

# Unreliable Batch Arrival Queuing Systems with SWV

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**ABSTRACT:** In this paper we analyze  $M^X/M/1/SWV/BD$  queueing model. For this model the total probability generating function is obtained through the Chapman-Kolmogorov balance equation satisfied by the steady state system size probabilities. The probability generating function is presented in closed form so that various performance measures are also analyzed.

**KEYWORDS:** Single Working Vacation, Breakdown, Working Vacation Policy, Batch Arrival, Total Probability Generating Function, Performance Measures.

## I. INTRODUCTION

A queuing system is described as one having a service facility at which units of some kind arrive for service and whenever there are more units in the system than the service facility can handle simultaneously, a queue or waiting line, develops. The waiting units take their turn for service according to a pre assigned rule and after service they leave the system. The study of classical queuing models have made by Saaty [13], Gross & Harris [3] and Medhi [12]. A system may have a singular server or a number of parallel servers. An arrival who finds more than one free server may choose at random any of them for receiving service. The batch arrival is described as the flow of arrivals in batches. Gaver [2] introduced bulk arrival queues, where the arrivals could be in batch. The study on the analysis of queueing model of bulk arrival in  $M^X/M/1$ ,  $M^X/G/1$  and  $M^X/M/\infty$  is found in Gross and Harris [3], Medhi [11]. Later Lee et al., [8] studied the batch arrival system and found that the system size decomposes into two random variables, one is the system size of the classical  $M^X/G/1$  queue and the other is the probability Generating Function (PGF) of the customers in the system when the server is idle.

In classical single server queueing models an idle server will start his service as soon as a customer enters the empty system. Among many control policies the N-policy is the most general one.

In N-policy the server does not start his service until there are N-customers in the queue. This policy is introduced by Yadin and Naor [23] and is designed to minimize server switch overs and to avoid excessive frequent use of setups. The N-policy queueing model is used in Wireless Sensor Network (WSN). Analytical study of the N-policy  $M/E_k/1$  queueing system was first analysed by Wang and Huang [19]. The N policy  $M/G/1$  queueing system was first studied by Heyman [5] and was developed by several researchers such as Bell [1], Teghem [15], Wang and Ke [17] and others. Ke and Wang [6] presented a recursive method using supplementary variable technique to develop analytical closed form solution for the N policy  $GI/M/1$  queueing system with finite capacity.

In the case of Single Vacation Policy if the server returns from vacation and finds fewer number of customers than N, he joins the system and waits until the system size reach or exceeds N and then begins to serve exhaustively. Servi and Finn [14] first studied the  $M/M/1$  queue with working vacations. In working vacation policy, a customer is served at a lower rate rather than completely stopping the service and such a policy may cause the loss or dissatisfaction of the customer. If there are customers in the system, the server will start a new regular busy period or he takes another busy period. But under working vacation policy, the server can still work during the vacation and may accomplish other assistant work simultaneously. So the working vacation is more reasonable than the classical vacation. Li and Tian [9], Xu et al., [22] considered  $M/M/1$  queue with different working vacation policies. Hence Xu et al., [21] studied the results of Liu et al., [10] to input model  $M^X/M/1$  MWV. Later Julia Rose Mary, K. and Afthab Begum [4] analyse  $M^X/M/1/WV$  queueing system.

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The classical queueing systems assume a reliable machine or server, the server may fail and can repair. This phenomena of server breakdowns can be encountered in the area of computers, communication networks, flexible manufacturing system etc., The performance of the system may be affected heavily by these breakdowns and limited repair capacity. Wang [17] first proposed Markovian queueing system under the N-policy and server breakdowns. Wang [18] and Wang et al., [20] extended the model proposed by to  $M/E_k/1$  and  $M/H_2/1$  queue system respectively. Later ke [7] considered the control policy for batch arrival queueing system under N-policy in which the server is subject to breakdown. Thus the unreliable server plays an important role in queueing system.

In this paper a batch arrival  $M^X/M/1$  queueing system is analysed under exponential distributed single working vacation with break down. And also we have made an attempt to obtain the steady state solution for  $M^X/M/1$  queue with single working vacation with breakdown by writing steady state equation and deduce the results for authors Servi And Finn [14] and Tian et al [16] by solving the difference equations directly.

## II. MODEL DISCRPTION

we consider  $M^X/M/1$  single working vaction with breakdown queueing model. The batch arrival rate be  $(\lambda)$ , and service rate be  $(\mu)$ , in regular busy period. During a working vacation an arrival unit is served at the rate of  $\mu_v$ . The server begins a working vacation of random length  $V$  at the instant when the queue becomes empty and vacation duration  $V$  follows an exponential distribution with parameter  $\eta$ . When the vacation ends if there are customer in the queue the server changes service rate from  $\mu_v$  to  $\mu$  and a regular busy period starts. Otherwise, the server enters an idle period, and a new regular busy period starts when arrival occurs.

If  $\lambda_k$  is the arrival rate or poisson process of batches of size  $k$  then  $g_k = \lambda_k / \lambda$   $k=1,2,3...$  where  $\lambda$  is the composite arrival rate of all batches equal to  $\sum_{i=1}^{\infty} \lambda_i$ . This total process, which arises from the overlap of the set of Poission processes with rates  $\{\lambda_k / k=1,2,...\}$  is compound Poission processes. Let  $X(Z)$ ,  $E(X)$  and  $E(X^2)$  denote the the PGF, first and second momend of random variables  $X$ .

$$\text{Then } X(Z) = \sum_{k=0}^{\infty} g_k z^k \quad E(X^k) = \sum_{n=1}^{\infty} g_n n^k$$

During the working vacation time and the arriving customers are served at rate  $\mu_v$ . The serve is subject to breakdown at any time while working with poission rate( $\alpha$ ) whenever the system fails, the server is sent immetiately for repair time is an indepent and identically distributed random variable ( $B_r$ ) following an exponential distribution where the repair time  $(1-e^{-\beta t})$ . The customer who is just being served when the server breaks down, join thead of the waiting line and resumes the service as soon as the server returns from the repair facility. This type of sevice continues untill the system becomes empty again. The server begins a working vacation of random length  $V$  at the instant, where queue becomes empty and vacation duration  $V$  follows an exponential distripution with parameter  $\eta$ . This model is denoted by  $M^X/M/1/SWV/BD$ .

## III.SYSTEM SIZE DISTRIBUTION

Let  $N_s(t)$  denotes the Number of customer in the system at time  $t$ , and

$$J(t) = \begin{cases} 0 & \text{the system is in a working vacation period at time } t \\ 1 & \text{the system is in regular busy period at time } t. \\ 2 & \text{the system is in break down period at time } t. \end{cases}$$

Then  $\{N_s(t), J(t)\}$  is a Markov Process.

Let  $Q_n(t) = P_r \{ N_s(t)=n; J(t)=0 \}$  ;  $n \geq 0$

$P_n(t) = P_r \{ N_s(t)=n; J(t)=1 \}$  ;  $n \geq 1$

And  $B_n(t) = P_r \{ N_s(t)=n; J(t)=2 \}$  ;  $n \geq 1$  denote the system size probabilities at time  $t$ .

Assuming the steady state system size probabilities as

$$P_n = \lim_{t \rightarrow \infty} P_n(t)$$

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$$Q_n = \lim_{t \rightarrow \infty} \int_t Q_n(t)$$

$$P_0 = \lim_{t \rightarrow \infty} \int_t P_0(t)$$

$$B_n = \lim_{t \rightarrow \infty} \int_t B_n(t) \quad \text{exist.}$$

The Steady state equation satisfied by the  $P_n$ 's,  $Q_n$ 's,  $B_n$ 's and  $P_0$  are given by

$$(\eta + \lambda)Q_0 = \mu_v Q_1 + \mu P_1 \tag{1}$$

$$(\lambda + \eta + \mu_v)Q_n = \lambda \sum_{k=1}^n Q_{n-k} g_k + \mu_v Q_{n+1} \tag{2}$$

$$(\lambda + \mu + \alpha)P_1 = \mu P_2 + \eta Q_1 + \lambda P_0 g_1 + \beta B_1 \tag{3}$$

$$(\lambda + \mu + \alpha)P_n = \lambda \sum_{k=1}^{n-1} P_{n-k} g_k + \mu P_{n+1} + \eta Q_0 + \lambda P_0 g_n + \beta B_n; n \geq 2 \tag{4}$$

$$\lambda P_0 = \eta Q_0 \tag{5}$$

$$(\lambda + \beta)B_1 = \alpha P_1 \tag{6}$$

$$(\lambda + \beta)B_n = \alpha P_n + \lambda \sum_{k=1}^{n-1} B_{n-k} g_k; n \geq 2 \tag{7}$$

To obtain the steady state distribution of the model, the partial PGFs are defined as,

$$Q(z) = \sum_{n=0}^{\infty} Q_n z^n$$

$$P(z) = \sum_{n=1}^{\infty} P_n z^n$$

$$B(z) = \sum_{n=1}^{\infty} B_n z^n$$

Multiplying equation (2) by  $z^n$  and summing over  $n=1,2,\dots,\infty$ , then adding with equation (1)

$$h(z) = (\lambda z(1 - X(z)) + \mu_v(z - 1) + \eta z)$$

$$Q(z) = \frac{\mu_v Q_0(z - z_1)}{z_1 [\lambda z(1 - X(z)) + \mu_v(z - 1) + \eta z]} \tag{8}$$

Multiplying equation (6) by  $z^n$  and summing over  $n=1,2,\dots,\infty$  then adding with equation (5)

$$B(z) = \frac{\alpha P(z)}{[\lambda(1 - X(z)) + \beta]} \tag{9}$$

Similarly we find that  $P(z)$  is given by,

$$P(z) = \left\{ Q_0 z \left[ \frac{\lambda \mu_v}{z_1} \left( \frac{z_1(z-1)(X(z_1)-1) - z(z_1-1)(X(z)-1)}{\lambda z(1-X(z)) + \mu_v(z-1) + \eta z} \right) - \eta(1-X(z)) \right] \left[ \frac{\lambda(1-X(z)) + \beta}{\beta [\lambda z(1-X(z)) + \mu(z-1)] + [\lambda z(1-X(z)) + \mu(z-1) + \alpha z] [\lambda(1-X(z))]} \right] \right\} \tag{10}$$

$$P_0 = \frac{\eta}{\lambda} Q_0 \tag{11}$$

Thus the total PGF  $P_{SWV}^{Br}(z)$  is obtained by adding  $Q(z)$ ,  $P(z)$ ,  $B(z)$  and  $P_0$

$$\text{i.e., } P_{SWV}^{Br}(z) = P(z) + Q(z) + B(z) + P_0$$

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$$P_{SWV}^{Br}(z) = \left\{ \frac{Q_0(z-1)}{\lambda z(1-X(z)) + \mu(z-1)} \left[ \frac{\mu_v}{z_1} \left[ \frac{\mu(z-z_1) + \lambda z z_1 (X(z_1) - X(z))}{\lambda z(1-X(z)) + \mu_v(z-1) + \eta z} \right] \frac{1}{R} \left[ C + \beta + \frac{\alpha D}{\mu z z_1 + D_1} \right] \right. \right. \\ \left. \left. + \frac{\eta \mu}{\lambda} \left[ \frac{z C^2 + z C(\beta + \eta) + C \mu_v(z-1) + \beta(\mu_v(z-1) + \eta z)}{R(z C + \mu_v(z-1) + \eta z)} \right] \right] \right\}$$

where  $C = \lambda(1-X(z))$ ,  $D = \lambda z z_1 (X(z_1) - X(z))$ ,  $R = \frac{1}{\lambda(1-X(z)) + \beta + \frac{\alpha \lambda z(1-X(z))}{\lambda z(1-X(z)) + \mu(z-1)}}$

By the normalizing condition we have  $P_{SWV}^{Br}(1) = 1$

Hence by applying the normality condition, we obtain

$$Q_0 = \left[ \frac{\mu(1-\rho)}{\frac{\mu_v}{\eta z_1} \left[ \mu(1-z_1) + \lambda z_1 (X(z_1) - 1) \left( \frac{\alpha}{\beta} + 1 \right) + \frac{\eta \mu}{\lambda} \right]} \right]$$

by substituting  $Q_0$  in  $P_{SWV}^{Br}$ , Then we get Total PGF  $P_{SWV}^{Br}$

$$P_{SWV}^{Br}(z) = \left[ \frac{\mu(1-\rho)}{\frac{\mu_v}{\eta z_1} \left[ \mu(1-z_1) + \lambda z_1 (X(z_1) - 1) \left( \frac{\alpha}{\beta} + 1 \right) + \frac{\eta \mu}{\lambda} \right]} \right] \\ \left\{ \frac{(z-1)}{\lambda z(1-X(z)) + \mu(z-1)} \left[ \frac{\mu_v}{z_1} \left[ \frac{\mu(z-z_1) + \lambda z z_1 (X(z_1) - X(z))}{\lambda z(1-X(z)) + \mu_v(z-1) + \eta z} \right] \frac{1}{R} \left[ C + \beta + \frac{\alpha D}{\mu z z_1 + D_1} \right] \right. \right. \\ \left. \left. + \frac{\eta \mu}{\lambda} \left[ \frac{z C^2 + z C(\beta + \eta) + C \mu_v(z-1) + \beta(\mu_v(z-1) + \eta z)}{R(z C + \mu_v(z-1) + \eta z)} \right] \right] \right\}$$

Thus the total PGF of  $P_{SWV}^{Br}(z)$  is obtained.

## IV. PERFORMANCE MEASURES

Let  $P_v$ ,  $P_{busy}$ ,  $P_{idle}$ , and  $P_{bd}$  denotes that the server is on working vacation, regular busy period, idle period, and break down in the system.

(i) The Probability that the server is on vacation  $P_v$  is given by,

$$P_v = \lim_{z \rightarrow 1} Q(z) \\ P_v = \frac{\mu_v(1-z_1)}{\eta z_1} \left[ \frac{\mu(1-\rho)}{\frac{\mu_v}{\eta z_1} \left[ \mu(1-z_1) + \lambda z_1 (X(z_1) - 1) \left( \frac{\alpha}{\beta} + 1 \right) + \frac{\eta \mu}{\lambda} \right]} \right]$$

(ii) The probability that the server is in regular busy period ( $P_{busy}$ ) is given by

$$P_{busy} = \lim_{z \rightarrow 1} P(z) \\ P_{busy} = \left\{ \frac{\mu(1-\rho)}{\frac{\mu_v}{\eta z_1} \left[ \mu(1-z_1) + \lambda z_1 (X(z_1) - 1) \left( \frac{\alpha}{\beta} + 1 \right) + \frac{\eta \mu}{\lambda} \right]} \right\} \left[ \frac{\lambda \mu_v}{z_1} \left( \frac{z_1 (X(z_1) - 1) - (z_1 - 1) E(X) + \eta^2 E(X)}{\mu(\beta - \rho(\alpha + \beta))} \right) \frac{\beta}{\eta} \right]$$

(iii) The probability that the server is in idle  $P_{idle}$  is given by

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$$P_{idle} = \lim_{z \rightarrow 1} P_0$$

$$P_{idle} = \frac{\eta}{\lambda} \left[ \frac{\mu(1-\rho)}{\frac{\mu_v}{\eta z_1} \left[ \mu(1-z_1) + \lambda Z_1(X(z_1)-1) \left( \frac{\alpha}{\beta} + 1 \right) + \frac{\eta\mu}{\lambda} \right]} \right]$$

(iv) The probability that the server is in breakdown period ( $P_{Br}$ ) is given by

$$P_{br} = \lim_{z \rightarrow 1} B(z)$$

$$P_{br} = \left\{ \left[ \frac{\mu(1-\rho)}{\frac{\mu_v}{\eta z_1} \left[ \mu(1-z_1) + \lambda Z_1(X(z_1)-1) \left( \frac{\alpha}{\beta} + 1 \right) + \frac{\eta\mu}{\lambda} \right]} \right] \left[ \frac{\lambda\mu_v}{z_1} \left( \frac{z_1(X(z_1)-1) - (z_1-1)E(X) + \eta^2 E(X)}{\mu(\beta - \rho(\alpha + \beta))} \right) \frac{\alpha}{\eta} \right] \right\}$$

(v) Expected system size of the model (L) is given by

$$L = \frac{d}{dz} (P_{SWV}^{Br})_{z=1}$$

$$L = \left\{ \left[ \frac{\rho\mu}{M} C \right] + \left[ \frac{D(\mu - \lambda z_1 E(X)) - \mu(1-z_1)(\mu_v + \eta - \lambda E(X)) - D(\mu_v - \lambda E(X))}{M\eta} \right] - \left[ \frac{\eta\mu z_1(1-\rho)}{\lambda\mu_v M} \right] \left[ \mu(\lambda E(X) - \mu_v)(\lambda + 1)E(X) \right] \right\}$$

$$C = \mu(1-z_1) + \lambda z_1(X(z_1) - X(z)), \quad D = \lambda z_1(X(z_1) - 1), \quad M = \mu(1-z_1) + \lambda z_1(X(z_1) - 1) + \frac{\eta^2 \mu z_1}{\lambda \mu_v}$$

Thus the expected system size of the model is evaluated.

## V.CONCLUSION

In this paper we have analyzed the batch arrival  $M^X/M/1/SWV$  with breakdowns queuing model. And the total PGF of  $P_{SWV}^{Br}(z)$  is obtained and various performance measures has derived and the expected system size of the model is evaluated.

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