

Economical Design of Prestressed Concrete Beams

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ABSTRACT: In this investigation four hundred and forty simply supported PSC beams have been designed for spans in the range of 5m to 25m considering the load in the range of 5 to 20 kN/m. The grades of concrete adopted are M30-M60. In order to study the effect of grades of concrete on the total cost of beams, the relative cost of simply supported beams of rectangular section and symmetrical I section have been studied. The design of beam and estimation of cost have been carried out using MS Excel. The cost of beams of I – section is compared to the cost of beams of the rectangular section.

Notations:

A	Area of cross section
A_p	Area of prestressing steel
B	Breadth of beam of rectangular section
b_f	Breadth of top and bottom flange
D	Depth of rectangular section
e	Eccentricity of prestressing cable
f_{ci}	Cube compressive strength of concrete at transfer
f_{ck}	Characteristic compressive strength of concrete
f_p	Characteristic compressive strength of prestressing steel
f_{pu}	Stress in concrete at strength limit state
I	Moment of inertia of cross section about horizontal centroidal axis
l	Effective span of beam
M_d	Maximum bending moment due to dead load
M_l	Maximum bending moment due to live load
M_t	Maximum bending moment due to total load
M_u	Ultimate moment of resistance of cross section
P	Effective prestressing force
P_i	Initial prestressing cable
t_f	Thickness of flange of I section
t_w	Thickness of web
W_d	Self weight of the beam per unit length
W_l	Live load acting on beam per unit length
X_u	Neutral axis depth at strength limit state
Y_1	Distance of top fibre from the horizontal centroidal axis of the cross section
Y_2	Distance of bottom fibre from the horizontal centroidal axis of the cross section

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Z	Section modulus of the cross section
η	Loss ratio
σ_c	Permissible compressive stress in concrete at transfer of prestress
σ_c	Permissible compressive stress in concrete at final stage(when live load is acting)
σ_{ct}	Permissible tensile stress in concrete at transfer
σ_{ct}	Permissible tensile stress in concrete at final stage (when live load is action)
σ_i	Maximum permissible stress in the wire initially (at transfer of prestress)
σ_1	Maximum compressive stress in concrete at mid span section

KEYWORDS: prestressing, post-tensioning, Symmetrical I-section, rectangular section.

I.INTRODUCTION

All the beams have been designed as Type 1 Post-tensioned beams.

Cost of materials adopted (obtained from SSS construction company Chennai.)

Cost of concrete:

Grade of concrete	Cost /m ³ in Rupees
M30	4800
M40	5200
M50	5940
M60	6770

Cost of high tensile steel:

12.7 mm (5#5mm wire) diameter strand – Rs 45.24 /m

Cost of Formwork - Rs465 / sqm

(including material and labour cost).

II. RELATED WORK

In their paper on ‘Design of prestressed concrete section for flexure’, **Bohdan K. Boczka et al** have attempted an analytical method for the design of a prestressed section with assigned efficiency for members working for flexure, which does not require iteration, use of tables or graphs or a solution of simultaneous equations. The section depth, the cover of tendons and effectiveness of prestress have to be evaluated at the start. The result of calculations can give a section fully stressed, or with stresses at the service stage satisfied exactly in case of a member with limited depth, or even a section with excessive capacity when it is desired. The section dimensioning is made in an approximate way, but required accuracy is easy to achieve. The introduction of the coefficient of dead load moments ratio in sections critical at transfer and at service allows it to take care of the differences between post-tensioned and pre-tensioned members with constant eccentricity tendons or with harping points, without the necessity of providing appropriate allowances in stresses. He has explained this method with examples of design of a post-tensioned member and a pre-tensioned one with harping points.

III. PROCEDURE FOLLOWED IN THE DESIGN OF PSC BEAMS

The design is based on reference No 5 and IS 1343:1980. The key steps involved in the design are:

2.1 Design Procedure for Rectangular section

Step1: Bending moment due to live load

The bending moment due to live load is calculated as $M_l = (wl^2/8)$

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Step2: Permissible stresses

The maximum permissible compressive stresses σ_c and σ_{cs} are calculated from IS1343-1980. Maximum permissible tensile stresses σ_{ct} and σ_{cts} are zero (Type 1 structure).

Step3: Trial section:

For a rectangular section, $\frac{I}{Y_1} = \frac{I}{Y_2}$ is

$$BD^2/6 = M_t / \sigma_{ct} + (1.5\eta - 0.5)\sigma_c$$

Assuming suitable relation between Band D and substituting the known values in the above equation, the values of B and D are calculated.

Step4: Properties of rectangular section

Area = BD; $Y_1 = Y_2 = D/2$

Moment of inertia, $I = (BD^3)/12$

Step 5: Bending moment due to dead load

Dead load bending moment is calculated as $M_d = (W_d l^2)/8$

Step6: Calculation of P_i and e

Initial stress at top fibre is equated to the permissible tensile stress

$$P_i/A - (P_i \times e \times Y_1)/I + (M_d \times Y_1)/I = -\sigma_{ct}$$

Final stress at bottom fibre is equated to the permissible tensile stress

$$\eta P_i/A + (\eta \times P_i \times e \times Y_2)/I - ((M_d + M_t) \times Y_2)/I = -\sigma_{ct}$$

Substituting the known values in the above two equations the values of P_i and e are determined.

Step7: Check for stresses

1. At transfer, the tensile stress at top fibre must be less than or equal to the allowable tensile stress of concrete at transfer.

$$P_i/A - (P_i \times e \times Y_1)/I + (M_d \times Y_1)/I \geq -\sigma_{ct}$$

2. At transfer, the compressive stress at bottom fibre must be less than or equal to the allowable compressive stress of concrete at transfer.

$$P_i/A + (P_i \times e \times Y_2)/I - (M_d \times Y_2)/I \leq \sigma_c$$

3. The compressive stress at the top fibre must be less than or equal to the allowable final compressive stress of concrete.

$$\eta P_i/A - (\eta \times P_i \times e \times Y_1)/I + ((M_d + M_t) \times Y_1)/I \leq \sigma_c$$

4. The tensile stress at the bottom fibre must be less than or equal to the allowable final tensile stress of concrete

$$\eta P_i/A + (\eta \times P_i \times e \times Y_2)/I - ((M_d + M_t) \times Y_2)/I \geq -\sigma_{ct}$$

Step 8: Amount of prestressing steel:

Amount of prestressing steel are calculated as $A_p = P_i / \sigma_i$

Step 9: check for available load factor:

If ratio between M_u and M_t is equal to or greater than 1.5, then the section is safe with respect to limit state of collapse.

$$M_u = F_{pu} \times A_p \times (d - (0.42 \times X_u))$$

F_{pu} and X_u can be calculated from Table 11 of IS1343-1980

$M_t = M_d + M_t$ under working condition.

2.2 Design Procedure for Symmetrical I section

Step1: Bending moment due to live load

The bending moment due to live load is calculated as $M_l = (wl^2)/8$

Step2: Permissible stresses

The maximum permissible compressive stresses σ_c and σ_{cs} are calculated from IS1343-1980.

Maximum permissible tensile stresses σ_{ct} and σ_{cts} are zero (Type 1 structure).

Step3: Trial section:

At first a rectangular section, $\frac{I}{Y_1} = \frac{I}{Y_2}$ is obtained.

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$$\text{i.e } BD^2/6 = M_1 / \sigma_{ct} + (1.5\eta - 0.5)\sigma_c'$$

Assuming suitable relation between Band D and substituting the known values in the above equation, the values of B and D are calculated.

Step4: Trial of symmetrical I section

The following dimensions are adopted.

The total depth are considered as 1.05D, 1.1D, 1.15D and 1.2D.

Breadth of bottom and top flange are taken as 0.6D

Thickness of top and bottom flange are taken as 0.2D

Thickness of web are varied as 0.15D, 0.175D and 0.2D

If the calculated thickness of web is less than 150mm, a minimum thickness of 150mm is provided.

Step5: Properties of I section

$$\text{Area of I section} = (2x_t x b_f) + (t_w x d_w)$$

$$\text{Moment of inertia } I = (I_1) + (2xI_2)$$

Step 6: Bending moment due to dead load

$$\text{Dead load bending moment is calculated as } M_d = (W_d l^2 / 8)$$

Step 7: Calculation of P_i and e

Initial stress at top fibre is equated to the permissible tensile stress

$$P_i / A - (P_i x e x Y_1) / I + (M_d x Y_1) / I = - \sigma_{ct}$$

Final stress at bottom fibre is equated to the permissible tensile stress

$$\eta P_i / A + (\eta x P_i x e x Y_2) / I - ((M_d + M_1) x Y_2) / I = - \sigma_{ct}$$

Substituting the known values in above two equations the values of P_i and e can be determined.

Step 8: Ceck for stresses

1. At transfer, the tensile stress at top fibre must be less than or equal to the allowable tensile stress of concrete at transfer.

$$P_i / A - (P_i x e x Y_1) / I + (M_d x Y_1) / I \geq - \sigma_{ct}'$$

2. At transfer, the compressive stress at bottom fibre must be less than or equal to the allowable compressive stress of concrete at transfer.

$$P_i / A + (P_i x e x Y_2) / I - (M_d x Y_2) / I \leq \sigma_c'$$

3. The compressive stress at the top fibre must be less than or equal to the allowable final compressive stress of concrete .

$$\eta P_i / A - (\eta x P_i x e x Y_1) / I + ((M_d + M_1) x Y_1) / I \leq \sigma_c$$

4. The tensile stress at the bottom fibre must be less than or equal to the allowable final tensile stress of concrete

$$\eta P_i / A + (\eta x P_i x e x Y_2) / I - ((M_d + M_1) x Y_2) / I \geq - \sigma_{ct}$$

step 9: Amount of prestressing steel:

Amount of prestressing steel is calculated as $A_p = P_i / \sigma_i$

Step 10: Check for available load factor:

If theratio between Mu and Mt is greater than or equal to 1.5 ,then the section is safe.

Case1

$$\text{Design ultimate moment} = 1.5x(M_d + M_l)$$

To examine whether NA lies inside the flange:

If $X_u = t$; Moment of resistance is given by

$$M_u = 0.36x f_{ck} x b_f x t_f x (d - 0.42x t_f)$$

If this moment of resistance is less than the ultimate bending moment to be resisted then the neutral axis will lie inside the top flange and the ultimate moment of resistance is calculated as per IS:1343:1980 code for a rectangular section of breadth equal to the width of the flange.

Case2 When the neutral axis is below the top flange

$$M_u' = M_{u1} + M_{u2}$$

$$M_{u1} = 0.447x f_{ck} x (b_f - t_w) x (d - 0.5x t_f), \text{ where } X_u \text{ is found by trials.}$$

$$M_{u2} = 0.36x f_{ck} x b_w x X_u x (d - 0.42x X_u)$$

If $M_t > M_u'$ then it is safe

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IV. RESULTS OF DESIGN

The eccentricity of prestressing cable at the end section is taken to be zero. The designed section and eccentricities of prestressing cable at mid span section of some of the beams are given below. The tensile stresses at mid-span section are zero.

Table 3.1 LL=5kN/m; M30 grade of concrete for rectangular section

Span m	Depth mm	Breadth mm	P_i KN	e mm	A_p mm ²	σ_1 (N/mm ²)		M_u/M_t
						top	bottom	
5	300	150	204	67	168	7.26	9.07	1.77
10	475	200	540	134	444	9.10	11.37	1.54
15	550	275	1104	188	908	11.68	14.60	1.65
20	675	350	1717	284	1412	11.63	14.54	1.85
25	800	400	2417	392	1988	12.08	15.11	1.92

Table 3.2 LL=5kN/m; Total Depth=1.2D;M30 grade of concrete for symmetrical I section

Span m	Depth mm	b_f mm	t_f mm	t_w mm	P_i KN	e mm	A_p mm ²	σ_1 (N/mm ²)		M_u/M_t
								top	bottom	
5	360	180	60	150	128	116	105	3.55	4.44	2.99
10	570	285	95	150	308	254	253	4.43	5.54	2.51
15	660	330	110	150	595	332	489	6.87	8.59	1.64
20	810	405	135	150	866	490	712	7.28	9.10	1.58
25	960	480	160	160	1181	683	972	7.38	9.23	1.56

V. ANALYSIS OF RESULTS

The following graphs are drawn for rectangular section of spans 5-25m and live load 5kN/m. The graphs are also drawn for symmetrical I section of spans 5-25m, live load 5kN/m and total depth 1.2D. M40 grade of concrete gives the most economical section among other grades of concrete such as M30, M50 and M60. Hence the total cost obtained by rectangular and symmetrical I section is compared for M40 grade of concrete. Finally the cost is compared for total depth of 1.05D, 1.1D, 1.15D and 1.2D for live load 5kN/m.

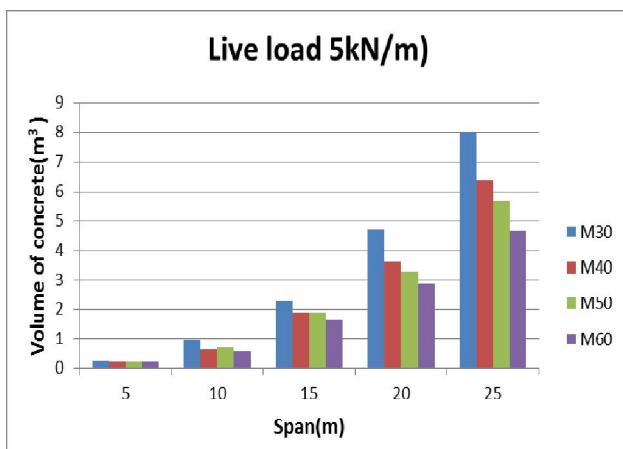


Fig 4.1 Volume of concrete of Rectangular section for spans 5m to 25m.

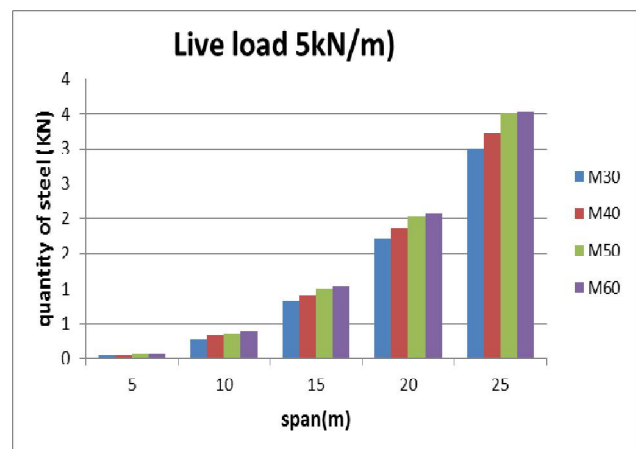


Fig 4.2 Steel quantity of Rectangular section for spans 5m to 25m.

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From fig 4.1 the graphs shows that the Volume of required concrete of rectangular section is found to be the least when M60 grade of concrete is used for span 5-25m and live load 5kN/m. From fig 4.2 the graphs shows that the required quantity of steel of rectangular section is found to be the least when M30 grade of concrete is used for span 5-25m and live load 5kN/m.

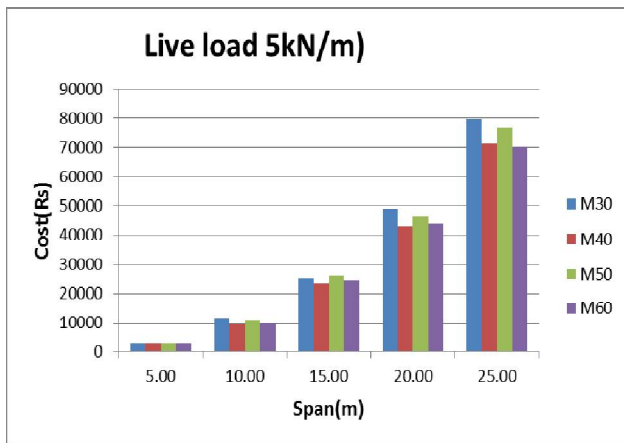


Fig 4.3 Total Cost of Rectangular section for spans 5m to 25m

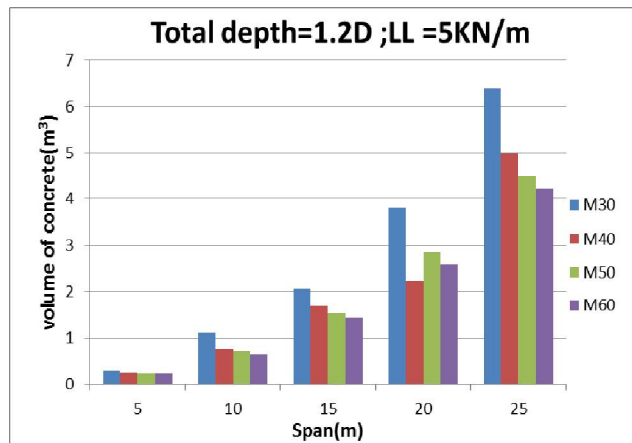


Fig 4.4 Volume of Concrete of Symmetrical I section 5m to 25m

From fig 4.3 the graphs shows that the total cost of rectangular section is found to be the least when M40 grade of concrete is used for span 5-25m and live load 5kN/m. From fig 4.4 the graphs shows that the volume of concrete of symmetrical I section is found to be the least when M60 grade of concrete is used for span 5-25m and live load 5kN/m.

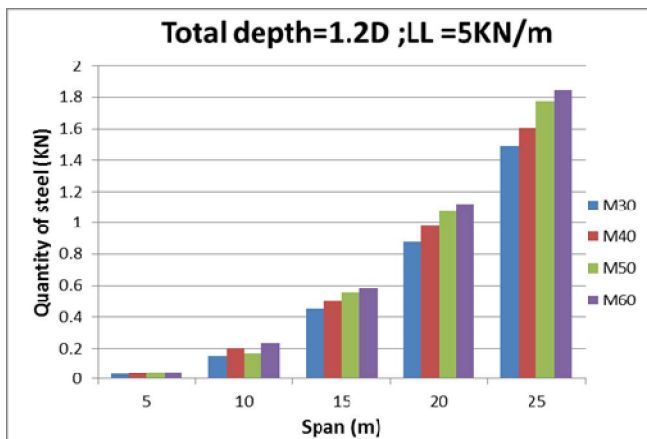


Fig 4.5 Steel quantity of Symmetrical I section for spans 5m to 25m

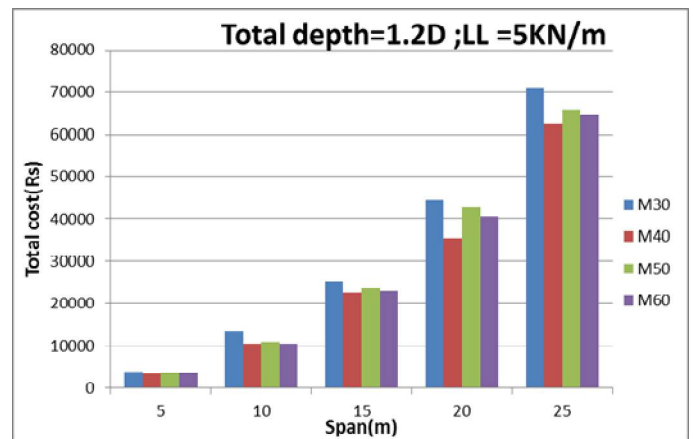


Fig 4.6 Total cost of Symmetrical I section for spans 5m to 25m

From fig 4.5 the graphs shows that required quantity of steel of symmetrical I section is found to be the least when M30 grade of concrete is used for span 5-25m and live load 5kN/m. From fig 4.6 the graphs shows that the total cost of symmetrical I section is found to be the least when 460 grade of concrete is used for span 5-25m and live load 5kN/m.

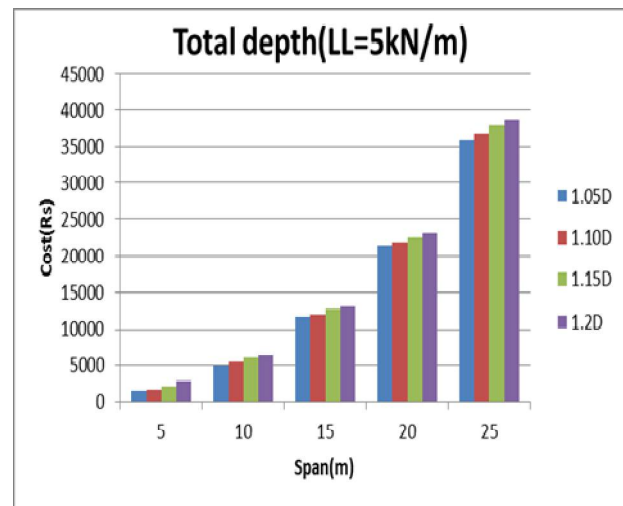
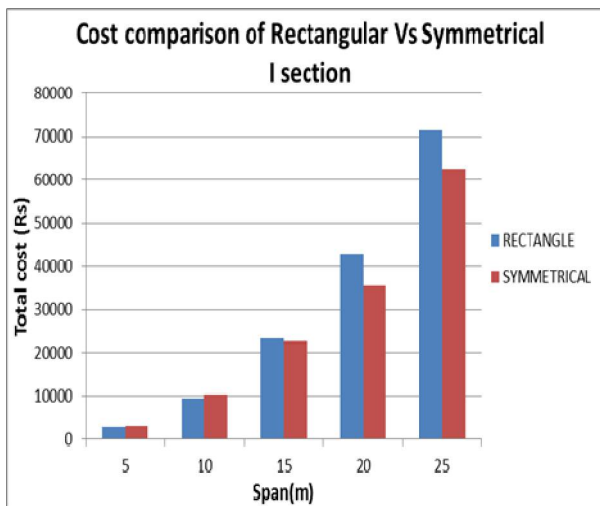


Fig 4.7 Cost comparison of Rectangular Vs Symmetrical I section (LL=5kN/m)
Fig 4.8 Cost Comparison of Total Depth (LL=5kN/m) Symmetrical I section (LL=5kN/m)

From fig 4.7 the graphs shows that the total cost of symmetrical I section is found to be the least for spans greater than 10m . From fig 4.8 The overall depth of 1.05D in beams of symmetrical I section results in the most economical section.

VI.CONCLUSIONS

The following conclusions are drawn for beams of rectangular and symmetrical I sections in the span range of 5m to 25m and for live loads of 5,10,15,20KN/m

- Volume of required concrete is found to be the least when M60 grade of concrete is used
- The required quantity of steel is found to be the least when M30 grade of concrete is used.
- M40 grade of concrete gives the most economical section.
- Symmetrical I section is found to be more economical than rectangular section for spans greater than 10m.
- The overall depth of 1.05D in beams of symmetrical section results in the most economical section.

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