\beta$  - continuous maps, Contra  $\pi g\beta$  - irresolute maps. Furthermore we extend and study their characterizations. At every places the new notions have been substantiated with suitable examples. We also investigate the relationships among such functions.

**KEYWORDS:** Contra  $\pi g\beta$  - continuous and Contra  $\pi g\beta$  - irresolute functions, totally continuous,  $\pi g\beta$  - open map.

## 1. INTRODUCTION AND PRELIMINARIES

N. Levine [6] introduced  $g$ -closed sets. The concept of generalized closed sets and generalized continuous maps of topological spaces were extended to closure spaces in [2]. Čech closure spaces were introduced by E. Čech [3] and then studied by many authors [4][5][7][8].

A map  $k: P(X) \rightarrow P(X)$  defined on the power set  $P(X)$  of a set  $X$  is called a closure operator on  $X$  and the pair  $(X, k)$  is called a closure space if the following axioms are satisfied.

- (i).  $k(\phi) = \phi$ ,
  - (ii).  $A \subseteq k(A)$  for every  $A \subseteq X$
  - (iii).  $k(A \cup B) = k(A) \cup k(B)$  for all  $A, B \subseteq X$
- A closure operator  $k$  on a set  $X$  is called idempotent if  $k(A) = k[k(A)]$  for all  $A \subseteq X$ .

**Definitions 1.1.** A subset  $A$  of a Čech closure space  $(X, k)$  is said to be

- (i) Čech closed if  $k(A) = A$
- (ii) Čech open if  $k(X-A) = X-A$
- (iii) Čech semi-open if  $A \subseteq k(\text{int}(A))$
- (iv) Čech pre-open if  $A \subseteq \text{int}[k(A)]$
- (v) Čech pre-closed if  $k[\text{int}(A)] \subseteq A$

**Definition 1.2.** A Čech closure space  $(Y, l)$  is said to be a subspace of  $(X, k)$  if  $Y \subseteq X$  and  $k(A) = k(A) \cap Y$  for each subset  $A \subseteq Y$ . If  $Y$  is closed in  $(X, k)$  then the subspace  $(Y, l)$  of  $(X, k)$  is said to be closed too.

**Definition 1.3.** Let  $(X, k)$  and  $(Y, l)$  be Čech closure spaces. A map  $f: (X, k) \rightarrow (Y, l)$  is said to be continuous, if  $f(kA) \subseteq l(f(A))$  for every subset  $A \subseteq X$ .

**Definition 1.4.** Let  $(X, k)$  and  $(Y, l)$  be Čech closure spaces. A map  $f: (X, k) \rightarrow (Y, l)$  is said to be closed (resp. open) if  $f(F)$  is a closed (resp. open) subset of  $(Y, l)$  whenever  $F$  is a closed (resp. open) subset of  $(X, k)$ .

**Definition 1.5.[1]**

Let  $(X, u)$  and  $(Y, v)$  be closure spaces. A map  $f: (X, u) \rightarrow (Y, v)$  is called  $\pi g\beta$  - continuous if the inverse image of every open set in  $(Y, v)$  is  $\pi g\beta$  - open in  $(X, u)$ .

**Definition 1.6.[1]**

Let  $(X, u)$  and  $(Y, v)$  be closure spaces and a map  $f: (X, u) \rightarrow (Y, v)$  is called  $\pi g\beta$  - irresolute, if  $f^{-1}(G)$  is  $\pi g\beta$  - open (closed) in  $(X, u)$  for every  $\pi g\beta$  - open set (closed set)  $G$  in  $(Y, v)$ .

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**Definition 1.7.[1]**

Let  $(X, u)$  and  $(Y, v)$  be closure spaces and a map  $f : (X, u) \rightarrow (Y, v)$  is called  $\pi g\beta$  - open map (closed map) if  $f(B)$  is  $\pi g\beta$  - open (closed) in  $(Y, v)$  for every open set (closed set)  $B$  in  $(X, u)$ .

**Definition 1.8.[1]**

A closure space  $(X, u)$  is said to be a  $T_f$  - space if every  $\pi g\beta$  - open set in  $(X, u)$  is open.

## II. CONTRA $\pi g\beta$ - CONTINUOUS AND CONTRA $\pi g\beta$ - IRRESOLUTE FUNCTIONS

**Definition 2.1.**

Let  $(X, u)$  and  $(Y, v)$  be closure spaces. A map  $f : (X, u) \rightarrow (Y, v)$  is called contra-  $\pi g\beta$  - continuous if the inverse image of every open set in  $(Y, v)$  is  $\pi g\beta$  - closed in  $(X, u)$ .

**Proposition 2.2.**

Let  $f : (X, u) \rightarrow (Y, v)$  be a map where  $(X, u)$  and  $(Y, v)$  are closure spaces. Then  $f$  is contra- $\pi g\beta$  - continuous if and only if the inverse image of every closed subset of  $(Y, v)$  is  $\pi g\beta$  - open in  $(X, u)$ .

**Proof:**

Consider  $F$  to be a closed subset in  $(Y, v)$ . Then  $Y - F$  is open in  $(Y, v)$ . Since  $f$  is contra  $\pi g\beta$  - continuous,  $f^{-1}(Y - F)$  is  $\pi g\beta$  - closed. But  $f^{-1}(Y - F) = X - f^{-1}(F)$ . Thus  $f^{-1}(F)$  is  $\pi g\beta$  - open in  $(X, u)$ . Conversely let  $G$  be an open subset in  $(Y, v)$ . Then  $Y - G$  is closed in  $(Y, v)$ . Since the inverse image of each closed subset in  $(Y, v)$  is  $\pi g\beta$  - open in  $(X, u)$ ,  $f^{-1}(Y - G)$  is  $\pi g\beta$  - open in  $(X, u)$ . But  $f^{-1}(Y - G) = X - f^{-1}(G)$ . Thus  $f^{-1}(G)$  is  $\pi g\beta$  - closed. Therefore  $f$  is contra  $\pi g\beta$  - continuous.

**Proposition 2.3.**

Let  $(X, u)$ ,  $(Y, v)$  and  $(Z, w)$  be closure spaces. If  $f : (X, u) \rightarrow (Y, v)$  and  $g : (Y, v) \rightarrow (Z, w)$  be maps. If  $g \circ f$  is contra  $\pi g\beta$  - continuous and  $g$  is a closed injection. Then  $f$  is contra  $\pi g\beta$  - continuous.

**Proof:**

Let  $H$  be a closed subset of  $(Y, v)$ . Since  $g$  is closed,  $g(H)$  is closed in  $(Z, w)$ . As  $g \circ f$  is contra  $\pi g\beta$  - continuous,  $(g \circ f)^{-1}(g(H)) = f^{-1}(g^{-1}(g(H)))$  is  $\pi g\beta$  - open in  $(X, u)$ . But  $g$  is injective, hence  $f^{-1}(g^{-1}(g(H))) = f^{-1}(H)$ ,  $f^{-1}(H)$  is  $\pi g\beta$  - open. Therefore,  $f$  is contra  $\pi g\beta$  - continuous.

**Proposition 2.4.**

Let  $(X, u)$  and  $(Z, w)$  be closure spaces and  $(Y, v)$  be a  $T_f$  - space. If  $f : (X, u) \rightarrow (Y, v)$  and  $g : (Y, v) \rightarrow (Z, w)$  are contra  $\pi g\beta$  - continuous maps. Then  $g \circ f$  is contra  $\pi g\beta$  - continuous.

**Proof :**

Let  $H$  be closed in  $(Z, w)$ . Since  $g$  is contra  $\pi g\beta$  - continuous,  $g^{-1}(H)$  is  $\pi g\beta$  - open in  $(Y, v)$ . But  $(Y, v)$  is a  $T_f$  - space, Hence  $g^{-1}(H)$  is open in  $(Y, v)$ . As  $f$  is contra  $\pi g\beta$  - continuous,  $f^{-1}(g^{-1}(H)) = (g \circ f)^{-1}(H)$  is  $\pi g\beta$  - closed in  $(X, u)$ . Therefore,  $g \circ f$  is  $\pi g\beta$  - continuous.

**Proposition 2.5.**

Assume  $(X, u)$ ,  $(Y, v)$  and  $(Z, w)$  be closure spaces, where  $f : (X, u) \rightarrow (Y, v)$  and  $g : (Y, v) \rightarrow (Z, w)$  be two maps. If  $f$  is contra  $\pi g\beta$  - continuous and  $g$  is continuous. Then  $g \circ f$  is contra  $\pi g\beta$  - continuous.

**Proof:**

Consider  $g \circ f : (X, u) \rightarrow (Z, w)$ . Let  $H$  be closed in  $Z$ . Since  $g$  is continuous,  $g^{-1}(H)$  is closed in  $Y$ . As  $f$  is contra  $\pi g\beta$  - continuous,  $f^{-1}(g^{-1}(H))$  is  $\pi g\beta$  - open in  $X$ . Hence  $(g \circ f)^{-1}(H)$  is  $\pi g\beta$  - open in  $X$ . Therefore,  $(g \circ f)$  is contra  $\pi g\beta$  - continuous.

**Proposition 2.6.**

Let  $(X, u)$  be a closure space,  $\{(Y_\alpha, v_\alpha) : \alpha \in J\}$  be a family of closure spaces

and  $f : (X, u) \rightarrow \prod_{\alpha \in J} (Y_\alpha, v_\alpha)$  be a map. If  $f$  is contra  $\pi g\beta$  - continuous and  $\pi_\alpha$  is a projection map, then  $\pi_\alpha \circ f$  is contra  $\pi g\beta$  - continuous for each  $\alpha \in J$ .

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**Proof:**

Let  $f: (X, u) \rightarrow \prod_{\alpha \in J} (Y_\alpha, v_\alpha)$  be a map. Let  $A$  be closed in  $\prod_{\alpha \in J} (Y_\alpha, v_\alpha)$ .  $\pi_\alpha$  is a projection map.  $(\pi_\alpha)^{-1}(A)$  is closed in

$\prod_{\alpha \in J} (Y_\alpha, v_\alpha)$ . As  $f$  is contra  $\pi g \beta$ -continuous,  $f^{-1}(\pi_\alpha)^{-1}(A)$  is  $\pi g \beta$ -open which implies  $(\pi_\alpha \circ f)^{-1}(A)$  is  $\pi g \beta$ -open which also implies  $(\pi_\alpha \circ f)$  is  $\pi g \beta$ -continuous for each  $\alpha \in J$ .

**Definition 2.7.**

A function  $f: (X, u) \rightarrow (Y, v)$  is said to be contra  $\pi g \beta$  irresolute if  $f^{-1}(V)$  is  $\pi g \beta$ -open in  $(X, u)$  for every  $\pi g \beta$ -closed set  $V$  in  $(Y, v)$ .

**Definition 2.8.**

A function  $f: (X, u) \rightarrow (Y, v)$  is said to be totally continuous if  $f^{-1}(V)$  is closed in  $(X, u)$  for every open set  $V$  in  $(Y, v)$ .

**Theorem 2.9.**

1. Every contra continuous function is contra  $\pi g \beta$ -continuous.
2. Every contra  $\beta$ -continuous function is contra  $\pi g \beta$ -continuous.
3. Every contra  $g$ -continuous function is contra  $\pi g \beta$ -continuous.
4. Every contra  $\pi g \beta$ -irresolute function is contra  $\pi g \beta$ -continuous.
5. Every totally continuous function is contra  $\pi g \beta$ -continuous.

**Proof:**

1. Let  $f: (X, u) \rightarrow (Y, v)$  be contra-continuous and  $V$  be any open set in  $Y$ . Then  $f^{-1}(V)$  is closed in  $X$  which implies that  $f^{-1}(V)$  is  $\pi g \beta$ -closed.
2. Suppose  $A$  is any open set in  $Y$ . If  $f$  is contra- $\beta$ -continuous, then  $f^{-1}(A)$  is  $\beta$ -closed in  $X$ . Hence,  $f^{-1}(A)$  is  $\pi g \beta$ -closed in  $X$ .

Proofs of the following statements are similar.

**Remark 2.10.**

The converse of the above need not be true may be seen by the following example.

**Example 2.11.**

Let  $X = \{1, 2, 3\}$ ,  $Y = \{a, b, c\}$ . Define a closure operator  $u$  on  $X$  by  $u(\emptyset) = \emptyset$ ,  $u(\{1\}) = u(\{3\}) = u(\{1, 2\}) = u(\{1, 3\}) = u(\{2, 3\}) = uX = X$  and  $u(\{2\}) = \{2\}$ . Define a closure operator  $v$  on  $Y$  by  $v(\emptyset) = \emptyset$ ,  $v(\{a\}) = \{a, c\}$ ,  $v(\{b\}) = \{b\}$ ,  $v(\{c\}) = \{a, c\}$  &  $v(\{a, b\}) = v(\{a, c\}) = v(\{b, c\}) = vY = Y$ .

Let  $f: (X, u) \rightarrow (Y, v)$  be defined by  $f(1) = b$ ,  $f(2) = a$  &  $f(3) = c$ .

Here  $f$  is contra  $\pi g \beta$ -continuous but not contra continuous. Since for the closed set  $\{b\}$  in  $Y$ , the inverse image  $f^{-1}(\{b\}) = \{1\}$  is not open in  $X$ .

**Example 2.12.**

Let  $X = \{1, 2, 3\}$ ,  $Y = \{a, b, c\}$ . Define a closure operator  $u$  on  $X$  by  $u(\emptyset) = \emptyset$ ,  $u(\{1\}) = u(\{3\}) = u(\{1, 2\}) = u(\{1, 3\}) = u(\{2, 3\}) = uX = X$  and  $u(\{2\}) = \{2\}$ . Define a closure operator  $v$  on  $Y$  by  $v(\emptyset) = \emptyset$ ,  $v(\{a\}) = v\{a, c\}$ ,  $v(\{b\}) = \{b\}$ ,  $v(\{c\}) = \{a, c\}$  &  $v(\{a, b\}) = v(\{a, c\}) = v(\{b, c\}) = vY = Y$ .

Let  $f: (X, u) \rightarrow (Y, v)$  be defined by  $f(1) = a$ ,  $f(2) = b$  &  $f(3) = c$ . Here

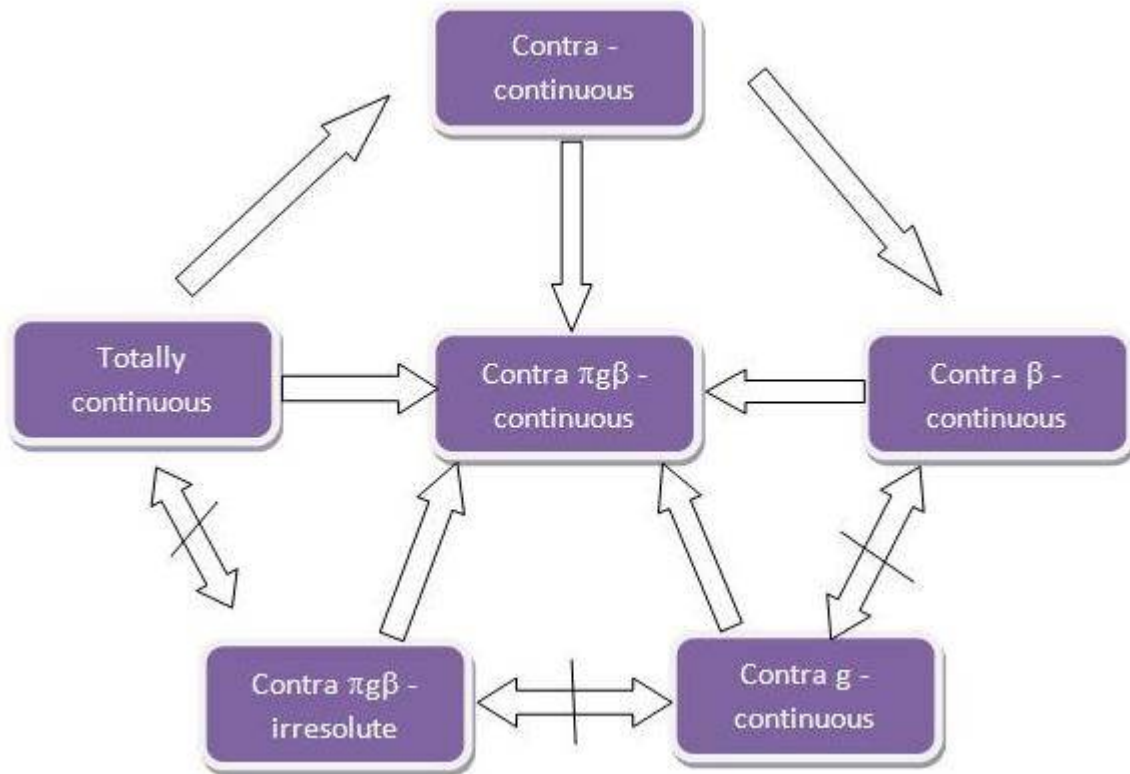
1.  $f$  is contra  $\pi g \beta$ -continuous but not contra  $\beta$ -continuous. Since for the closed set  $\{b\}$  in  $Y$ , the inverse image  $f^{-1}(\{b\}) = \{2\}$  is not  $\beta$ -open in  $X$ .
2.  $f$  is contra  $\pi g \beta$ -continuous but not contra  $g$ -continuous. Since for the open set  $\{a, c\}$  in  $Y$ , the inverse image  $f^{-1}(\{a, c\}) = \{1, 3\}$  is not  $g$ -closed in  $X$ .

From the above propositions we have the following diagram.

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**Proposition 2.13.**

Let  $f: (X,u) \rightarrow (Y,v)$  be a surjection function. Suppose  $\pi g\beta(X)$  is closed under arbitrary union. Then the following statements are equivalent.

1.  $f$  is contra  $\pi g\beta$ - continuous.
2. For every Čech closed subset  $F$  of  $Y$ ,  $f^{-1}(F)$  is  $\pi g\beta$  open in  $X$
3. For each  $x \in X$  and each Čech closed set  $F$  in  $Y$  containing  $f(x)$ , there exists  $\pi g\beta$ -open set  $U$  in  $X$  containing  $x$  such that  $f(U) \subseteq F$

**Proof:**

(1) $\Rightarrow$ (2)

Let  $F$  be Čech closed in  $Y$ . Then  $Y-F$  is Čech open in  $Y$ .

By (1),  $f^{-1}(Y-F) = X - f^{-1}(F)$  is  $\pi g\beta$ - closed in  $X$  which implies  $f^{-1}(F)$  is  $\pi g\beta$ -open.

(2) $\Rightarrow$ (1)

Suppose  $V$  is Čech open in  $Y$ . Then  $Y-V$  is Čech closed in  $Y$

By (2),  $f^{-1}(Y-V) = X - f^{-1}(V)$  is  $\pi g\beta$ - open in  $X$  which implies  $f^{-1}(V)$  is  $\pi g\beta$ -closed in  $X$

Thus  $f$  is contra  $\pi g\beta$ - continuous.

(2) $\Rightarrow$ (3)

Let  $F$  be Čech closed set in  $Y$  containing  $f(x)$ . By (2),  $f^{-1}(F)$  is  $\pi g\beta$ -open for  $x \in f^{-1}(F)$

Take  $U = f^{-1}(F)$  then  $f(U) \subseteq F$

(3) $\Rightarrow$ (2)

Let  $F$  be any Čech closed set in  $Y$  and  $x \in f^{-1}(F)$

By (3), there exists  $\pi g\beta$ -open set  $U_x \subseteq f^{-1}(F)$ , Hence  $f^{-1}(F) = \bigcup_{x \in X} U_x$ . Thus  $f^{-1}(F)$  is

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$\pi g\beta$ -open.

## Proposition 2.14.

If  $f: (X,u) \rightarrow (Y,v)$  be a map, then the following are equivalent

1.  $f$  is contra- $\pi g\beta$ - irresolute
2. For each  $x \in X$  and any  $\pi g\beta$ -open set  $V$  of  $(Y,u)$  containing  $f(x)$  there exists a  $\pi g\beta$ -closed set  $U$  such that  $x \in U$  and  $f(U) \subseteq V$
3. The inverse image of every  $\pi g\beta$ -closed set in  $(Y,v)$  is  $\pi g\beta$ -open in  $(X,u)$ .

The proof is obvious.

## III.CONCLUSION

The results in this paper help us to know the relationship between contra- $\pi g\beta$ - continuity and contra- $\pi g\beta$ - irresolute functions. The theorems and examples given along with the diagrammatic representation helps to have a clear idea about the concept introduced and for further investigations.

## REFERENCES

- [1] I.Arockiarani and A.FrancinaShalini, "New functions in Cech  $\pi g\beta$  - Closure Spaces", International Journal of Recent scientific Research, Accepted.
- [2] C.Boonpok, "Generalized closed sets in cech closed space", Acta math Univ, Apulensis, no-22, pp133-140,2010.
- [3] E.Čech, "Topological Spaces", Topological papers of Eduard Čech, Academia Prague, pp 436-472,1968.
- [4] J.Chvalina, "On homeomorphic topologies and equivalent systems", Arch Math.2, Scripta Fac.Sci.Nat. UJEP Brunensis, vol12, pp 107- 116,1976.
- [5] J.Chvalina, "Stackbases in power sets of neighbourhood spaces Preserving the continuity of mappings", Arch Math.2, Scripta Fac.Sci.Nat. UJEP Brunensis, vol 17, pp 81-86,1981.
- [6] N.Levine, "Generalized closed sets in topology", Rend, Circ. Mat.Palermo, vol 19, pp 89-96,1970.
- [7] L.Skula, Systeme von stetigenabbildungen.Caech.Math. J. 17. 92, pp45-52,1967.
- [8] J.Slapal, "Closure operations for digital topology", Theoret. Comput.Sci, vol305, pp 457 – 471,2003.