

Mathematical Modeling on the Two-Phase Hepatic Blood Flow through Arterioles with Special Reference of Hepatitis C

U.S.Mishra¹, V.Upadhyay², R.Khare³, P.N.Pandey⁴

Research Scholar, Department of Physical Science, M.G.C.Gramodaya, Vishwavidyalay, Chitrakoot, Satna, M.P, India¹

Associate Professor, Department of Physical Science, M.G.C.Gramodaya, Vishwavidyalay, Chitrakoot, Satna, M.P,
India²

Professor, Department of Mathematics & Statistics SHIATS Allahbad, U.P, India³

Professor & Head, Department of Mathematics, University of Allahabad, Allahabad, U.P, India⁴

ABSTRACT: In this paper we observe the hepatic blood flow in arterioles keeping in the nature of hepatic circulatory system in human body. We know that a human being have one of red blood cells and other are plasma. P.N.Pandey and V.Upadhyay have considered that blood flow in two phase . They have also applied the Harschel Bulkley Non- Newtonian Model in bio fluid observation. We have collected a clinical data related to hepatitis C for hematocrit versus blood pressure clinical observation and graphical presentation both value is very close. All mathematical presentation is tensorial form using analytical as well as numerical technique. In this hepatic dieses of hepatitis C the role of hematocrit is explicit in the determination of blood pressure.

KEYWORDS: Structure of liver, Circulatory system, Harschel Bulkley Non-Newtonian Model, Hepatitis C, Hematocrit, Blood flow.

I. INTRODUCTION (DESCRIPTION OF BIO-PHYSICAL PROBLEM)

The liver is a large, meaty organ of the body that sits on the right side of the belly.^[1] The liver is reddish-brown in color and feels rubbery to the touch.^[2] It weight about 3 pounds. Normally you can't feel the liver, because it's protected by the rib cage.^[3] The liver has two large sections, called the right and the left lobes. The gallbladder sits under the liver, along with parts of the pancreas and intestines.^[4] The liver and these organs work together to digest, absorb and process food.^[5]

The liver's main job is to filter the blood coming from the digestive tract, before passing it to the rest of the body. The liver also detoxifies chemicals and metabolizes drugs.^[6] As it does so, the liver secretes bile that ends up back in the intestines. The liver also makes proteins important for blood clotting and other functions.^[7] The liver is the body's chemical factory, converting molecules into simpler or more complex forms as the body requires. For example, nutrients are modified in the liver, then stored there or distributed throughout the body toxins such as alcohol are broken down into less harmful substances. The liver also produces the digestive juice bile.^[8]

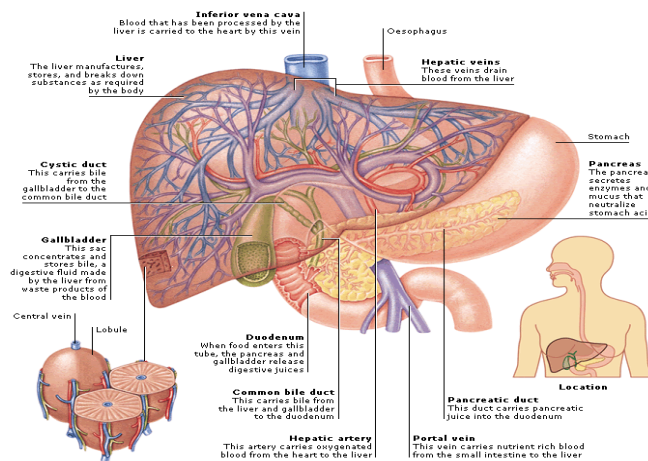


Fig.1 Structure of Liver

The liver regulates most chemical levels in the blood and excretes a product called bile. Bile helps to break down fats, preparing them for further digestion and absorption. All of the blood leaving the stomach and intestines passes through the liver. The liver processes this blood and breaks down, balances, and creates nutrients for the body to use.^[9] It also metabolized drugs in the blood into forms that are easier for the body to use. Many vital functions have been identified with the liver. Some of the more well-known functions include the following.^[10]

Production of bile, which helps carry away waste and break down fats in the small intestine during digestion. Production of certain proteins for blood plasma. Production of cholesterol and special proteins to help carry fats through the body. Store and release glucose as needed. Processing of hemoglobin for use of its iron content (the liver stores iron). Conversion of harmful ammonia to urea (urea is one of the end products of protein metabolism that is excreted in the urine). Clearing the blood of drugs and other harmful substances. Regulating blood clotting. Resisting infections by producing immune factors and removing bacteria from the blood stream.^[11] Some liver diseases are like Hepatitis A, B and C, cirrhosis, liver cancer, liver failure etc.^[12]



Fig. 2 Hepatitis C Virus

Hepatitis C is a liver disease caused by virus that infects the liver. In time, it can lead to permanent liver damage as well as cirrhosis, liver cancer, and liver failure.^[13] Many people do not know that they have hepatitis C have it for a short time and then get better. This is called acute hepatitis C. But most people who are infected with the virus go on to develop long term, or chronic, hepatitis C. Although hepatitis C can be very serious, most of people can manage the disease and lead active, full lives. It is spread by contact with and infected person's blood.^[14] This work will focus on two phase hepatic blood flow in arterioles with special reference in hepatitis C. In two phase hepatic blood flow study

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of measuring the blood pressure if hemoglobin known^[15] The percentage of volume covered by blood cells in the whole blood is called hematocrite. Hematocrite is three times of hemoglobin concentration.^[16]

REAL MODEL

We have to select a frame of reference for mathematical modeling of the state of moving blood keeping in view the difficulty and generality of the problem of blood flow, we select generalized three dimensional orthogonal curvilinear co-ordinate system. Briefly described as E called as Euclidean space. According to Mishra biophysical laws thus expressed fully hold good in any coordinate system which is the compulsion for the truthfulness of the laws (1990). According to Sherman I.W and Sherman V.G blood is mixed fluid. Mainly there are two phase in blood .The first phase is plasma, while the other phase is that of the blood cells are in closed with a semi permeable membrane whose density is greater than that of the plasma .These blood cells are uniformly distributed in plasma. Thus, blood can be considered as a homogenous mixture of two phase(1989).

(A) EQUATION OF CONTINUITY FOR TWO PHASE BLOOD FLOW

The blood flow is affected by the presence of blood cells. This effect is directly proportional to the volume occupied by blood cells. Let the volume portion covered by blood cells in unit volume be X , this X is replaced by $H/100$, where H is the hematocrit the volume percentage of blood cells.^[17] Then the volume portion covered by the plasma will be $(1 - X)$. The mass ratio of blood cells to plasma is given below:

$$r = \frac{X \rho_c}{(1 - X) \rho_p} \quad \dots (2.1)$$

where ρ_c and ρ_p are densities of blood cells and blood plasma respectively. Usually this mass ratio is not constant , even then it may be supposed to constant in present context[Singh and Upadhyay (1986)]^[18]

The both phase of blood ,i.e. blood cells and plasma move with the common velocity. Campbell and Pitcher has presented a model for two phase of blood separately (1958). The equation of continuity for two phases according to the principle of conservation of mass as follow [Kapur and Gupta (1963)].^[19]

$$\frac{\partial (X \rho_c)}{\partial t} + (X \rho_c v^i), i = 0 \quad \dots (2.2)$$

$$\text{and } \frac{\partial (1 - X) \rho_p}{\partial t} + ((1 - X) \rho_p v^i), i = 0 \quad \dots (2.3)$$

where, v is the common velocity of two phase blood cells and plasma . If we define the uniform density of the

$$\text{blood } \rho_m \text{ as follow } \frac{1 + r}{\rho_m} = \frac{r}{\rho_c} + \frac{1}{\rho_p} \quad \dots (2.4)$$

From equation (2.3) and (2.4) we can written as,

$$\frac{\partial \rho_m}{\partial t} + (\rho_m v^i), i = 0 \quad \dots (2.5)$$

(B) EQUATION OF MOTION FOR TWO PHASE BLOOD FLOW

According to [Ruch and Patten (1973)] the hydro dynamical pressure p between the two phases of blood can be supposed to be uniform because the both phases i.e. blood cells and plasma are always in equilibrium state in blood (1973).^[20] Taking viscosity coefficient of blood cells to be η_c and applying the principle of conservation of momentum, we get the equation of motion for the phase of blood cells as follows:

$$X \rho_c \frac{\partial v^j}{\partial t} + (X \rho_c v^j) v^i_{,j} = -X p_{,j} g^{ij} + X \eta_c (g^{jk} v^i_{,k})_{,j} \quad \dots (2.6)$$

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The equation of motion for plasma will be as follows:

$$(1-X)\rho_p \frac{\partial v^i}{\partial t} + \{(1-X)\rho_p v^i\}_{,j} v^i = -(1-X)p_{,j} g^{ij} + (1-X)\eta_c (g^{jk} v^i_{,k})_{,j} \quad \dots (2.7)$$

Now adding equation (2.6) and (2.7) and using relation (2.4), the equation of motion for blood flow with the both phases will be as follows:

$$\rho_m \frac{\partial v^i}{\partial t} + (\rho_m v^j)_{,j} v^i = -p_{,j} + \eta_m (g^{jk} v^i_{,k})_{,j} \quad (2.8)$$

where $\eta_m = X\eta_c + (1-X)\eta_p$ is the viscosity coefficient of blood as a mixture of two phases.

III. MATHEMATICAL MODELING

As the velocity of blood flow decreases, the viscosity of blood increases. The velocity of blood decreases successively. The Herschel Bulkley law hold good on the two phase blood flow through veins arterioles, veinules and whose constitutive equation is as follows:

$$T' = \eta_m e^n + T_p (T' \geq T_p) \text{ and } e = 0(T' < T_p) \text{ where, } T_p \text{ is the yield stress.}$$

When strain $e = 0(T' < T_p)$ a core region is formed which is flows just like a plug. Let the radius of the plug be r_p . The stress acting on the surface of plug will be T_p . Equating the forces acting on the plug, we get,

$$P\pi r_p^2 = T_p 2\pi r_p \Rightarrow r_p = 2 \frac{T_p}{P} \quad \dots (2.9)$$

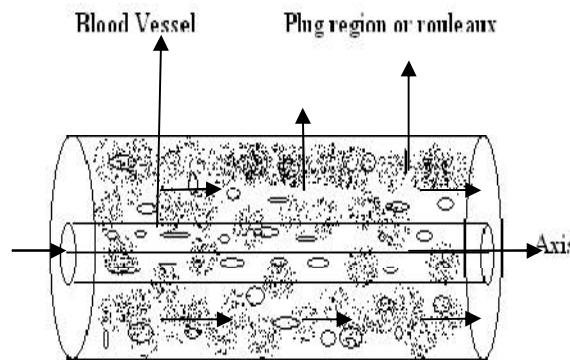


Fig. 3 Geometical structure of Herschel Bulkley blood flow Model

The Constitutive equation for test part of the blood vessel is given by where, T_e = effective stress, whose generalized

$$T' = \eta_m e^n + T_p \text{ or } T' - T_p = \eta_m e^n = T_e \text{ Form are given below } T^{ij} = -Pg^{ij} + T_e^{ij} \text{ w } T_e^{ij} = \eta_m (e^{ij})^n \text{ while } e^{ij} = g^{jk} v^i_{,k}$$

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Now we describe the basic equations for Herschel Bulkley blood flow as follows:

(A) EQUATION OF CONTINUITY

$$\frac{1}{\sqrt{g} \sqrt{(gV^i)_{,i}}} = 0 \quad \dots (3.0)$$

(B) EQUATION OF MOTION IS

$$\rho_m \frac{\partial v^i}{\partial t} + \rho_m V^j V_{,j}^i = -T_{e,j}^{ij} \dots (3.1)$$

where all the symbols have their usual meanings.

IV. ANALYSIS (SOLUTION)

The blood vessels are cylindrical; the above governing equations have to be transformed into cylindrical co-ordinates.

As we know, $X^1 = r, X^2 = \theta, X^3 = z$, Matrix of metric tensor in cylindrical co-ordinates is $[g_{ij}]$ and matrix of

conjugate metric tensor is $[g^{ij}]$ whereas the Christoffel's symbols of 2nd kind are as follows:

$$\left\{ \begin{matrix} 1 \\ 2 \end{matrix} \right\} = -r, \left\{ \begin{matrix} 1 \\ 2 \end{matrix} \right\} = \left\{ \begin{matrix} 1 \\ 2 \end{matrix} \right\} = \frac{1}{r} \text{ Remaining others are zero.}$$

The governing tensorial equations can be transformed into cylindrical forms which are as follows:

The equation of continuity are given below $\frac{\partial v}{\partial z} = 0$

The equation of motion- r-component $-\frac{\partial p}{\partial z} = 0, \Theta$ -component = 0 and

Z-component $0 = -\frac{\partial p}{\partial z} + \frac{\eta_m}{r} \left[r \left(\frac{\partial v_z}{\partial r} \right)^n \right] \dots (3.2)$

Here, this fact has been taken in view that the blood flow in axially symmetric in arteries concerned, i.e. $v_\theta = 0$ and v_r, v_z and p do not depend upon θ .

We get $v_z = v(r)$ and $p = p(z)$ and $0 = -\frac{dp}{dz} + \frac{\eta_m}{r} \left[r \left(\frac{dv}{dz} \right)^n \right] \dots (3.3)$

And the pressure gradient $-\frac{dp}{dz} = p$

$r \left(\frac{dv}{dz} \right)^n = -\frac{pr^2}{2\eta_m} + A$, We apply boundary condition: at $r = 0, V = V_0$ then $A = 0$

Or $-\frac{dv}{dr} = -\left(\frac{p_r}{2\eta_m} \right)^{\frac{1}{n}}$ Replace r from $r - r_p$

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$$-\frac{dv}{dr} = \left(\frac{\frac{1}{2}Pr - \frac{1}{2}Pr_p}{\eta_m} \right)^{\frac{1}{n}} \Rightarrow \frac{dv}{dr} = - \left(\frac{P}{2\eta_m} \right)^{\frac{1}{n}} (r - r_p)^{\frac{1}{n}} \quad \dots (3.4)$$

Integrating above equation (13) under the no slip boundary condition: $v = 0$ at $r = R$ we get:

$$V = \left(\frac{P}{2\eta_m} \right)^{\frac{1}{n}} \frac{n}{n+1} \left[(R - r_p)^{\frac{1}{n}+1} - (r - r_p)^{\frac{1}{n}+1} \right] \quad \dots (3.5)$$

This is the formula for velocity of blood flow in arterioles, veinules and veins. Putting $r = r_p$ to get the velocity V_p of plug flow are given by,

$$V_p = \frac{n}{n+1} \left(\frac{P}{2\eta_m} \right)^{\frac{1}{n}} (R - r_p)^{\frac{1}{n}+1} \quad \dots (3.6)$$

Where the value of r_p is taken from equation (2.7), we have

V.RESULT&DISCUSSION (BIO-PHYSICAL INTERPRETATION)

Observations: Hematocrit Vs blood pressure is taken from **Jeevan Jyoti hospital** Allahabad

Patient Name: Mr. Amit Shukla Age-26Years

Diagnosis:Dr Sunil Singla

S.No	Date	HB(g)	BP	Hematocrit	B.P(Pascal)
1	26.08.13	8.9	132/80	26.7	17598.5/10665.8
2	28.08.13	6.2	110/72	18.6	14665.2/9599.04
3	30.08.13	7.1	120/65	21.3	15998.4/8665.8
4	02.09.13	7.3	115/60	21.9	15331.8/7999.2

According to Berkow,Robert, The hematocrit (expressed as percentage points) is normally about three times the hemoglobin concentration(report as per deciliter).^[21]

The flux of two phased blood flow in arterioles, veinules and veins is given below:

$$Q = \int_0^{r_p} 2\pi r V_p dr + \int_{r_p}^R 2\pi r V dr$$

$$= \int_0^{r_p} 2\pi r \frac{n}{n+1} \left(\frac{P}{2\eta_m} \right)^{\frac{1}{n}} (R - r_p)^{\frac{1}{n}+1} dr +$$

$$\int_0^{r_p} 2\pi r \frac{n}{n+1} \left(\frac{P}{2\eta_m} \right)^{\frac{1}{n}} \left[(R - r_p)^{\frac{1}{n}+1} - (r - r_p)^{\frac{1}{n}+1} \right] dr \quad \text{Using (3.4) and (3.6), we get}$$

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$$\begin{aligned}
 &= \int_0^{r_p} 2\pi r \frac{n}{n+1} \left(\frac{p}{2\eta_m} \right)^{\frac{1}{n}} (R-r_p)^{\frac{1}{n}+1} dr + \\
 &\int_0^{r_p} 2\pi r \frac{n}{n+1} \left(\frac{p}{2\eta_m} \right)^{\frac{1}{n}} \left[(R-r_p)^{\frac{1}{n}+1} - (r-r_p)^{\frac{1}{n}+1} \right] dr \\
 Q &= \frac{2\pi n}{(n+1)} \left(\frac{p}{2\eta_m} \right)^{\frac{1}{n}} (R-r_p)^{\frac{1}{n}+1} \left[\frac{r^2}{2} \right]_0^{r_p} + \frac{2\pi n}{(n+1)} \left(\frac{p}{2\eta_m} \right)^{\frac{1}{n}} \left[\frac{r^2}{2} (R-r_p)^{\frac{1}{n}+1} - \frac{r(r-r_p)^{\frac{1}{n}}}{\frac{1}{n}+2} + \frac{(r-r_p)^{\frac{1}{n}+3}}{\left(\frac{1}{n}+2\right)\left(\frac{1}{n}+3\right)} \right]_{r_p}^R \dots(3.7)
 \end{aligned}$$

$Q=1000\text{m.l/min} = 0.01666\text{lit/sec}$, $R=1, r_p = 1/3$ [22]

According to Gustofson, Daniel R(1980). $\eta_p = 0.0015$ (Pascal-Sec) [23]

$\eta_m = 0.035$ (Pascal-Sec)

$H = 26.7$, $P = 17598.5$ Pascal

By using relation

$\eta_m = \eta_{cX} + \eta_p (1-X)$ Where $X=H/100$

$\eta_c = 0.12692$, and using same above relation, we get

$\eta_m = 0.0012696H + 0.0015$

Substituting the value of R, r_p in equation (3.7), we get

$$Q = \frac{2\pi}{27} \left(\frac{P}{3\eta_m} \right)^{\frac{1}{n}} \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right]$$

$$\frac{Q \times 27}{2\pi} = \left(\frac{P}{3\eta_m} \right)^{\frac{1}{n}} \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right]$$

$$\frac{27 \times 0.01666}{6.28} = \left(\frac{17598.5}{3 \times 0.035} \right)^{\frac{1}{n}} \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right]$$

$$0.071627 = (167604.762)^{\frac{1}{n}} \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right]$$

Solved by numerical method, we get $n = -2.7746$

$$P = 3\eta_m \left(\frac{Q \times 27}{2\pi} \right)^{\frac{1}{n}} \times \left(\frac{6n^3 + 11n^2 + 6n + 1}{26n^3 + 33n^2 + 9n} \right)^n$$

Substituting the value of Q and n we get

$P = (64.70H + 76.44)$

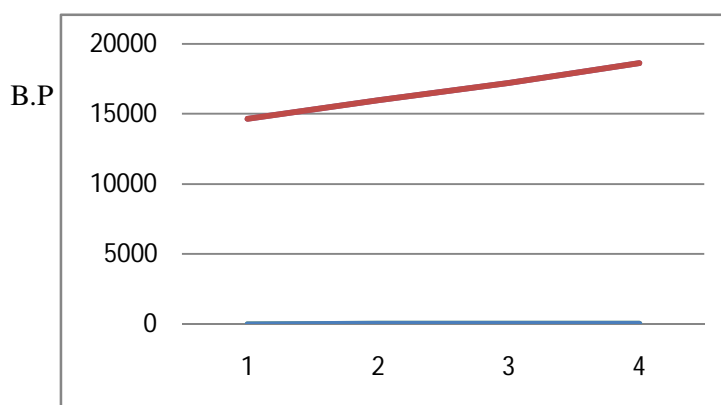
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Relation between Blood Pressure and Hematocrit

S.N	H (Hematocrit)	B.P(Blood Pressure)
1	18.6	1279.86
2	21.3	1454.55
3	21.9	1493.37
4	26.7	1803.93



Hematocrit Graph

VI. CONCLUSION

A simple survey of the graph between blood pressure and hematocrit in hepatitis C patient shows that when hematocrit increased then blood pressure is also increased. That is the hematocrit is proportional to blood pressure .

VII. ACKNOWLEDGEMENT

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VIII. REMARK

If this would have been possible to get blood pressure on the particular tissue (liver) during operation of hepatitis C patient then the relation blood pressure and hemoglobin has been measured more accurately.

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