

Equations of Motion for Radial Fluid Flow in Porous Media

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ABSTRACT: The multiphase flow in porous media is a matter of great problems with a much wide history in the field of fluid mechanics. This is the subject of important technical applications, most probably in the oil recovery from petroleum reservoirs and others also. The single phase fluid flow through the porous medium is analyzed by Darcy's law. In petroleum industry and in other applications, the transport is modeled by postulating the multiphase generalization of Darcy's law. In this relation, distinct pressures are defined for every constituent phase with difference, said as capillary pressure and is determined by micro pore geometry, interfacial tension and surface chemistry of dense medium. For the flow rates, the relative permeability is defined which relates the volume flow rate of every fluid to its pressure gradient. In present paper, there is the derivation and analysis of the diffusion equation in radial coordinates for the fluid flow in the porous rocks and some useful results have been founded. The permeability is the function of rock type which varies with temperature, stress etc., and is independent on fluid. The effect of fluid on the flow rate is calculated for by the viscosity. The numerical value of the permeability for the given rock varies with the size of the pores in the rock and also on the degree of interconnectivity of void space. The pressure pulses satisfy the diffusion equation instead of the wave equation. Then they move at the speed which continually decreases with the time instead of moving at constant speed. The results shown in the paper are very useful in earth sciences, petroleum industry and others.

KEYWORDS: Darcy's law, Diffusion equation, Porous rock, Radial coordinates.

I. INTRODUCTION

The concept of porous media within a multi scale framework is a concept that takes advantage of much mature state of understanding that applies at low length scales as the method to give the description of the larger scale systems. Several physical systems can be linked with a series of length scales; everyone is associated with particular mathematical formulation that describes the behavior at that scale. The multi scale frame works give a relationship between these various descriptions that give a sequence of mathematical formulations.

When those are applied to the porous media the approach can be utilized to tie macro thermo dynamical forms and conservation equations to those that apply at pore scale, otherwise known as the micro scale. This is beneficial when macroscopic closure relationships are incomplete or unreliable; microscopic closure relationships are better known. The microscopic analysis can be applied to give insights into macroscopic behavior, judges simplifying assumptions and generate suitable macroscopic relationships. These analyses rely heavily on computational methods for giving the actual solutions for microscopic analysis of porous medium flows. The computational analysis gives the opportunities to incorporate larger and more realistic descriptions of micro analysis behavior into macroscopic modeling analysis.

For the flow in porous media, the Darcy's equation has been used. The Darcy equation is normally based on principle of a linear relation between pressure gradient and velocity in the porous media. The linear factor is expressed as the porosity and is representing resistant to flow in solid media. The flow process in porous media is governed by various physical phenomena like as viscous forces and other forces coming from surface tensions between fluid and solid and

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also surface tensions between the different phases of fluid. The flow process is involved in principle modeled by use of momentum equation, but it takes more simulation analysis to solve momentum equation in place of the Darcy's equation. Due to this reason the Darcy's equation is much commonly used in the simulations of fluid flow through porous media.

II. GOVERNING LAWS AND EQUATIONS

The basic law for governing the fluid flow through porous media is the Darcy's law that was given by the Henry Darcy in 1856 by the experiments on vertical water filtration through the sand beds. The thorough study related to diffusion was performed by R. W Zimmerman in 2002 [14]. The Darcy had given the equation as follows:

$$Q = \frac{C A \Delta(P - \rho g z)}{L} \quad (1)$$

Where

P = pressure [Pa]

ρ = density [kg/m^3]

g = gravitational acceleration [m/s^2]

z = vertical coordinate (measured downwards) [m]

L = length of sample [m]

Q = volumetric flow rate [m^3/s]

C = constant of proportionality [$m^2/Pa s$]

A = cross-sectional area of sample [m^2]

The consistent set of units was used in Darcy's law like SI units, C.G.S. units etc. In the oil industry it is generally use "oilfield units", that are inconsistent. The Darcy's law is same as other linear transport laws in mathematical manner, such as Ohm's law for the electrical conduction, Fick's law for the solute diffusion and Fourier's law for the heat conduction. By the laws of fluid mechanics we know that Bernoulli's equation is as follows:

$$\frac{P}{\rho} - g z + \frac{V^2}{2} = \frac{1}{\rho} \left(P - \rho g z + \rho \frac{v^2}{2} \right) \quad (2)$$

where P/ρ is related to enthalpy per unit mass, $g z$ is gravitational energy per unit mass, $v^2/2$ is kinetic energy per unit mass. But the fluid velocities in the reservoir are small so the third term will be negligible. And we can see that the term $(P - \rho g z)$ represents an energy type term. So it seems reasonable that fluid can flow from the regions of higher energy to lower energy so the driving force for the flow must be the gradient of $(P - \rho g z)$. But due to the Darcy's theory, it has been found that all the factors are equal, Q varies inversely to fluid viscosity. It is now suitable to factor out the term μ , and put $C = k/\mu$, where k is known as permeability. By taking the volumetric flow per unit area, i. e., $q = Q/A$. Hence the Darcy's law can be shown as:

$$q = \frac{Q}{A} = \frac{k \Delta(P - \rho g z)}{\mu L} \quad (3)$$

Where flux q has the dimensions of [m/s]. It is now easier to say of the units as [m^3/m^2s].

For the transient processes in which flux varies from point to point, we can show a differential form of the Darcy's law. In vertical direction, this equation can be shown as:

$$q_v = \frac{Q}{A} = - \frac{k}{\mu} \frac{d(P - \rho g z)}{dz} \quad (4)$$

Where suffix v is using for vertical flow. The negative sign is taken due to the fluid flows in direction from higher potential to lower potential. The differential form of the Darcy's law for one dimensional horizontal flow will be

$$q_h = \frac{Q}{A} = - \frac{k}{\mu} \frac{d(P - \rho g z)}{dx} = - \frac{k}{\mu} \frac{dP}{dx} \quad (5)$$

Where suffix v is taken for vertical flow. In many rocks the permeability k_h in horizontal plane is different from the vertical permeability, k_v ; in these cases, $k_h > k_v$. The permeabilities in the two orthogonal directions within the horizontal plane have some difference. Then in that course we will generally take: $k_h = k_v$. The permeability is some

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function of rock type and varies with temperature, stress etc., but it is independent of the fluid; the effect of the fluid due to flow rate is accounted for by the viscosity term in the above equations. The permeability has units of m^2 , but in the mathematical sense it is commonly use as “Darcy” units, defined as:

$$1 \text{ Darcy} = 0.987 \times 10^{-12} \text{ m}^2 \approx 10^{-12} \text{ m}^2$$

The Darcy unit is defined in such a way that a rock having the permeability of 1 *Darcy* would transmit 1 c.c. of the water with viscosity 1 *cP* per second, through the region of 1 sq. cm. cross-sectional area, if pressure drop along the direction of flow were 1 *atm* per cm. Many soils and sands that analyzers must deal with have permeabilities on the order of a few *Darcies*. The main purpose of the “*Darcy*” definition was to avoid the need for using small prefixes like 10^{-12} etc. But a Darcy is like a round number in SI units, so conversion between the two is very easy. The numerical value of k for the given rock depends on size of pores in the rock, d on the degree of interconnectivity of the void space. So that

$$k \approx d^2 / 1000$$

where d is the diameter of the pores. The permeabilities of various types of soils and rocks vary over many orders of magnitude. Then the permeabilities of petroleum reservoir rocks lie in the range of 0.001-1.0 *Darcies*. So it is suitable to refer the permeability of the reservoir rocks in the units of “*milliDarcies*” (*mD*) that equal 0.001 *Darcies*. In many reservoirs, the permeability is mainly due to an interconnected network of the fractures. The permeabilities of the fractured rocks lie in the range 1 *mD* to 10 *Darcies*. In the fractured reservoir, the reservoir scale permeability is not much related to core scale permeability, so that anyone can measure it.

If the fluid is in the static equilibrium then we have $q = 0$, hence the differential equation can be written as:

$$\frac{d(P - \rho g z)}{dx} = 0 \Rightarrow P - \rho g z = \text{constant} \quad (6)$$

If we put $z = 0$ i.e., at sea level, where fluid pressure is equal to atmospheric pressure, then we can write

$$P_s = P_a + \rho g z \quad (7)$$

where P_s, P_a show the static pressure and atmospheric pressure respectively. But we always observe the pressure above the atmospheric pressure, so we can neglect the atmospheric pressure P_a in above equation (7). We can found by comparing equation (7) with equation (4) that only pressure above and beyond the static pressure given by equation (2) plays an important role in the derivation of flow. So the term $\rho g z$ is not of any use, as it only contributes to the static pressure and but does not play a role to driving force for flow. Then after neglecting the said term, the equation for correct pressure can be written as:

$$P_c = P - \rho g z \quad (8)$$

So the Darcy’s law in terms of corrected pressure for the horizontal flow can be written as:

$$q = \frac{Q}{A} = -\frac{k}{\mu} \frac{dP_c}{dx} \quad (9)$$

Instead by using sea level i.e., $z = 0$, we can also take $z = z_0$ as a datum i.e., the amounts of initial oil in place lie above and below $z = z_0$. So we have

$$P_c = P - \rho g (z - z_0) \quad (10)$$

The choice of datum level is immaterial; it means that it gives a constant term to corrected pressure so it does not contribute to pressure gradient. The pressure P_c defined in the equation (10) can be interpreted as pressure of a analytical fluid at the depth $z = z_0$ that will be in the equilibrium with fluid and exists at actual pressure at depth z . The Darcy’s law is supposed to be macroscopic law that is intended to be meaningful over regions that are much bigger than the size of the single pore. Now we can discuss about the permeability at the point in the reservoir, we cannot be referring to the permeability at the mathematically infinitesimal point since the given point can be in a sand grain and not in the pore space. The permeability property is now defined for a porous medium, not for the single pore. So the permeability is property that is in a meaning of the averaged out on the certain region of space surrounded by the point (x, y, z) . Now the region must be sufficiently big to encompass a significant number of pores. The pressure P that is used in Darcy’s law is mainly an average pressure taken on the small region of space.

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The Darcy's law does not give a sufficient information to calculate time-dependent i.e., transient problems that involving the subsurface flow. So to find out the complete governing equation that applies to these types of problems, we can first derive the mathematical expression for the principle of mass conservation. The mass conservation equation states the balance between rate of change of mass in an arbitrary volume and inflow of the mass through the boundary surface area. In the integral form we can write this as follows:

$$\frac{\partial}{\partial t} \iiint \rho \phi dV + \iint \rho \vec{v} \cdot \vec{n} dS = \iiint q dV \quad (11)$$

In the above equation the double and triple integrals are taken over the surface and the volume respectively and \vec{v} , \vec{n} , ρ , q and ϕ represent the velocity vector, unit normal vector, fluid density, external mass flow rate and porosity respectively. The right hand side term of equation (11) can be changed into the volume integral form by using the Gauss' divergence theorem as:

$$\iint \rho \vec{v} \cdot \vec{n} dS = \iiint \nabla \cdot (\rho \vec{v}) dV \quad (12)$$

So for a fixed control volume, the integral form of conservation law can be

$$\iiint \left[\frac{\partial(\rho \phi)}{\partial t} + \nabla \cdot (\rho \vec{v}) - q \right] dV \quad (13)$$

Now the differential form of the mass conservation equation can be written in the coordinate invariant form as:

$$\frac{\partial(\rho \phi)}{\partial t} + \nabla \cdot (\rho \vec{v}) = q \quad (14)$$

For the multiphase phase flow, it is necessary to account for saturation of each phase. So that the equation (14) within the every phase α can be written as:

$$\frac{\partial(\rho_\alpha S_\alpha \phi)}{\partial t} + \nabla \cdot (\rho_\alpha \vec{v}_\alpha) = q_\alpha \quad (15)$$

III. DIFFUSION EQUATION IN RADIAL COORDINATES

The transient flow of a fluid through a porous medium is governed by a certain type of partial differential equation known as a diffusion equation. The detailed derivation and discussion related to diffusion was given by R. W. Zimmerman in 2002 [14] i.e., discussed in this paper as follows. So to derive the equation, we combine Darcy's law with the law of mass conservation and an equation that describes the process for which the fluid is stored inside a porous rock. Now using the differentiation for the product function $(\rho \phi)$, we get

$$\begin{aligned} D(\rho \phi) &= \rho D\phi + \phi D\rho, D \equiv \frac{d}{dt} \\ &= \rho \frac{d\phi}{dP} \frac{dP}{dt} + \phi \frac{d\rho}{dP} \frac{dP}{dt} \\ &= \phi \left[\left(\frac{1}{\phi} \frac{d\phi}{dP} \right) \frac{dP}{dt} + \rho \left(\frac{1}{\rho} \frac{d\rho}{dP} \right) \frac{dP}{dt} \right] \end{aligned} \quad (16)$$

$$D(\rho \phi) = \rho \phi (c_R + c_f) D(P) \quad (17)$$

where c_R , c_f are the compressibility of the rock and the fluid respectively.

The equation for mass conservation for the fluid flow is taken as:

$$-\frac{d(\rho q)}{dx} = \frac{d(\rho \phi)}{dt} \quad (18)$$

By using the Darcy's law, we have

$$\begin{aligned} -\frac{d(\rho q)}{dx} &= -\frac{d}{dx} \left[-\frac{\rho k}{\mu} \frac{dP}{dx} \right] \\ -\frac{d(\rho q)}{dx} &= \frac{k}{\mu} \left[\rho \frac{d^2 P}{dx^2} + \frac{d\rho}{dx} \frac{dP}{dx} \right] \\ -\frac{d(\rho q)}{dx} &= \frac{k}{\mu} \left[\rho \frac{d^2 P}{dx^2} + \frac{d\rho}{dP} \frac{dP}{dx} \frac{dP}{dx} \right] \end{aligned}$$

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$$-\frac{d(\rho q)}{dx} = \frac{k\rho}{\mu} \left[\frac{d^2 P}{dx^2} + \frac{1}{\rho} \frac{d\rho}{dP} \left(\frac{dP}{dx} \right)^2 \right]$$

$$-\frac{d(\rho q)}{dx} = \frac{k\rho}{\mu} \left[\frac{d^2 P}{dx^2} + c_f \left(\frac{dP}{dx} \right)^2 \right] \tag{19}$$

From the equations (17) and (19), we get

$$\frac{d^2 P}{dx^2} + c_f \left(\frac{dP}{dx} \right)^2 = \frac{1}{K} \rho \phi (c_R + c_f) D(P) \tag{20}$$

But we know that,

$$\frac{d^2 P}{dx^2} \gg c_f \left(\frac{dP}{dx} \right)^2$$

On neglecting the term $c_f \left(\frac{dP}{dx} \right)^2$, i. e.

$$c_f \left(\frac{dP}{dx} \right)^2 \approx c_f \left[\frac{\mu Q}{2 \pi k H R} \right]^2 \tag{21}$$

$$\frac{d^2 P}{dx^2} \approx \frac{\mu Q}{2 \pi k H R^2} \tag{22}$$

So we can neglect the nonlinear term in equation (20), we get the diffusion equation as given

$$\frac{dP}{dt} = \frac{K}{\phi \mu (c_\phi + c_f)} \frac{d^2 P}{dx^2} \tag{23}$$

where $(c_\phi + c_f) = c$ is the total compressibility.

The parameter which governs the rate at which fluid pressure diffuses through the rock is the hydraulic diffusivity D_H , that is defined by

$$D_H = \frac{k}{\phi \mu (c_\phi + c_f)} \tag{24}$$

The distance d at which a pressure disturbance will travel during an elapsed time t is given as

$$d = \sqrt{4 D_H t} \tag{25}$$

For the multi-phase flow if we have assume that the pores of the rock are filled with two components, oil and water, and often also contain some hydrocarbons in the gaseous phase. So we have to find the governing flow equations for an oil and water system, in the general form. From the rock properties module that Darcy's law can be generalized for two-phase flow by including a relative permeability factor for each phase, we have

$$q_w = \frac{-k k_{rw}}{\mu_w} \frac{dP_w}{dx} \tag{26}$$

$$q_o = \frac{-k k_{ro}}{\mu_o} \frac{dP_o}{dx} \tag{27}$$

where the subscripts w and o are used for oil and water respectively. The two relative permeability functions are supposed to be known functions of the phase saturations. For the oil-water system, the two saturations are necessarily related to each other by the relationship

$$S_w + S_o = 1 \tag{28}$$

The pressures in the two phases at every point in the reservoir must be different. If the reservoir is oil-wet then the two pressures will be given by

$$P_o - P_w = P_c S_o \tag{29}$$

where the capillary pressure P_c is given by the rock-dependent function of oil saturation.

But the volume of the oil in a given region is equal to the total pore volume multiplied by the oil saturation then by the equations of the conservation of mass for the two phases can be written by inserting a saturation factor in the storage term as given

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$$-\frac{d(\rho_0 q_0)}{dx} = \frac{d(\phi \rho_0 S_0)}{dt} \tag{30}$$

$$-\frac{d(\rho_w w)}{dx} = \frac{d(\phi \rho_w S_w)}{dt} \tag{31}$$

The densities of the two phases are related to their respective phase pressures with the equation of state as given by

$$\rho_o = \rho_o(P_o) \tag{32}$$

$$\rho_w = \rho_w(P_w) \tag{33}$$

where the temperature are taken as constant.

Lastly, the porosity must be the function of the phase pressures P_o and P_w . The above two pressures independently affect the porosity. Now, the capillary pressure P_c is generally small so that

$$P_o \approx P_w$$

From which we can use the pressure-porosity relationship that would be obtained under single-phase conditions, *i.e.*,

$$\phi = \phi(P_o) \tag{34}$$

If the fluid is taken as slightly compressible or if the pressure variations are small then the equations of state are written as

$$\rho(P_o) = \rho_{oi} [1 + c_o (P_o - P_{oi})] \tag{35}$$

where the subscript “i” is used for the initial state, and the compressibility c_o is taken as a constant.

In the field of engineering there is much interest in the case of fluid flow towards a well, in which case it is more convenient to use the cylindrical (radial) coordinates in the place of Cartesian coordinates. To derive the diffusion equation in radial coordinates, it is considered that the fluid flowing radially towards (or away from) a vertical well, in the radially symmetric manner. Now replace x with R , and taking $A(R) = 2 \pi RH$:

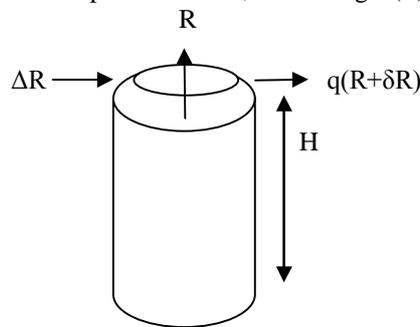


Fig.1 (A region used in deriving the diffusion equation in radial coordinates)

$$[2 \pi RH \rho(R) q(R) - 2 \pi (R + \delta R) H \rho(R + \delta R) q(R + \delta R)] \delta t = m(t + \delta t) - m(t) \tag{36}$$

Now dividing the above equation by δt , and taking limit as $\delta t \rightarrow 0$, we have

$$2 \pi H [R \rho(R) q(R) - (R + \delta R) \rho(R + \delta R) q(R + \delta R)] = dm/dt \tag{37}$$

On the right-hand side:

$$M = \rho \phi V = \rho \phi 2 \pi H R \delta R \tag{38}$$

$$\frac{Dm}{dt} = \frac{d(\rho \phi 2 \pi H R \delta R)}{dt} = 2 \pi H R \frac{d(\rho \phi)}{dt} \delta R \tag{39}$$

Equate eqs. (37) and (39), divide by δR , and taking $\delta R \rightarrow 0$, we have

$$-\frac{d(\rho q R)}{dR} = \frac{R d(\rho \phi)}{dt} \tag{40}$$

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Eq. (40) is the radial flow version of the continuity (*i.e.*, conservation of mass) equation. Now use the Darcy's law we have

$$\frac{K}{\mu} \frac{d}{dR} \left(\rho R \frac{dP}{dR} \right) = \rho \varphi (c_f + c_\varphi) R \frac{dP}{dt} \quad (41)$$

Follow the same procedure, we have

$$\frac{1}{R} \frac{d}{dR} \left(R \frac{dP}{dR} \right) + c_f \left(\frac{dP}{dR} \right)^2 = \frac{\varphi \mu (c_f + c_\varphi) dP}{k dt} \quad (42)$$

For the liquids, we again neglect the term $c_f (dP / dR)^2$, to arrive at

$$\frac{dP}{dt} = \frac{k}{\varphi \mu c_t} \frac{1}{R} \frac{d}{dR} \left(R \frac{dP}{dR} \right) \quad (43)$$

Eq. (43) is the governing equation for transient, radial flow of a liquid through porous rock.

IV. DISCUSSION AND RESULTS

The permeability is a function of rock type that varies with stress, temperature etc., and does not depend on the fluid. The effect of the fluid on the flow rate is accounted for by the term of viscosity. The numerical value of k for a given rock depends on the diameter of the pores in the rock " d " as well as on the degree of interconnectivity of the void space. The parameter that governs the rate at which fluid pressure diffuses through a rock mass is the hydraulic diffusivity which is defined by

$$D_H = \frac{k}{\phi \mu c}$$

The distance d at which a pressure disturbance will travel during an elapsed time t is given as

$$d = \sqrt{4 D_H t}$$

The time required for a pressure disturbance to travel a distance d is found by

$$t = \frac{\phi \mu c d^2}{4k}$$

The pressure pulses obey a diffusion equation not a wave equation. So, they travel at a speed that continually decreases with time rather than travelling at a constant speed.

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