

# Variance of Time to Recruitment for a Single Grade Manpower System with Different Epochs for Exits and Two Types of Decisions Having Two Thresholds

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**ABSTRACT:** In this paper, the problem of time to recruitment is studied using a univariate policy of recruitment involving two thresholds for a single grade manpower system with attrition generated by its policy decisions. Assuming that the policy decisions and exits occur at different epochs, a stochastic model is constructed and the system characteristics namely the variance of time to recruitment is obtained when (i) the inter-exit times are independent and identically distributed exponential random variables (ii) the inter-policy decision times are of two types, one with high rate of attrition and the other having low rate of attrition and (iii) order statistics and geometric process are associated separately with the population of inter-policy decision times.

**KEYWORDS:** Single grade manpower system; different decision and exit epochs; geometric process; order statistics; univariate policy of recruitment involving two thresholds and variance of time to recruitment.

## I. INTRODUCTION

Attrition is a common phenomenon in many organizations. This leads to the depletion of manpower. Recruitment on every occasion of depletion of manpower is not advisable since every recruitment involves cost. Hence the cumulative depletion of manpower is permitted till it reaches a level, called the threshold. If the total loss of manpower exceeds this threshold, the activities in the organization will be affected and hence recruitment becomes necessary. In [1, 2, 3] the authors have discussed the manpower planning models by Markovian and renewal theoretic approach. In [4] the author has studied the problem of time to recruitment for a single grade manpower system and obtained the variance of the time to recruitment when the loss of manpower forms a sequence of independent and identically distributed random variables, the inter-decision times form a geometric process and the mandatory breakdown threshold for the cumulative loss of manpower is an exponential random variable by using univariate cum policy of recruitment. In [5, 6] the authors have studied the problem of time to recruitment for a single grade manpower system and obtained the variance of the time to recruitment when the inter-decision times form a geometric process. In [7] the author has initiated the study of the problem of time to recruitment for a single grade manpower system by incorporating alertness in the event of cumulative loss of manpower due to attrition crossing the threshold, by considering optional and mandatory thresholds for the cumulative loss of manpower in this manpower system. In [8] the authors have studied this problem when inter-decision times form an order statistics.

In all the above cited work, it is assumed that attrition takes place instantaneously at decision epochs. This assumption is not realistic as the actual attrition will take place only at exit points which may or may not coincide with decision points. This aspect is taken into account for the first time in [9] and the variance of time to recruitment is obtained when the inter-decision times and exit times are independent and identically distributed exponential random variables using univariate policy for recruitment and Laplace transform in the analysis. In [11, 12] the authors have studied the problem

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in [9] when inter-decision times form a geometric process and an order statistics respectively. Recently, in [13, 14, 15] the authors have studied the work in [9, 11, 12] by considering optional and mandatory thresholds which is a variation from the work of [7] in the context of considering non-instantaneous exits at decision epochs.

In the present paper, for a single grade manpower system, a mathematical model is constructed in which attrition due to policy decisions take place at exit points and there are optional and mandatory thresholds as control limits for the cumulative loss of manpower. A univariate policy of recruitment based on shock model approach is used to determine the variance of time to recruitment when the system has different epochs for policy decisions and exits. The present paper extends the research work in [14, 15] when the inter- policy decision times are of two types, one with high rate of attrition and the other having low rate of attrition.

## II. MODEL DESCRIPTION

Consider an organization taking decisions at random epochs in  $(0, \infty)$  and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower if a person quits. It is assumed that the loss of manpower is linear and cumulative. Let  $X_i$  be the continuous random variable representing the amount of depletion of manpower (loss of man hours) caused at the  $i^{th}$  exit point and  $S_k$  be the total loss of manpower up to the first  $k$  exit points. It is assumed that  $X_i$ 's are independent and identically distributed random variables with probability density function  $m(\cdot)$ , distribution function  $M(\cdot)$  and mean  $\frac{1}{\alpha}(\alpha > 0)$ . Let  $U_k$  be the continuous random variable representing the time between the  $(k-1)^{th}$  and  $k^{th}$  policy decisions. Let  $W_i$  be the continuous random variable representing the time between the  $(i-1)^{th}$  and  $i^{th}$  exit times. It is assumed that  $W_i$ 's are independent and identically distributed random variables with probability density function  $g(\cdot)$ , probability distribution function  $G(\cdot)$ . Let  $N_e(t)$  be the number of exit points lying in  $(0, t]$ . Let  $Y$  be the optional threshold level and  $Z$  the mandatory threshold level ( $Y < Z$ ) for the cumulative depletion of manpower in the organization with probability density function  $h(\cdot)$  and distribution function  $H(\cdot)$ . Let  $p$  be the probability that the organization is not going for recruitment when optional threshold is exceeded by the cumulative loss of manpower. Let  $q$  be the probability that every policy decision has exit of personnel. As  $q = 0$  corresponds to the case where exits are impossible, it is assumed that  $q \neq 0$ . Let  $T$  be the random variable denoting the time to recruitment with probability distribution function  $L(\cdot)$ , density function  $l(\cdot)$ , mean  $E(T)$  and variance  $V(T)$ . Let  $\bar{a}(\cdot)$  be the Laplace transform of  $a(\cdot)$ . The univariate CUM policy of recruitment employed in this paper is stated as follows: Recruitment is done whenever the cumulative loss of manpower in the organization exceeds the mandatory threshold. The organization may or may not go for recruitment if the cumulative loss of manpower exceeds the optional threshold.

## III. MAIN RESULT

As in [14], we get

$$P(T > t) = \sum_{k=0}^{\infty} P[N_e(t) = k] P(S_k \leq Y) + p \sum_{k=0}^{\infty} P[N_e(t) = k] P(S_k > Y) P(S_k \leq Z) \quad (1)$$

$$L(t) = \bar{a} \sum_{k=1}^{\infty} G_k(t) a^{k-1} + p \left\{ \bar{b} \sum_{k=1}^{\infty} G_k(t) b^{k-1} - \bar{ab} \sum_{k=1}^{\infty} G_k(t) (ab)^{k-1} \right\} \quad (2)$$

and

$$\bar{l}(s) = \frac{\bar{a} \bar{g}(s)}{1 - a \bar{g}(s)} + p \left\{ \frac{\bar{b} \bar{g}(s)}{1 - b \bar{g}(s)} - \frac{\bar{ab} \bar{g}(s)}{1 - ab \bar{g}(s)} \right\} \quad (3)$$

$$\text{where } \bar{a} = 1 - a, \bar{b} = 1 - b, \bar{ab} = 1 - ab \quad (4)$$

$$\text{It is known that } E(T^r) = (-1)^r \left[ \frac{d^r}{ds^r} \bar{l}(s) \right]_{s=0}, \quad r = 1, 2, 3, \dots \quad (5)$$

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From (3) and (5), we get

$$E(T) = -\bar{g}'(0) \left\{ \frac{1}{a} + p \left[ \frac{1}{b} - \frac{1}{ab} \right] \right\} \quad (6)$$

$$E(T^2) = \frac{\bar{a} \bar{g}''(0) + 2a [\bar{g}'(0)]^2}{[\bar{a}]^2} + p \left\{ \frac{\bar{b} \bar{g}''(0) + 2b [\bar{g}'(0)]^2}{[\bar{b}]^2} - \frac{\bar{ab} \bar{g}''(0) + 2ab [\bar{g}'(0)]^2}{[\bar{ab}]^2} \right\} \quad (7)$$

and

$$V(T) = E(T^2) - [E(T)]^2 \quad (8)$$

Assuming  $M(x) = 1 - e^{-\alpha x}$ ,  $G(x) = 1 - e^{-\delta x}$ ,  $H(y) = 1 - e^{-\theta_1 y}$  and  $H(z) = 1 - e^{-\theta_2 z}$ , we now determine variance of time to recruitment for two different cases on inter-policy decision times.

Case(i):  $\{U_k\}_{k=1}^{\infty}$  form a geometric process with rate  $c, (c > 0)$ . The distribution  $F(\cdot)$  of  $U_1$  is  $F(t) = 1 - [p_1 e^{-\lambda_1 t} + (1 - p_1) e^{-\lambda_2 t}]$ ,  $\lambda_1, \lambda_2 > 0$ , where  $p_1$  and  $1 - p_1$  are proportions of policy decisions with high and low rates of attritions  $\lambda_1$  and  $\lambda_2$  respectively.

From (6), we get

$$E(T) = -\bar{g}'(0) \left\{ \frac{1}{a} + p \left[ \frac{1}{b} - \frac{1}{ab} \right] \right\}, \quad (9)$$

$$\text{where } \bar{g}'(0) = \frac{-c}{(c-1+q)} \left( \frac{p_1}{\lambda_1} + \frac{1-p_1}{\lambda_2} \right) \quad (10)$$

$$\bar{g}''(0) = \frac{c^2}{(c^2-1+q)} \bar{f}''(0) + \frac{2c^2(1-q)}{(c^2-1+q)(c-1+q)} (\bar{f}'(0))^2, \text{ where } \bar{f}''(0) = 2 \left( \frac{p_1}{\lambda_1^2} + \frac{1-p_1}{\lambda_2^2} \right)$$

$$\text{and } \bar{f}'(0) = - \left( \frac{p_1}{\lambda_1} + \frac{1-p_1}{\lambda_2} \right) \quad (11)$$

Equations (9), (8) together with (7), (10) and (11) give the mean and variance on the time to recruitment for case (i).

Case(ii):  $\{U_k\}_{k=1}^{\infty}$  form an order statistics where the sample of size  $r$  associated with this order statistics is selected from a hyper-exponential population of independent and identically distributed inter-policy decision times, where the common distribution  $F(\cdot)$  is given as in case(i).

Let  $F_{u(j)}(\cdot)$  and  $f_{u(j)}(\cdot)$  be the distribution and the probability density function of the  $j^{th}$  order statistic selected from the sample of size  $r$  from the exponential population  $\{U_k\}_{k=1}^{\infty}$ . From the theory of order statistics [15],

it is known that

$$f_{u(j)}(t) = j \binom{r}{j} [F(t)]^{j-1} f(t) [1-F(t)]^{r-j}, \quad j = 1, 2, \dots, r. \quad (12)$$

Suppose  $f(t) = f_{u(1)}(t) = r [1-F(t)]^{r-1} f(t)$ , where  $f(t) = p_1 \lambda_1 e^{-\lambda_1 t} + (1-p_1) \lambda_2 e^{-\lambda_2 t}$ .

From (6), (7) and (8), we get

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$$E(T) = \frac{A_r}{q} \left\{ \frac{1}{a} + p \left[ \frac{1}{b} - \frac{1}{ab} \right] \right\} \quad (13)$$

and

$$E(T^2) = \frac{2B_r}{q} \left\{ \frac{1}{a} + \frac{p}{b} - \frac{p}{ab} \right\} - \frac{2A_r^2}{q^2} \left\{ \frac{(\bar{a}q-1)}{a^2} + \frac{p(\bar{b}q-1)}{b^2} - \frac{p(\bar{ab}q-1)}{(ab)^2} \right\}^2 \quad (14)$$

$$\left[ \frac{d}{ds} \bar{f}_{u(1)}(s) \right]_{s=0} = -A_r, \text{ where } A_r = \sum_{n=0}^r \frac{{}^r C_n p_1^n (1-p_1)^{r-n}}{[\lambda_1 n + (r-n)\lambda_2]} \quad (15)$$

$$\left[ \frac{d^2}{ds^2} \bar{f}_{u(1)}(s) \right]_{s=0} = 2B_r, \text{ where } B_r = \sum_{n=0}^r \frac{{}^r C_n p_1^n (1-p_1)^{r-n}}{[\lambda_1 n + (r-n)\lambda_2]^2} \quad (16)$$

Equations (13), (8) together with (14), (15) and (16) give the mean and variance on the time to recruitment when  $f(t) = f_{u(1)}(t)$ .

Suppose  $f(t) = f_{u(r)}(t) = r [F(t)]^{r-1} f(t)$ .

From (6), (7) and (8), we get

$$E(T) = -\frac{C_r}{q} \left\{ \frac{1}{a} + \frac{p}{b} - \frac{p}{ab} \right\} \quad (17)$$

and

$$E(T^2) = \frac{-2D_r}{q} \left\{ \frac{1}{a} + \frac{p}{b} - \frac{p}{ab} \right\} - \frac{2C_r^2}{q^2} \left\{ \frac{(\bar{a}q-1)}{a^2} + \frac{p(\bar{b}q-1)}{b^2} - \frac{p(\bar{ab}q-1)}{(ab)^2} \right\}^2 \quad (18)$$

$$\left[ \frac{d}{ds} \bar{f}_{u(r)}(s) \right]_{s=0} = C_r, \text{ where } C_r = \sum_{n=0}^r \sum_{n_1=0}^{r-n} \frac{{}^r C_n (-1)^{r-n} {}^{r-n} C_{n_1} p_1^{n_1} (1-p_1)^{r-n-n_1}}{[\lambda_1 n_1 + (r-n-n_1)\lambda_2]} \quad (19)$$

$$\left[ \frac{d^2}{ds^2} \bar{f}_{u(r)}(s) \right]_{s=0} = -2D_r, \text{ where } D_r = \sum_{n=0}^r \sum_{n_1=0}^{r-n} \frac{{}^r C_n (-1)^{r-n} {}^{r-n} C_{n_1} p_1^{n_1} (1-p_1)^{r-n-n_1}}{[\lambda_1 n_1 + (r-n-n_1)\lambda_2]^2} \quad (20)$$

Equations (17), (8) together with (18), (19) and (20) give the mean and variance on the time to recruitment when  $f(t) = f_{u(r)}(t)$ .

### Remark:

1. Computation of  $V(T)$  for extended exponential and SCBZ Property possessing thresholds is similar for both the cases as their distribution will have just additional exponential terms.
2. Variance of time to recruitment for the case when  $\{U_k\}_{k=1}^{\infty}$  is a sequence of independent and identically distributed hyper exponential random variables can be obtained by taking  $c = 1$  in case (i).

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## Note:

- (i) When  $p=0$ , our results for case (i) agree with the results in [11] for the manpower system having only one threshold which is the mandatory threshold.
- (ii) When  $q = c = p_1 = 1$ , our results for case (i) agree with the results in [7] for the manpower system having certain instantaneous exits in the decision epochs.
- (iii) When  $c = p_1 = 1, p \neq 0, q \neq 1$  our results for case (i) agree with the results in [13] for the manpower system having the inter-decision times as an ordinary renewal process.
- (iv) When  $p = 0$  and  $c = p_1 = 1$ , our results in case (i) agree with the results in [9] for the manpower system having only one threshold which is the mandatory threshold.
- (v) When  $p = 0$  and  $p_1 = 1$ , our results in case (ii) agree with the results in [12] for the manpower system having only one threshold which is the mandatory threshold.
- (vi) When  $q = r = p_1 = 1$ , our results in case (ii) agree with the results in [7] for the manpower system having certain instantaneous exits in the decision epochs.
- (vii) When  $r = p_1 = 1, p \neq 0, q \neq 1$ , our results in case (ii) agree with the results in [13] for the manpower system having inter-decision times as an ordinary renewal process.

## IV. FINDINGS

From the above results, the following observations are presented which agree with reality.

1. When  $\alpha$  increases and keeping all the other parameters fixed, the average loss of manpower increases. Therefore the mean and variance of time to recruitment increase.
2. As  $\lambda$  increases, on the average, the inter-decision time decreases and consequently the mean and variance of time to recruitment decrease when the other parameters are fixed.
3. The mean and variance of the time to recruitment decrease or increase according as  $c > 1$  or  $c < 1$ , since the geometric process of inter-policy decision times is stochastically decreasing when  $c > 1$  and increasing when  $c < 1$ .

## V. CONCLUSION

The models discussed in this paper are new in the context of considering (i) separate points (exit points) on the time axis for attrition, thereby removing a severe limitation on instantaneous attrition at decision epochs (ii) associating a probability for any decision to have exit points (iii) considering two types of policy decisions, one with high rate of attrition and the other having low rate of attrition and (iv) provision of optional and mandatory thresholds. From the organization's point of view, our models are more suitable than the corresponding models with instantaneous attrition at decision epochs, as the provision of exit points at which attrition actually takes place, postpone the time to recruitment.

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