

Steady State Analysis of M/M(a,b)/1/MWV/Br Queuing Model

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ABSTRACT: In this paper Steady state Analysis of M/M(a,b)/1/MWV Queueing model for an unreliable server is evaluated. The general bulk service rule introduced by Neuts[17] is considered. The Steady state queue size Problem are obtained by solving difference equations. Further their performance measures are also analysed.

KEYWORDS: Breakdown, Bulk Service, Multiple Working Vacation, Mean queue length, Performance measures.

I.INTRODUCTION

In queueing models, there are situations particularly in transportation system, where the service provided is such that a group of customers can be served simultaneously. Examples include shuttle-bus service, freight trains, express elevators, tour guides and batch servicing in manufacturing process. The theory of batch queues originated with the work of Bailey [3]. He considered a queue with Poisson arrival and fixed size service. Numerous authors (Jaishwal [5]&Madil and Choudhury [15]) have investigated a variety of extensions of the basic model. Neuts[16] has introduced the most general bulk service rule in considering a queueing system with Poisson arrival and general service time distribution.

Most of the general bulk service queueing models with servers vacation have been analysed by many authors using matrix geometric method. Some of the notable works for matrix geometric method can be seen in Neuts [18]. Analytic solution of bulk service queueing models can be found in (Madhil and choudhury [15] and Medhi [16]. But these works do not include vacations. In general it is difficult to obtain a closed form solution for bulk service queueing models with servers vacation. Afthab Begum [1] has obtained analytic solutions for M/M(a,b)/1 queue, $E_k/M(a,b)/1$ queue and GI/M(a,b)/1 queue with servers single and multiple vacation in her Ph.D. thesis and presented the steady state results.

In 2002, Servi and Finn introduced a class of semi vacation policies, in which servers work at a lower rate rather than completely stopping primary service during vacation. Such a vacation is called **Working vacation (WV)**. In working vacation queues, the server works at a lower service rather than completely stopping service during the vacation period. At the vacation termination epochs, if there are customers in the system, the server will start a new regular busy period, otherwise, he takes another working vacation or joins the system and stays idle according as, he follows multiple or single working vacation policy. Zhang and Xu [30] have analysed the M/M/1 queue with multiple working vacations and N-Policy. The M/M/1 queueing system with working vacation has been analysed by Servi and Finn [20] and Tian .N, Zhao .X and Wang .K[22] Most of the general bulk service queueing models with servers vacation have been analysed by many authors using matrix geometric method. Tian, Li and Zhang [23] provided a survey of the results of working vacation queues. Moreover, Julia Rose Mary and Afthab Begum [7] have analysed the Markovian M/M(a,b)/1 queueing model under multiple working vacation and derived the steady state probability distribution and the mean queue length for the model.

Tian et al., [21], Li and Tian [11], Xu et al., [28] considered M/M/1 queue with different working vacation policies. Baba[2], Li et al., [13], Banik[4] studied the GI/M/1 type working vacation queues. Using different methods, Kim et al., [9], Wu and Takagi [27], Li et al., [14] discussed several M/G/1 type working vacation queues. Due to the wide applications in the performance analysis of communication and computer systems, the discrete-time queues with various working vacation policies have been considered in Tian et al., [21], Li and Tian [12], Li et al.,[10] and in Yi et al., [29]. Recently Tian et al., [23] provided a survey of the results of working vacation queues and demonstrated that

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the matrix analytic methods developed by Neuts([18] & [19]) are powerful tools for analyzing the working vacation queues and can be considered as a unified approach to this class of queueing models.

Wang[24] first proposed Markovian queueing system under the N-policy and server break downs. Wang[25] and Wang et al., [26] extended the model proposed by Wang [24] to $M/E_k/1$ and $M/H_2/1$ queue system respectively. Ke, J.C and Pearn, W. L [8] discussed the optimal management policy of heterogenous arrival $M/M/1$ queue with server's break down and vacations. Later Ke, J.C[6] studied the operating system characteristics of $M^X/G/1$ queueing system under vacation policies with start up and break down times in which the server may breakdown according to a Poisson process while working and his repair time has general distribution. With the help of available literature in this paper we are analysing $M/M(a,b)/1/MWV$ queueing model, for an unreliable server.

II. MODEL DESCRIPTION

In this model, it is assumed that the arrival process is Poisson with parameter λ . The server processes the customers in batches according to the general bulk service rule (GBSR) introduced by Neuts [17].

According to this rule the server starts service only when a minimum of 'a' customers are present in the system. If the server after a service completion finds a (or) more but at most **b** customers present in the system, then he takes them all in a batch, and if he finds more than **b**, then he takes in the batch the first **b**- customers for service, while others wait. Thus each batch for service contains a minimum of 'a' units and a maximum of 'b' units. This rule is called general bulk service rule(GBSR).The service time of batches of size S ($a \leq s \leq b$) is assumed to be independent identically distributed random variable with exponential distribution of parameter μ .

The server subject to break downs at any time while working, with Poisson rate α . Whenever the system fails, the server is sent immediately for repair facility where the repair time is an independent and identically distributed random variable B_r following an exponential distribution $(1 - e^{-\beta t})$ the customer, who is just being served when the server break down, joins the head of waiting line and resumes the service as soon as the server returns from the repair facility. This type of service continues until the system becomes empty again.

Whenever the server completes a service and finds less than 'a' customers in the queue he begins a vacation which is an exponentially distributed random variable V with parameter η . After completing a vacation, if the system length is still less than 'a' he takes another vacation and the vacations are continued until the server finds at least 'a' customers in the queue (i.e) multiple vacation is adopted. Suppose during vacation if the queue size becomes at least 'a' the server starts his service under the GBSR with service rate μ_v which is different from the regular service rate μ . When the vacation ends he switches his service rate from μ_v to μ when the server is working, the size of the batch in service is x with $a \leq x \leq b$ and the service rates are independent of the size of the batch in service, The server may breakdown at any time, and it is sent for repair and thus it is completing repaired. Then the server continues the service.

The above queueing model is denoted by $M/M(a,b)/1/MWV/Br$. The steady state results including the probability distribution of the queue size and the expected number of units in the queue are obtained.

III. SYSTEM SIZE DISTRIBUTION

Let $N_Q(t)$ =the number of customers in the queue at time t and $J(t)=0,1,2$ or 3 according as the server is idle on vacation, busy on vacation or in regular busy state or in break down period respectively.

Let $R_n(t)=Pr\{N_Q(t)=n, J(t)=0\}; \quad 0 \leq n \leq a-1$

$Q_n(t)=Pr\{N_Q(t)=n, J(t)=1\}; \quad n \geq 0$

$P_n(t)=Pr\{N_Q(t)=n, J(t)=2\}; \quad n \geq 0$

$B_n(t)=Pr\{N_Q(t)=n, J(t)=3\}; \quad n \geq 0$

When $J(t)=0$ the queue size and the system size are same. When $J(t)=1,2$ or 3 the number of customers in the system is the sum of the number of customers in queue and the size of the service batch containing $a \leq x \leq b$ customers also by assuming the steady state probabilities as,

$$R_n = \lim_{t \rightarrow \infty} R_n(t); Q_n = \lim_{t \rightarrow \infty} Q_n(t); P_n = \lim_{t \rightarrow \infty} P_n(t); B_n = \lim_{t \rightarrow \infty} B_n(t);$$

Thus, the chapman kolmogrov equations satisfied by them in the steady state are given by,

$$\lambda R_0 = \mu P_0 + \mu_v Q_0 \tag{1}$$

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$$\lambda R_n = \lambda R_{n-1} + \mu P_n + \mu_v Q_n ; (1 \leq n \leq a-1) \tag{2}$$

$$(\lambda + \eta + \mu_v) Q_0 = \lambda R_{a-1} + \mu \sum_{n=a}^b Q_n \tag{3}$$

$$(\lambda + \eta + \mu_v) Q_n = \lambda Q_{n-1} + \mu_v Q_{n+b} ; (n \geq 1) \tag{4}$$

$$(\lambda + \mu + \alpha) P_0 = \mu \sum_{n=a}^b P_n + \eta Q_0 + \beta B_0 \tag{5}$$

$$(\lambda + \mu + \alpha) P_n = \lambda P_{n-1} + \mu P_{n+b} + \eta Q_n + \beta B_n ; (n \geq 1) \tag{6}$$

$$(\lambda + \beta) B_0 = \alpha P_0 \tag{7}$$

$$(\lambda + \beta) B_n = \alpha P_n + \lambda \sum_{k=1}^n B_{n-k} g_k ; (n \geq 1) \tag{8}$$

IV. STEADY STATE SOLUTION

To solve the steady state equations, the forward shifting operator E on P_n and Q_n are introduced as,

$$E(P_n) = P_{n+1}; E(Q_n) = Q_{n+1}; E(B_n) = B_{n+1}; (n \geq 0)$$

Thus the Equation (4) gives homogeneous difference equation,

$$(\lambda + \eta + \mu_v) Q_n = \lambda Q_{n-1} + \mu_v Q_{n+b}; (n \geq 1) \\ [\mu_v E^{b+1} - (\lambda + \eta + \mu_v) E + \lambda] Q_n = 0; (n \geq 0) \tag{9}$$

The characteristic equation of the difference equation is given by,

$$h(z) = [\mu_v z^{b+1} - (\lambda + \eta + \mu_v) z + \lambda] = 0 \text{ By taking } f(z) = (\lambda + \eta + \mu_v) z; g(z) = \mu_v z^{b+1} + \lambda;$$

It is found that $|g(z)| < |f(z)|$ on $|z|=1$ and hence by Rouché's theorem h(z) has only one root r_v inside the contour $|z|=1$. Further the root is real since h(0)=λ and h(1)=−η are of opposite sign and hence the solution of the

$$\text{homogeneous difference equation is given by, } Q_n = r_v^n Q_0; (n \geq 0) \tag{10}$$

Further the equation (6) can be written as,

$$(\lambda + \mu + \alpha) P_n = \lambda P_{n-1} + \mu P_{n+b} + \eta Q_n + \beta B_n; (n \geq 1) \\ [\mu E^{b+1} - (\lambda + \mu + \alpha) E + \lambda] P_n = -\eta Q_{n+1} - \beta E(B_n); (n \geq 0) \tag{11}$$

Again by applying Rouché's theorem, we find that the equation,

$$\mu z^{b+1} - (\lambda + \mu + \alpha) z + \lambda = 0 \text{ has unique root with } |r| < 1 \text{ provided by } \frac{\lambda}{b\mu} < 1$$

Thus the equation (8) can be written as,

$$(\lambda + \beta) B_n = \alpha P_n + \lambda B_{n-1}; (n \geq 1) \\ B_n = \frac{\alpha E(P_n)}{(\lambda + \beta) E - \lambda} \tag{12}$$

By Substituting (12) in (11), we get

$$[\mu E^{b+1} - (\lambda + \mu + \alpha) E + \lambda] P_n = -\eta r_v^{n+1} Q_0 - \beta \left[\frac{\alpha E^2(P_n)}{(\lambda + \beta) E - \lambda} \right] \tag{13}$$

Hence the solution of the Non homogenous difference equation of (13) is given by,

$$P_n = \left[Ar^n - \frac{\eta r_v^{n+1}}{\mu r_v^{b+1} - (\lambda + \mu + \alpha) r_v + \lambda + \frac{\alpha \beta r_v^2}{(\lambda + \beta) r_v - \lambda}} \right] Q_0 \\ \text{(i.e) } P_n = (Ar^n + Br_v^n) Q_0 \tag{14}$$

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$$\text{where } B = \frac{\eta r_v (\lambda(r_v - 1) + \beta r_v)}{[\lambda(r_v - 1) + \mu r_v (1 - r_v^b) + \alpha r_v][\lambda(r_v - 1) + \beta r_v] - \alpha \beta r_v^2}$$

The expression for R_n ($0 \leq n \leq a-1$) is obtained by adding equations (1) and (2) over 0 to n. Hence,

$$\lambda \sum_{k=0}^n R_n = \lambda \sum_{k=0}^{n-1} R_k + \mu \sum_{K=0}^n P_K + \mu_v \sum_{K=0}^n Q_k$$

$$\lambda R_n = \mu \sum_{k=0}^n P_K + \mu_v \sum_{K=0}^n Q_K$$

By Substituting for Q_n and P_n from equations (9) and (14) it is found that

$$\lambda R_n = [\mu(Ar^n + Br_v^n) + \mu_v(r_v^n)]Q_0$$

$$R_n = \left[\frac{\mu}{\lambda} \left(\frac{A(1-r^{n+1})}{(1-r)} + \frac{B(1-r_v^{n+1})}{(1-r_v)} \right) + \frac{\mu_v}{\lambda} \left(\frac{1-r_v^{n+1}}{1-r_v} \right) \right] Q_0$$

Hence the steady state queue size probabilities are expressed in terms of the unknowns A and Q_0 . Now to calculate A, we consider the equation (5),

$$(\lambda + \mu + \alpha)P_0 = \mu \sum_{n=a}^b P_n + \eta Q_0 + \beta B_0$$

By substituting for P_n from equation (14) and (12) it is found that

$$(\lambda + \mu + \alpha)(A + B)Q_0 = \mu \left(A \left[\frac{r^a - r^{b+1}}{(1-r)} \right] + B \left[\frac{r_v^a - r_v^{b+1}}{1-r_v} \right] \right) Q_0 + \eta Q_0 + \beta \left(\frac{\alpha P_1}{(\lambda + \beta)r_v - \lambda} \right)$$

which can be further simplified as,

$$A \frac{\mu(1-r^a)}{(1-r)} = \frac{\eta}{(1-r_v)} - \frac{B\mu(1-r_v^a)}{(1-r_v)} + \beta \left(\frac{\alpha P_1}{(\lambda + \beta)E - \lambda} \right)$$

$$A = \left[\frac{\mu(1-r^a)}{(1-r)} - \frac{\alpha \beta r}{\lambda(r-1) + \beta r} \right]^{-1} \left[\frac{\eta}{(1-r_v)} - \frac{B\mu(1-r_v^a)}{(1-r_v)} + \frac{\alpha \beta B r_v}{\lambda(r_v - 1) + \beta r_v} \right]$$

Hence the steady state queue size probabilities of the model are expressed in terms of Q_0 and are given by,

$$Q_n = r_v^n Q_0; \quad (n \geq 0) \tag{15}$$

$$P_n = (Ar^n + Br_v^n)Q_0; \quad (n \geq 0) \tag{16}$$

$$A = \left[\frac{\mu(1-r^a)}{(1-r)} - \frac{\alpha \beta r}{\lambda(r-1) + \beta r} \right]^{-1} \left[\frac{\eta}{(1-r_v)} - \frac{B\mu(1-r_v^a)}{(1-r_v)} + \frac{\alpha \beta B r_v}{\lambda(r_v - 1) + \beta r_v} \right]$$

$$B = \frac{\eta r_v (\lambda(r_v - 1) + \beta r_v)}{[\lambda(r_v - 1) + \mu r_v (1 - r_v^b) + \alpha r_v][\lambda(r_v - 1) + \beta r_v] - \alpha \beta r_v^2}$$

$$R_n = \left[\frac{\mu}{\lambda} \left(\frac{A(1-r^{n+1})}{(1-r)} + \frac{B(1-r_v^{n+1})}{(1-r_v)} \right) + \frac{\mu_v}{\lambda} \left(\frac{1-r_v^{n+1}}{1-r_v} \right) \right] Q_0; \quad (0 \leq n \leq a-1) \tag{17}$$

Now the expression for Q_0 can be calculated by using the normalizing condition

$$\sum_{n=0}^{\infty} Q_n + \sum_{n=0}^{\infty} P_n + \sum_{n=0}^{a-1} R_n + \sum_{n=0}^{\infty} B_n = 1$$

which yields,

$$Q_0^{-1} = F(r_v, \mu_v) + AF(r, \mu) + BF(r_v, \mu) + AS(r, \alpha) + BS(r_v, \alpha)$$

$$\text{where } F(x, y) = \frac{1}{(1-x)} \left[1 + \frac{y}{\lambda} \left(a - \frac{x(1-x^a)}{(1-x)} \right) \right]$$

$$\text{and } S(x, y) = \frac{1}{(1-x)} \left[\frac{xy}{\lambda(x-1) + \beta x} \right]$$

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Thus the value Q_0^{-1} is evaluated.

V. PERFORMANCE MEASURES

In this section, the performance measures of M/M(a,b)/1/MWV/Br are evaluated.

The expected queue length (L_q) of the model is calculated as,

$$L_q = \sum_{n=1}^{\infty} n(Q_n + P_n + B_n) + \sum_{n=1}^{a-1} nR_n \quad (18)$$

By Substituting the values of Q_n , P_n , B_n and R_n from (15) to (17), we get

$$L_q = \sum_{n=1}^{\infty} n(r_v^n + (Ar^n + Br_v^n)) + \sum_{n=1}^{a-1} n \left[\frac{\mu}{\lambda} \left(A \frac{(1-r^{n+1})}{(1-r)} + B \frac{(1-r_v^{n+1})}{(1-r_v)} \right) + \frac{\mu_v}{\lambda} \left(\frac{(1-r_v^{n+1})}{(1-r_v)} \right) \right] + \sum_{n=1}^{\infty} n \left(\frac{\alpha r A r^n}{(\lambda + \beta)r - \lambda} + \frac{\alpha r_v B r_v^n}{(\lambda + \beta)r_v - \lambda} \right)$$

Moreover L_q can be simplified as,

$$L_q = AH(r, \mu) + BH(r_v, \mu) + H(r_v, \mu_v) + AJ(r, \alpha) + BJ(r_v, \alpha) \quad (19)$$

$$\text{where } H(x, y) = \frac{x}{(1-x)^2} + \frac{y}{\lambda(1-x)} \left\{ \frac{a(a-1)}{2} + \frac{ax^{a+1}(1-x) - x^2(1-x^a)}{(1-x)^2} \right\}$$

$$J(x, y) = \frac{x}{(1-x)^2} \left[\frac{xy}{\lambda(x-1) + \beta x} \right]$$

Let P_v , P_{busy} , P_{idle} and P_{br} respectively denote the probability that the server is busy in vacation state, regular busy state idle state and in break down busy state respectively.

Thus, P_v , P_{busy} , P_{idle} and P_{br} are given by,

$$(i) P_v = \sum_{n=0}^{\infty} Q_n = \frac{Q_0}{(1-r_v)}$$

$$(ii) P_{\text{busy}} = \sum_{n=0}^{\infty} P_n = \sum_{n=0}^{\infty} (Ar^n + Br_v^n) Q_0 = \left[\frac{A}{(1-r)} + \frac{B}{(1-r_v)} \right] Q_0$$

$$(iii) P_{\text{idle}} = \sum_{n=0}^{a-1} R_n$$

$$(iv) P_{\text{br}} = \sum_{n=0}^{\infty} B_n = \frac{A\alpha r}{(\lambda + \beta)r - \lambda} \left[\frac{r}{(1-r)^2} \right] + \frac{B\alpha r_v}{(\lambda + \beta)r_v - \lambda} \left[\frac{r_v}{(1-r_v)^2} \right]$$

VI. CONCLUSION

In this paper we deal with M/M(a,b)/1/Br queueing model under Multiple Working Vacations For this Model the total probability generating is obtained in a closed form by using Chapman-Kolmogorov balance equations. The various Performance measures are analysed.

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