

Relativistic Magneto Hydrodynamic Shocks in Gyro Tropic Plasma

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ABSTRACT: We investigate the jump conditions of a relativistic magneto hydrodynamic shock wave in a gyro tropic plasma and analyze these relations for a parallel and a perpendicular shock. The main purpose of this article is to study the relativistic shock theory for a plasma with gyro tropic but anisotropic pressures.

KEYWORDS: Magneto Hydrodynamic, Cartesian coordinate system

I. INTRODUCTION

Shocks are produced by an abrupt, nearly discontinuous change in characteristics of medium. In several space plasmas, a common particle density is nearly a few particle per c.c. So, in such thin plasmas, the formation for collision less shocks is more frequent than the generation of conventional hydrodynamic shocks due to elastic collision frequency which is less than plasma and gyro frequency. Such, shocks were studied theoretically and experimentally by Colburn and Sonett (1966), Kunkel and Brown (1967). The Earth's bow shocks and interplanetary shock were studied by Treumann and Jaroschek (2011) and showed that velocity-anisotropic distributions (VADs) of plasma particles play an important role in the shock formation process. Such velocity-anisotropic distributions form in the upstream medium by shock reflection. The important role of shock-reflected charged particles in the shock formation process also observed from numerical particle-in-cell (PIC) simulations of relativistic shock acceleration as suggested by Spitkovsky (2008), and Sironi and Spitkovsky (2009). Double et al (2004) studied relativistic magneto hydrodynamic (MHD) fast shocks for the case of anisotropic pressure in the downstream region by using an approximate form for the adiabatic index.

The main purpose of this article is to study the relativistic shock theory for a plasma with gyro tropic but anisotropic pressures. The relativistic shock theory plays an important role in understanding and interpreting the results of the PIC simulations. It is demonstrated that the use of instability criteria combined with the solutions of the jump conditions resulted in thresholds for the downstream anisotropy for stable relativistic shock structures to exist.

II. MAGNETO HYDRODYNAMIC JUMP CONDITIONS AND SHOCK EQUATIONS

Lichnerowicz (1967) presented relativistic magneto hydrodynamic equations for isotropic pressure. Further Cissoko (1975) studied for a relativistic gyro tropic plasma. Let u^μ be the four-velocity of the fluid and ρ be the density such that

$$u^\mu = \gamma(1, \underline{\beta}) \quad (1)$$

where $\gamma = (1 - \beta^2)^{-1/2}$ and $\underline{\beta} = \underline{u}/c$ with c as the speed of light.

Then one may write the mass four-flow of a plasma as

$$M^\mu = \rho u^\mu, \quad (2)$$

with mass continuity equation

$$M^\mu{}_{;\mu} = 0. \quad (3)$$

For the case of perfect magneto hydrodynamics the dual of the electromagnetic tensor $*F_{\mu\nu}$ assumes the form

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$${}^*F^{\mu\nu} = h^\mu u^\nu - h^\nu u^\mu, \tag{4}$$

where

$$h_\nu = u^\mu {}^*F_{\mu\nu}, \tag{5}$$

be the magnetic induction four vector, such that

$$h^\mu = \left(-\gamma\beta \cdot \underline{B} - \underline{B} - \frac{\gamma-1}{\beta^2} \beta (\beta \cdot \underline{B}) \right). \tag{6}$$

Hence, the conservation of magnetic flux reads

$${}^*F_{\mu\nu;\nu} = 0. \tag{7}$$

Chew et al (1956) presented the pressure tensor for an ideal collision less no relativistic plasma in strong parallel magnetic field for gyro tropic form. Tsikarishvile et al (1994), generalized this model for the relativistic case and obtained four pressure tensor as

$$\pi^{\mu\nu} = (P_\parallel - P_\perp) \frac{h^\mu h^\nu}{h^2} - P_\perp (\eta^{\mu\nu} - u^\mu u^\nu), \tag{8}$$

where $\eta^{\mu\nu} = \text{dia} (1, -1, -1, -1)$ as the Murkowski metric for flat space-time, $h^2 = -h^\mu h_\mu$, $P_\parallel (P_\perp)$

denotes pressure parallel (perpendicular) to the magnetic field. Hence the total energy momentum tensor $T^{\mu\nu}$ for a relativistic gyro tropic plasma assumes the form

$$\begin{aligned} T^{\mu\nu} &= T_f^{\mu\nu} + T_{em}^{\mu\nu} \\ &= e u^\mu u^\nu + \pi^{\mu\nu} + \frac{h^2}{4\pi} (u^\mu u^\nu - \frac{1}{2} \eta^{\mu\nu}) - \frac{h^\mu h^\nu}{4\pi}, \end{aligned} \tag{9}$$

where e as the total energy density. Therefore, we get

$$T^{\mu\nu}_{;\nu} = 0. \tag{10}$$

Double et al (2004) obtained the equation state as

$$e = \frac{2P_\perp + P_\parallel}{3(K-1)} + \rho c^2, \tag{11}$$

where K denotes the adiabatic index for perfect gyro tropic plasma.

Now, in general all state variables of a plasma change across a shock front. Hence, the jump conditions for a relativistic magneto hydrodynamics shock in gyro tropic plasma read

$$[M^\nu] n_{\nu,s} = 0, \tag{12}$$

$$[T^{\mu\nu}] n_{\nu,s} = 0, \tag{13}$$

$$[{}^*F^{\mu\nu}] n_{\nu,s} = 0, \tag{14}$$

where $[Q] = Q_2 - Q_1$ being the jump the quantity Q across the shock, and $n_{\nu,s} = (n_0, \underline{n})$ be the shock normal. The subscripts 1 and 2 denote the upstream and downstream states respectively and subscript s for the quantity is measured in the shock frame.

Let us consider a Cartesian coordinate system such that z-axis is pointing toward the downstream direction. So, upstream magnetic field has components in x and z directions. Let us consider a plane shock is infinite expanded in the x-y plane and in the steady state. Further, let us consider a perfect collision less upstream plasma with infinite conductivity. Let us also assume that the heat flow be unimportant.

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Erkaev et al (2000) and Liu et al (2007) presented that for no relativistic shock the downstream anisotropy $\alpha_2 = P_{\perp 2}/P_{\parallel 2}$ be a fixed quantity and K_2 as the downstream adiabatic index is not known for an isotropic plasma. In view of Double et al (2004) we use an approximate adiabatic index.

III. THE PARALLEL SHOCK SOLUTION

In view of the works of Hau and Wang ((2007), Parker (1958), Hasegawa (1975), Chandrasekhar et al (1958), Siewert and Fahr (2008) and Chew et al (1956) and by using the constraints from the firehose and mirror instability, the jump conditions for a parallel shock assume the form

$$[\beta\gamma\rho] = 0, \tag{15}$$

$$[\beta\gamma^2(e + P_{\parallel})] = 0, \tag{16}$$

$$[\beta^2\gamma^2(e + P_{\parallel}) + P_{\parallel}] = 0, \tag{17}$$

$$[h] = 0. \tag{18}$$

Let us now define dimensionless parameters,

$$r = \frac{\beta_1}{\beta_2}, M_{A1} = \left(\frac{4\pi\rho_1\beta_1^2c^2}{h_1^2}\right)^{1/2}, M_{S1} = \left(\frac{\rho_1\beta_1^2c^2}{K_1P_1}\right)^{1/2}, \tag{19}$$

where r be the compression ratio, M_{A1} as the Alfvén Mach number, and M_{S1} be the sonic Mach number. In view of eqs. (15)-(19), one may obtain analytic solution for the parallel downstream plasma beta as,

$$\xi_{\parallel 2} = \frac{3\xi_1}{2\alpha_1 + 1} + \gamma_1^2\beta_1^2\left(1 - \frac{1}{r}\right)\left[\left(\frac{1}{K_1 - 1} + \frac{3}{2\alpha_1 + 1}\right)\xi_1 + \frac{2M_{A1}^2}{\beta_1^2}\right], \tag{20}$$

where $\xi_1 = (\xi_{\parallel 1} + 2\xi_{\perp 1})/3$. The parallel and perpendicular plasma beta are given,

$$\xi_{\parallel} = \frac{8\pi P_{\parallel}}{h^2}, \xi_{\perp} = \frac{8\pi P_{\perp}}{h^2}.$$

The downstream anisotropy reads

$$\alpha_2 = \left[\frac{\gamma_1^2}{r}(r^2 - \beta_1^2)\left\{\left(\frac{1}{K_1 - 1} + \frac{3}{2\alpha_1 + 1}\right)\xi_1 + \frac{2M_{A1}^2}{\beta_1^2}\right\} - 2\gamma_1\sqrt{r^2 - \beta_1^2}\frac{M_{A1}^2}{\beta_1^2} - \frac{(3K_2 - 2)}{3(K_2 - 1)}\xi_{\parallel 2}\right]\frac{3(K_2 - 1)}{2\xi_{\parallel 2}} \tag{21}$$

where

$$K_2 = \frac{1}{3}\left(4 + \frac{\sqrt{r^2 - \beta_1^2}}{\gamma_1(r - \beta_1^2)}\right) \tag{22}$$

For the case of parallel shock, the downstream anisotropy is independent of magnetic field strength. Now, by eliminating the Alfvén number, we get

$$\alpha_2 = \left[\frac{\gamma_1^2}{r}(r^2 - \beta_1^2)\left\{\left(\frac{1}{K_1 - 1} + \frac{3}{2\alpha_1 + 1}\right) + \frac{K_1}{\beta_1^2}M_{S1}^2\right\} - 2\gamma_1\sqrt{r^2 - \beta_1^2}\frac{M_{S1}^2}{\beta_1^2} - \frac{(3K_2 - 2)}{3(K_2 - 1)}\xi_{\parallel 2}\right]\frac{3(K_2 - 1)}{2\xi_{\parallel 2}}$$

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$$\begin{aligned}
 & -\frac{\gamma_1}{\beta_1^2} \sqrt{r^2 - \beta_1^2} K_1 M_{s1}^2 - \frac{(3K_2 - 2)}{3(K_2 - 1)} \left\{ \frac{3}{2\alpha_1 + 1} + \gamma_1^2 \beta_1^2 \right. \\
 & \times \left(1 - \frac{1}{r} \right) \left(\left(\frac{1}{K_1 - 1} + \frac{3}{2\alpha_1 + 1} \right) + \frac{K_1}{\beta_1^2} M_{s1}^2 \right) \left. \right\} \\
 & \times 3(K_2 - 1) \left\{ \frac{6}{2\alpha_1 + 1} + 2\gamma_1^2 \beta_1^2 \left(1 - \frac{1}{r} \right) \right\} \\
 & \times \left(\left(\frac{1}{K_1 - 1} + \frac{3}{2\alpha_1 + 1} \right) + \frac{K_1}{\beta_1^2} M_{s1}^2 \right)^{-1} \tag{23}
 \end{aligned}$$

For the nonrelativistic limiting cases the downstream parallel plasma beta, one obtains

$$\xi_{\parallel 2} \square \frac{3\xi_1}{2\alpha_1 + 1} + 2M_{A1}^2 \left(1 - \frac{1}{r} \right). \tag{24}$$

We get the downstream anisotropy parameter

$$\alpha_2 \square \frac{1}{\xi_{\parallel 2}} \left[\frac{3\xi_1 r}{2\alpha_1 + 1} \left(\alpha_1 + \frac{3}{2} - \frac{3}{2r} \right) + M_{A1}^2 \left(r + \frac{2}{r} - 3 \right) \right] \tag{25}$$

Again for high sonic Mach number i.e. $M_{A1}^2 \square \xi_1$, one obtains

$$\alpha_2 \square \frac{r + \frac{2}{r} - 3}{2 \left(1 - \frac{1}{r} \right)}. \tag{26}$$

By solving for r, we get

$$r \square 2\alpha_2 + 2. \tag{27}$$

Now for ultra relativistic limiting cases $\gamma_1 \square 1, \beta_1 \square 1, K_1 = 5/3$ and $K_2 = 4/3$, so downstream parallel plasma beta assumes the form

$$\xi_{\parallel 2} \square \gamma_1^2 \left(1 - \frac{1}{r} \right) \left[\left(\frac{3}{2} + \frac{3}{2\alpha_1 + 1} \right) \xi_1 + 2M_{A1}^2 \right]. \tag{28}$$

and the downstream anisotropy parameter

$$\alpha_2 \square \left[\frac{\gamma_1^2}{r} (r^2 - 1) \left\{ \left(\frac{3}{2} + \frac{3}{2\alpha_1 + 1} \right) \xi_1 + 2M_{A1}^2 \right\} - 2\xi_{\parallel 2} \right] \frac{1}{2\xi_{\parallel 2}}. \tag{29}$$

Again by solving this relation for r, we get

$$r \square 2\alpha_2 + 1. \tag{30}$$

IV. PERPENDICULAR SHOCK SOLUTION

In this case the shock normal be perpendicular to the magnetic field and therefore the normal magnetic field component vanishes. Now jump conditions reduce to

$$[\gamma\beta\rho] = 0, \tag{31}$$

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$$\left[\gamma^2 \beta \left(e + P_{\perp} + \frac{h^2}{4\pi} \right) \right] = 0, \quad (32)$$

$$\left[\gamma^2 \beta^2 \left(e + P_{\perp} + \frac{h^2}{4\pi} \right) + P_{\perp} + \frac{\gamma^2 h^2}{8\pi} \right] = 0, \quad (33)$$

$$[\gamma \beta h] = 0. \quad (34)$$

By solving above set of equations, we get the perpendicular downstream plasma beta

$$\begin{aligned} \xi_{\perp 2} = & \frac{3\alpha_1}{2\alpha_1 + 1} \frac{\xi_1}{\gamma_1^2 (r^2 - \beta_1^2)} - \frac{r^2 - 1}{r^2 - \beta_1^2} + \frac{r - 1}{r(r^2 - \beta_1^2)} \\ & \times \left[\left(\frac{1}{K_1 - 1} + \frac{3\alpha_1}{2\alpha_1 + 1} \right) \xi_1 \beta_1^2 + 2 + \frac{2M_{A1}^2}{\beta_1^2} \right], \end{aligned} \quad (35)$$

and the down stream anisotropy

$$\begin{aligned} \alpha_2 = & \frac{\xi_{\perp 2}}{3(K_2 - 1)} \left[\frac{1}{r} \left\{ \left(\frac{1}{K_1 - 1} + \frac{3\alpha_1}{2\alpha_1 + 1} \right) \xi_{\perp} + 2 + \frac{2M_{A1}^2}{\beta_1^2} \right\} \right. \\ & \left. - \frac{2M_{A1}^2}{\gamma_1 \beta_1^2 \sqrt{r^2 - \beta_1^2}} - 2 - \frac{3K_2 - 1}{3(K_2 - 1)} \xi_{\perp 2} \right]^{-1}. \end{aligned} \quad (36)$$

The non-relativistic limiting cases for the downstream perpendicular plasma beta

$$\xi_{\perp 2} \square \frac{2\alpha_1 \xi_1}{(2\alpha_1 + 1)r^2} + \frac{2M_{A1}^2}{r^2} \left(1 - \frac{1}{r} \right) - 1 + \frac{1}{r^2}, \quad (37)$$

and the downstream anisotropy parameter reads

$$\alpha_2 \square \frac{\frac{3\alpha_1 \xi_1}{2\alpha_1 + 1} + 2M_{A1}^2 \left(1 - \frac{1}{r} \right) + 1 - r^2}{\frac{3\alpha_1 \xi_1}{2\alpha_1 + 1} \left(4r - 4 + \frac{r}{\alpha_1} \right) + 2M_{A1}^2 \left(r + \frac{3}{r} - 4 \right) + 4(r - 1)}. \quad (38)$$

Again for $M_{A1}^2 \square \xi_{\perp}$, one obtains

$$\alpha_2 \square \frac{1 - \frac{1}{r}}{r + \frac{3}{r} - 4}. \quad (39)$$

By solving eq. (39), we get

$$r \square \frac{1}{\alpha_2} + 2 \quad (40)$$

Again for ultra-relativistic limiting cases the perpendicular downstream plasma beta read

$$\xi_{\perp 2} \square \frac{1}{r(r+1)} \left[\left(\frac{3}{2} + \frac{3\alpha_1}{2\alpha_1 + 1} \right) \xi_1 + 2M_{A1}^2 - 2 \right] - 1. \quad (41)$$

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and anisotropy parameter reads

$$\alpha_2 \square \xi_{\perp 2} \left[\frac{1}{r} \left\{ \left(\frac{3}{2} + \frac{3\alpha_1}{2\alpha_1 + 1} \right) \xi_1 - 2 + 2M_{A1}^2 \right\} - 1 - 3\xi_{\perp 2} \right]^{-1} \quad (42)$$

Again eq. (42) in view of eq. (42) assumes the form

$$\alpha_2 \square \frac{\left[\left(\frac{3}{2} + \frac{3\alpha_1}{2\alpha_1 + 1} \right) \xi_1 + 2M_{A1}^2 - 2 \right] - r(r+1)}{(r-2) \left[\left(\frac{3}{2} + \frac{3\alpha_1}{2\alpha_1 + 1} \right) \xi_1 + 2M_{A1}^2 - 2 \right] + r(r+1)}. \quad (43)$$

After solving eq. (43) for r, one obtains

$$r \square \frac{\left[(\alpha_2 + x\alpha_2 + 1)^2 + 4x(\alpha_2 + 1)(2\alpha_2 + 1) \right]^{\frac{1}{2}} - \alpha_2 - x\alpha_2 - 1}{2(\alpha_2 + 1)}, \quad (44)$$

where

$$x = \left[\left(\frac{3}{2} + \frac{3\alpha_1}{2\alpha_1 + 1} \right) \xi_1 + 2M_{A1}^2 - 2 \right]. \quad (45)$$

V. CONCLUSION

We have studied and solved analytically jump conditions of a relativistic magneto hydrodynamic shock wave in gyro tropic plasma for a parallel and perpendicular shock. The jump conditions are obtained by using constraints of the fire hose and mirror stability as given by Double et al (2004) along with $\alpha_2 = P_{\perp 2} / P_{\parallel 2}$ as an equation of state, which operates in anisotropic, gyro tropic plasma. All post shock quantities are given in terms of the compression ratio and dimensionless upstream quantities. The results of this investigation is directly applicable to already existing models which deal with test particle acceleration in gamma-ray bursts as well as for shock fronts which exist in the jets of active galactic nuclei.

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