

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 3, March 2016

Numerical Solution of Higher Order Linear Fuzzy Differential Equations using Generalized STWS Technique

Emimal Kanaga Pushpam A¹, Anandhan P²

Associate Professor, Department of Mathematics, Bishop Heber College, Tiruchirappalli, Tamil Nadu, India¹

Assistant Professor, Department of Mathematics, Ganesar College of Arts and Science, Pudukkottai, Tamil Nadu,
India²

ABSTRACT: This paper presents the numerical solution of higher order linear fuzzy differential equations (FDEs) using generalized Single Term Walsh Series Technique (STWS). The applicability of this technique is illustrated by examples. The numerical results are compared with their exact solutions.

KEYWORDS: Fuzzy differential equations, linear system, IVP, Generalized STWS.

I. INTRODUCTION

Recently the study of fuzzy differential equations has been growing rapidly. The fuzzy derivative was introduced by Chang and Zadeh [1]. Seikkala and Kaleva have studied Fuzzy IVPs [2 - 4]. Further many researchers have suggested various numerical methods for solving fuzzy differential equations [5-9].

STWS technique was introduced by Rao et al. [10]. Balachandran and Murugesan applied STWS technique to solve first order system of IVPs [11, 12]. Murugesan and Paul Dhayabaran extended STWS technique for solving second order singular system of IVPs [13]. Emimal et al. [14] proposed the generalized STWS Technique to solve system of IVPs of any order 'n' with 'p' variables.

In this paper we adopt this generalized STWS technique to solve higher order linear fuzzy IVPs. In Section 2, we provide definitions and results on fuzzy numbers and fuzzy derivatives. In Section 3, we define fuzzy Cauchy problem. In Section 4, we give the generalized Single Term Walsh Series Technique which is capable of determining the discrete solutions of the linear systems of IVPs of any higher order 'n' with 'p' variables. In Section 5 we provide numerical examples to illustrate the applicability of the generalized STWS technique in solving fuzzy IVPs.

II. PRELIMINARIES

According to Zadeh, a fuzzy set is a generalization of classical set that allows membership function to take any value in the unit interval [0,1].

Definition 1

Let A be a fuzzy set defined in R. A is called a fuzzy interval if

1. A is normal, that is there exists $x_0 \in R$ such that $A(x_0) = 1$
2. A is convex, that is for all $x, y \in R$ and $0 \leq \lambda \leq 1$, it holds that

$$A(\lambda x + (1 - \lambda)y) \geq \min(A(x), A(y))$$

3. A is upper semi-continuous, that is for any $x_0 \in R$ it holds that

$$A(x_0) \geq \lim_{x \rightarrow x_0^+} A(x)$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 3, March 2016

4. $[A]^0 = \overline{\{x \in R / (x) > 0\}}$ is a compact subset of R.

Definition 2

Let A be a fuzzy interval defined in R. The α - cut of A is the set $[A]^\alpha$ that contains all elements in R such that the membership values of A is greater than or equal to α , that is

$$[A]^\alpha = \{x \in R / A(x) > \alpha\}, \quad \alpha \in (0,1].$$

For a fuzzy interval A, it's α - cut are closed intervals in R and we denote them by

$$[A]^\alpha = [\underline{A}^\alpha, \bar{A}^\alpha], \quad \alpha \in (0,1].$$

Definition 3

A fuzzy interval A is called a triangular fuzzy interval if its membership has the following form

$$A(x) = \begin{cases} 0, & \text{if } x < a \\ \frac{x-a}{b-a}, & \text{if } a \leq x \leq b \\ \frac{c-x}{c-b}, & \text{if } b \leq x \leq c \\ 0, & \text{if } x > c \end{cases}$$

and its α -cuts are simply $[A]^\alpha = [a + \alpha(b-a), c - \alpha(c-b)]$, $\alpha \in (0,1]$. It is clear that

1. \underline{A}^α is a bounded left continuous non decreasing function over $[0,1]$,
2. \bar{A}^α is a bounded right continuous non increasing function over $[0,1]$,
3. $\underline{A}^\alpha \leq \bar{A}^\alpha \quad \forall \alpha \in [0,1]$.

Definition 4

The supremum metric d_∞ on E is defined by $d_\infty(U, V) = \sup\{d_H[U]^\alpha, [V]^\alpha) : \alpha \in I\}$, and (E, d_∞) is a complete metric space.

Definition 5

A mapping $F : I \rightarrow E$ is Hukuhara differentiable at $t_0 \in T \subseteq R$ if for some $h_0 > 0$ the Hukuhara difference

$$F(t_0 + \Delta t) \sim_h F(t_0), \quad F(t_0) \sim_h F(t_0 - \Delta t),$$

exist in E for all $0 < \Delta t < h_0$ and if there exists an $F'(t_0) \in E$ such that

$$\lim_{\Delta t \rightarrow 0^+} d_\infty \frac{(F(t_0 + \Delta t) \sim_h F(t_0))}{\Delta t} - F'(t_0) = 0 \text{ and } \lim_{\Delta t \rightarrow 0^+} d_\infty \frac{(F(t_0) \sim_h F(t_0 - \Delta t))}{\Delta t} - F'(t_0) = 0,$$

the fuzzy set is called the Hukuhara derivative of F at t_0 .

Definition 6

The fuzzy integral

$$\int_a^b y(t) dt, \quad 0 \leq a \leq b \leq 1$$

is outlined by

$$\left[\int_a^b y(t) dt \right]^\alpha = \left[\int_a^b \underline{y}^\alpha(t) dt, \int_a^b \bar{y}^\alpha(t) dt \right]$$

provided the Lebesgue integrals on the proper exist.

Definition 7

If $y : I \rightarrow E$ is called fuzzy process. We denote

$$[y(t)]^\alpha = [\underline{y}^\alpha(t), \bar{y}^\alpha(t)], \quad t \in I, \quad 0 < \alpha \leq 1$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 3, March 2016

The Seikkala derivative $y'(t)$ of a fuzzy process y is defined by

$$[y'(t)]^\alpha = [(\underline{y}^\alpha)'(t), (\overline{y}^\alpha)'(t)], \quad 0 < \alpha \leq 1, \text{ provided by this equation defines a fuzzy number } y'(t) \in E.$$

III. FUZZY INITIAL VALUE PROBLEM

Consider the fuzzy initial value problem in the following form:

$$\begin{cases} y'(t) = f(t, y(t)), & t \in [t_0, T] \\ y(t_0) = y_0 \end{cases}$$

where y is a fuzzy function of t , $f(t, y)$ is a fuzzy operation of the crisp variable t and the fuzzy variable y , y' is the fuzzy derivative of y and $y(t_0) = y_0$ is a fuzzy number. Therefore we have a fuzzy Cauchy problem.

We denote the fuzzy function y by $[\underline{y}, \overline{y}]$. It means that the α -level set of $y(t)$ for $t \in [t_0, T]$ is

$$[y(t)]_\alpha = [\underline{y}(t; \alpha), \overline{y}(t; \alpha)], \quad [y'(t)]_\alpha = [\underline{y}'(t; \alpha), \overline{y}'(t; \alpha)], \quad \alpha \in (0, 1].$$

By using the extension principle of Zadeh, we have the membership function

$$f(t, y(t))(s) = \sup\{y(t)(\tau) \mid f(t, \tau) = s\}, \quad s \in R$$

so $f(t, y(t))$ is a fuzzy number. From this it follows that

$$[f(t, y(t))]_\alpha = [\underline{f}(t, y(t); \alpha), \overline{f}(t, y(t); \alpha)], \quad \alpha \in (0, 1]$$

where

$$\underline{f}(t, y(t); \alpha) = \min\{f(t, u) \mid u \in [\underline{y}(t; \alpha), \overline{y}(t; \alpha)]\}$$

$$\overline{f}(t, y(t); \alpha) = \max\{f(t, u) \mid u \in [\underline{y}(t; \alpha), \overline{y}(t; \alpha)]\}$$

We define

$$\underline{f}(t, y(t); \alpha) = F[t, \underline{y}(t; \alpha), \overline{y}(t; \alpha)], \quad \overline{f}(t, y(t); \alpha) = G[t, \underline{y}(t; \alpha), \overline{y}(t; \alpha)].$$

In this paper we solve higher order linear system of fuzzy IVPs using STWS technique. Consider an n^{th} order linear system of fuzzy differential equations of the form

$$A_0 X^{(n)}(t) = A_1 X^{(n-1)}(t) + A_2 X^{(n-2)}(t) + \dots + A_{n-1} X^{(1)}(t) + A_n X(t) + Bu(t)$$

with $X(0) = X_0$ and $X^{(j)}(0) = X_0^{(j)}$, $j = 1, 2, \dots, (n-1)$

where $A_0, A_1, \dots, A_n \in R^{p \times p}$, $B \in R^{p \times r}$, $X(t)$ is the p -state vector, $u(t)$ is an r -input vector.

Here $X^{(j)}$ is the j^{th} derivative of X , X_0 's are fuzzy numbers and X 's are fuzzy variables.

IV. GENERALIZED STWS TECHNIQUE FOR SOLVING LINEAR TIME INVARIANT SYSTEMS [14]

Consider an n^{th} order time-invariant linear system of the form

$$A_0 X^{(n)}(t) = A_1 X^{(n-1)}(t) + A_2 X^{(n-2)}(t) + \dots + A_{n-1} X^{(1)}(t) + A_n X(t) + Bu(t)$$

with $X(0) = X_0$ and $X^{(j)}(0) = X_0^{(j)}$, $j = 1, 2, \dots, (n-1)$

where $A_0, A_1, \dots, A_n \in R^{p \times p}$, $B \in R^{p \times r}$, $X(t)$ is the p -state vector, $u(t)$ is an r -input vector.

Here $X^{(j)}$ is the j^{th} derivative of X .

The n^{th} order system given by Eqn. (1) is called a *singular system* while A_0 is singular. The given functions are multiplied as a single term Walsh series in the normalized interval $s \in [0, 1]$ which corresponds to $t \in [0, 1/m]$ by defining $t = s/m$, m being any integer. Within the normalized interval, Eqn. (1) becomes

$$A_0 X^{(n)}(s) = \frac{A_1}{m} X^{(n-1)}(s) + \frac{A_2}{m^2} X^{(n-2)}(s) + \dots + \frac{A_{n-1}}{m^{(n-1)}} X^{(1)}(s) + \frac{A_n}{m^n} X(s) + \frac{B}{m^n} u(s) \quad (2)$$

Expressing Eqn. (2) in STWS with

$$X^{(n-j)}(s) = (C_j)_i \Psi(s), \quad (j=0, 1, 2, \dots, n-1)$$

$$X(s) = (C_n)_i \Psi(s)$$

$$u(s) = H_i \Psi(s)$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 3, March 2016

where C_j 's ($j=0, 1, 2, \dots, n-1$) are BPF values of n^{th} rate, $(n-1)^{\text{th}}$ rate, \dots , 1^{st} rate, state and input vector respectively in $s \in [0,1]$, the following set of recursive equations with the operational matrix for integration $E = \frac{1}{2}$ has been obtained:

$$(C_1)_i = \left[A_0 - \frac{A_1}{2m} - \frac{A_2}{(2m)^2} - \dots - \frac{A_n}{(2m)^n} \right]^{-1} G_i \tag{3}$$

$$(C_j)_i = \frac{1}{2} (C_{(j-1)})_i + X^{(n+j)}(i-1), \quad (j = 2, 3, \dots, n)$$

$$X^{(n-j)}(i) = (C_j)_i + X^{(n-j)}(i-1), \quad (j = 1, 2, \dots, (n-1))$$

$$X(i) = (C_n)_i + X(i-1)$$

where

$$G_i = \left(\frac{A_1}{m} + \frac{A_2}{2m^2} + \dots + \frac{A_n}{2^{(n-1)}m^n} \right) X^{(n-1)}(i-1) + \left(\frac{A_2}{m^2} + \frac{A_3}{2m^3} + \dots + \frac{A_n}{2^{(n-2)}m^n} \right) X^{(n-2)}(i-1) + \dots + \frac{A_n}{m^n} X(i-1) + \frac{B}{m^n} H_i \text{ and } i = 1, 2, 3, \dots$$

Using these recursive equations, the discrete values of $X, X^{(1)}, X^{(2)}, \dots, X^{(n-1)}$ in Eqn. (1) can be determined with the initial conditions.

These equations give solutions for all integer values of 'm'. In the case of singular system, that is, if A_0 is singular, the matrix

$$\left[A_0 - \frac{A_1}{2m} - \frac{A_2}{(2m)^2} - \dots - \frac{A_n}{(2m)^n} \right]$$

becomes non-singular, which is an added advantage of this method. The value of 'm' may be decided on large enough to growth the accuracy of results. The STWS approach is rather solid because it's far based totally on the Trapezoidal rule, that's an A-stable approach.

V. NUMERICAL EXAMPLES

Example 1

Consider the second order fuzzy differential initial value problem

$$\begin{cases} y''(t) = -y(t), & t \in [0,1] \\ y(0) = 0 \\ y'(0) = [0.9 + 0.1\alpha, 1.1 - 0.1\alpha], \text{ where } 0 \leq \alpha \leq 1. \end{cases}$$

The exact solution is given by

$$y(t; \alpha) = [(0.9 + 0.1\alpha) \sin(t), (1.1 - 0.1\alpha) \sin(t)].$$

The results are shown in the following Table 1 and Fig. 1.

Table 1 Results of Example 1 (at t = 1)

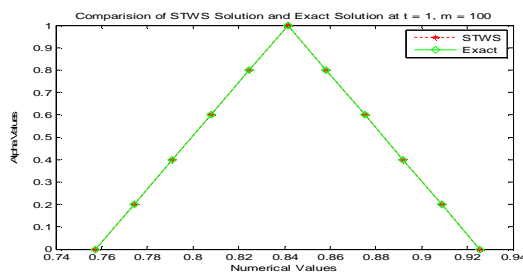
| α | Numerical Solution | | Exact solution | | Absolute Error | |
|----------|----------------------------|---------------------------|----------------------------|---------------------------|----------------------------|---------------------------|
| | $\underline{y}(t; \alpha)$ | $\overline{y}(t; \alpha)$ | $\underline{Y}(t; \alpha)$ | $\overline{Y}(t; \alpha)$ | $\underline{y}(t; \alpha)$ | $\overline{y}(t; \alpha)$ |
| 0.0 | 0.757320 | 0.925613 | 0.757324 | 0.925618 | 4.052233e-06 | 4.952729e-06 |
| 0.2 | 0.774149 | 0.908784 | 0.774153 | 0.908789 | 4.142282e-06 | 4.862679e-06 |
| 0.4 | 0.790978 | 0.891954 | 0.790983 | 0.891959 | 4.232332e-06 | 4.772630e-06 |
| 0.6 | 0.807808 | 0.875125 | 0.807812 | 0.875130 | 4.322382e-06 | 4.682580e-06 |
| 0.8 | 0.824637 | 0.858296 | 0.824642 | 0.858300 | 4.412431e-06 | 4.592531e-06 |
| 1.0 | 0.841466 | 0.841466 | 0.841471 | 0.841471 | 4.502481e-06 | 4.502481e-06 |

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 3, March 2016

Fig. 1 Solution Graph of Example 1



Example 2

Consider the following fuzzy IVP

$$\begin{cases} y'''(t) = 2y''(t) + 3y(t), & (0 \leq t \leq 1) \\ y(0) = (3 + \alpha, 5 - \alpha), & y'(0) = (-1 - \alpha, -3 - \alpha), & y''(0) = (8 + \alpha, 10 - \alpha) \end{cases}$$

The exact solution is as follows:

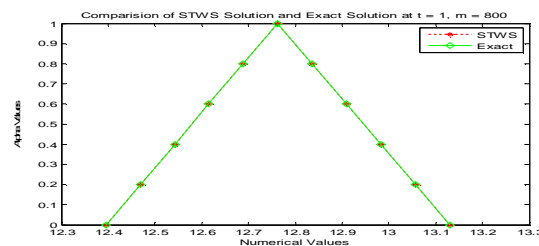
$$y(t; \alpha) = \left[-\frac{1}{3} + \frac{7}{12} e^{3t} + \left(\frac{11}{4} + \alpha \right) e^{-t}, -\frac{1}{3} + \frac{7}{12} e^{3t} + \left(\frac{19}{4} - \alpha \right) e^{-t} \right]$$

The results are shown in the following Table 2 and Fig. 2.

Table 2 Results of Example 2 (at t = 1)

| α | Numerical Solution | | Exact solution | | Absolute Error | |
|----------|----------------------------|---------------------------|----------------------------|---------------------------|----------------------------|---------------------------|
| | $\underline{y}(t; \alpha)$ | $\overline{y}(t; \alpha)$ | $\underline{Y}(t; \alpha)$ | $\overline{Y}(t; \alpha)$ | $\underline{y}(t; \alpha)$ | $\overline{y}(t; \alpha)$ |
| 0.0 | 12.394939 | 13.130698 | 12.394898 | 13.130657 | 4.105947e-05 | 4.096367e-05 |
| 0.2 | 12.468515 | 13.057122 | 12.468474 | 13.057081 | 4.104989e-05 | 4.097325e-05 |
| 0.4 | 12.542091 | 12.983546 | 12.542050 | 12.983505 | 4.104031e-05 | 4.098283e-05 |
| 0.6 | 12.615667 | 12.909971 | 12.615626 | 12.909930 | 4.103073e-05 | 4.099241e-05 |
| 0.8 | 12.689243 | 12.836395 | 12.689202 | 12.836354 | 4.102115e-05 | 4.100199e-05 |
| 1.0 | 12.762819 | 12.762819 | 12.762778 | 12.762778 | 4.101157e-05 | 4.101157e-05 |

Fig. 2 Solution Graph of Example 2



International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 3, March 2016

Example 3

Consider the following fourth Order fuzzy linear differential equation

$$\begin{cases} y^{(4)}(t) - y = 0, & t \in [0,1] \\ y(0) = (r-1, 1-r), & y'(0) = (r-1, 1-r), \\ y''(0) = (r-1, 1-r), & y'''(0) = (r-1, 1-r) \end{cases}$$

with the exact fuzzy solution

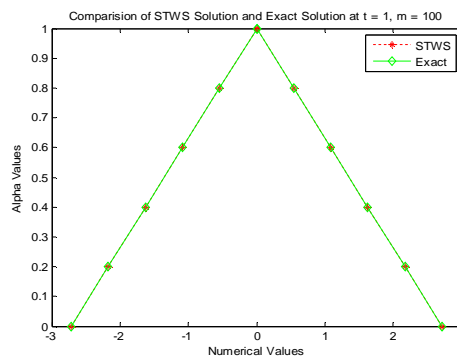
$$y(t; r) = \left[(r-1)e^t, (1-r)e^t \right].$$

The results are shown in the following Table 3 and Fig. 3.

Table 3 Results of Example 3 (at t = 1)

| α | Numerical Solution | | Exact solution | | Absolute Error | |
|----------|----------------------------|---------------------------|----------------------------|---------------------------|----------------------------|---------------------------|
| | $\underline{y}(t; \alpha)$ | $\overline{y}(t; \alpha)$ | $\underline{Y}(t; \alpha)$ | $\overline{Y}(t; \alpha)$ | $\underline{y}(t; \alpha)$ | $\overline{y}(t; \alpha)$ |
| 0.0 | -2.718304 | 2.718304 | -2.718282 | 2.718282 | 2.265278e-05 | 2.265278e-05 |
| 0.2 | -2.174644 | 2.174644 | -2.174625 | 2.174625 | 1.812223e-05 | 1.812223e-05 |
| 0.4 | -1.630983 | 1.630983 | -1.630969 | 1.630969 | 1.359167e-05 | 1.359167e-05 |
| 0.6 | -1.087322 | 1.087322 | -1.087313 | 1.087313 | 9.061113e-06 | 9.061113e-06 |
| 0.8 | -0.543661 | 0.543661 | -0.543656 | 0.543656 | 4.530557e-06 | 4.530557e-06 |
| 1.0 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000e+00 | 0.000000e+00 |

Fig. 3 Solution Graph of Example 3



Example 4

Consider the following system of first order fuzzy differential equations

$$\begin{cases} y_1'(t) = y_2, \\ y_2'(t) = -4y_1 + 4y_2, & t \in [0,1] \\ y_1(0) = (2+r, 4-r), & y_2(0) = (5+r, 7-r) \end{cases}$$

The exact solution is as follows:

$$\begin{aligned} y_1(t; r) &= [(2+r)e^{2t} + (1-r)te^{2t}, (4-r)e^{2t} + (r-1)te^{2t}] \\ y_2(t; r) &= [(4+2r)e^{2t} + (1-r)e^{2t} + (2-2r)te^{2t}, (r-1)e^{2t} + (8-2r)e^{2t} + (2r-2)te^{2t}] \end{aligned}$$

The results of Y_1 are shown in the following Tables 4 and Fig. 4.

International Journal of Innovative Research in Science, Engineering and Technology

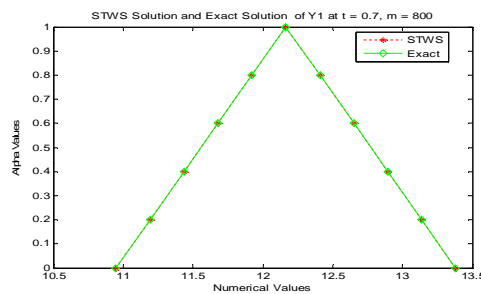
(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 3, March 2016

Table 4 Results of Y_1 in Example 4 (at $t = 0.7$)

| α | Numerical Solution | | Exact solution | | Absolute Error | |
|----------|--------------------|------------------------|------------------|------------------------|------------------|------------------------|
| | $y_1(t; \alpha)$ | $\bar{y}_1(t; \alpha)$ | $Y_1(t; \alpha)$ | $\bar{Y}_1(t; \alpha)$ | $y_1(t; \alpha)$ | $\bar{y}_1(t; \alpha)$ |
| 0.0 | 10.949052 | 3.382165 | 10.949040 | 3.382160 | 1.241907e-05 | 5.322453e-06 |
| 0.2 | 11.192364 | 3.138854 | 11.192352 | 3.138848 | 1.170941e-05 | 6.032114e-06 |
| 0.4 | 11.435675 | 2.895543 | 11.435664 | 2.895536 | 1.099975e-05 | 6.741776e-06 |
| 0.6 | 11.678986 | 2.652231 | 11.678976 | 2.652224 | 1.029009e-05 | 7.451438e-06 |
| 0.8 | 11.922297 | 2.408920 | 11.922288 | 2.408912 | 9.580423e-06 | 8.161100e-06 |
| 1.0 | 12.165609 | 2.165609 | 12.165600 | 2.165600 | 8.870761e-06 | 8.870761e-06 |

Fig. 4 Solution Graph for Y_1 of Example 4

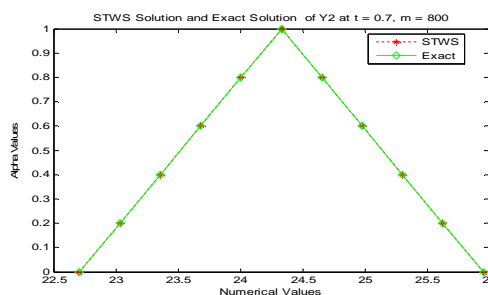


The results of Y_2 are shown in the following Tables 5 and Fig. 5.

Table 5 Results of Y_2 in Example 4 (at $t = 0.7$)

| α | Numerical Solution | | Exact solution | | Absolute Error | |
|----------|--------------------|------------------------|------------------|------------------------|------------------|------------------------|
| | $y_2(t; \alpha)$ | $\bar{y}_2(t; \alpha)$ | $Y_2(t; \alpha)$ | $\bar{Y}_2(t; \alpha)$ | $y_2(t; \alpha)$ | $\bar{y}_2(t; \alpha)$ |
| 0.0 | 25.953308 | 22.709128 | 25.953280 | 22.709120 | 2.779506e-05 | 7.687984e-06 |
| 0.2 | 25.628890 | 23.033546 | 25.628864 | 23.033536 | 2.578435e-05 | 9.698692e-06 |
| 0.4 | 25.304472 | 23.357964 | 25.304448 | 23.357952 | 2.377365e-05 | 1.170940e-05 |
| 0.6 | 24.980054 | 23.682382 | 24.980032 | 23.682368 | 2.176294e-05 | 1.372011e-05 |
| 0.8 | 24.655636 | 24.006800 | 24.655616 | 24.006784 | 1.975223e-05 | 1.573082e-05 |
| 1.0 | 24.331218 | 24.331218 | 24.331200 | 24.331200 | 1.774152e-05 | 1.774152e-05 |

Fig. 5 Solution Graph for Y_2 of Example 4



International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 3, March 2016

VI. CONCLUSION

In this paper, the generalized STWS method has been adopted to solve the linear systems of fuzzy IVPs. The effectiveness of this approach has been illustrated via examples. The numerical outcome had been compared with exact solutions. From the tables, it is observed that the absolute error is negligibly small. This suggests that the generalized STWS technique is suitable for linear systems of fuzzy IVPs of any order with any number of variables. Since STWS method is an A-Stable method, one can get the results for any length of time.

REFERENCES

- [1] Chang S. L., and Zadeh L. A., "On Fuzzy Mapping and Control", IEEE Trans. System Man Cybernet, Vol. 2, pp. 30-34, 1972.
- [2] Kaleva O., "Fuzzy Differential Equations", Fuzzy Sets Systems, Vol, 24, pp. 301-317, 1987.
- [3] Seikkala S., "On the Fuzzy Initial Value Problem", Fuzzy Sets Systems, Vol, 24, pp. 319-330, 1987.
- [4] Kaleva O., "The Cauchy problem for Fuzzy Differential Equations", Fuzzy Sets Systems, Vol, 35, pp. 389-396, 1990.
- [5] Abbasbandy S., Allahviranloo T., "Numerical solution of fuzzy differential equation by Taylor method", Journal of Computational Methods in Applied Mathematics, Vol. 2, pp. 113-124, 2002.
- [6] Abbasbandy S., Allahviranloo T., "Numerical solution of fuzzy differential equation, Mathematical and Computational Applications", Vol. 7, pp. 41-52, 2002.
- [7] Abbasbandy S., Allahviranloo T., "Numerical solution of fuzzy differential equation by Runge-Kutta method", Nonlinear Studies, Vol.11, pp. 117-129, 2004.
- [8] Allahviranloo T., Ahmady N., Ahmady E., "Numerical solution of fuzzy differential equations by predictor-Corrector method", Information Sciences, Vol. 177, pp. 1633-1647, 2007.
- [9] Allahviranloo T., Abbasbandy S., Ahmady N., Ahmady E., "Improved predictor-Corrector method for solving fuzzy initial value problems", Information Sciences, Vol. 179, pp. 945-955, 2009.
- [10] Rao G.P., Palanisamy K.R., Srinivasan T., "Extension of computation beyond the limit of normal interval in Walsh series analysis of dynamical systems", IEEE. Trans. Automat. Contr., Vol. 25, pp. 317-319, 1980.
- [11] Balachandran K., Murugesan K., "Analysis of different systems via single term Walsh series method", International Journal of Computer Mathematics, Vol. 33, pp. 171-179, 1990.
- [12] Balachandran K., Murugesan K., "Analysis of non-linear singular systems via STWS method", International Journal of Computer Mathematics, Vol. 36, pp. 9-12, 1990.
- [13] Murugesan K., Dhayabaran D.P., Evans D.J., "Analysis of second order multivariable linear system using single-term Walsh series technique and Runge-Kutta method", International Journal of Computer Mathematics, Vol. 72, pp. 367-374, 1999.
- [14] Emimal Kanaga Pushpam A., Paul Dhayabaran D., Henry Amirtharaj E.C., "Numerical solution of higher order systems of IVPs using generalized STWS technique", Applied Mathematics and Computation, Vol. 180, pp. 200-205, 2006.