

Fourier-Stieltjes Transform of Some Functions

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ABSTRACT: The Integral transform expressing the equation of physical system in the new form that can be solved by simple computation. An example is the Fourier transform which convert the Cartesian Coordinates into the functions of frequencies and The Stieltjes transform is depend on parameter $p > 0$ which is useful in large random matrices. This paper includes the generalization of Fourier-Stieltjes Transform and we have obtained the Fourier-Stieltjes Transform of some functions.

KEYWORDS: Text Fourier Transform, Stieltjes Transform, Fourier-Stieltjes Transform, Generalized function.

I. INTRODUCTION

Integral transform provides way to solve the problems in different manner. The Integral transform expressing the equation of physical system in the new form that can be solved by simple computation. The integral transform express in the following way as-

$$F(s) = \int_{t_1}^{t_2} K(t,s)f(t) dt ,$$

Where, $F(s)$ is output, $K(t,s)$ is Kernel, $f(t)$ is input. In this, the Kernel and Interval remain the same for a transform irrespective of the input function. The input and output functions changes depending on the problems we are solving. Integral Transforms are Linear Operators. This is a key point because it provides a way to understand exactly what an integral transform does [1]. There are various integral transform such as Fourier Transform, Laplace Transform, Mellin Transform, Stieltjes Transform, Hankle Transform etc. We know that generalized form of the Integral Transform is very much important in engineering sciences, applied mathematics, Mathematical physics.

The Fourier Transform is an important image processing tool which is used to decompose an image into its sine and cosine components [4]. Fourier transform is essential to understand how a signal behaves when it passes through filters, amplifiers and communications channels [2]. Fourier transform can be used as high-pass, low-pass, and band-pass filters and it can also be applied to signal and noise estimation by encoding the time series [3]. The Fourier transform applicable for image segmentation and pattern recognition [5].

Similarly, the Stieltjes transform which is very much important in limiting behavior of certain classes of random matrices [7]. The spectral analysis of large dimensional random matrices is often carried out with the help of the Stieltjes transform. The Stieltjes transform of a matrix X is known, then one can retrieve its eigenvalue distribution, which is often more difficult to obtain in the spectral domain than in the Stieltjes transform domain[8]. In the present work we have obtained Fourier-Stieltjes transform of some function.

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II. FOURIER-STIELTJES TRANSFORM OF SOME FUNCTION

2.1 If $FS\{f(t, x)\}$ denotes generalized Fourier-Stieltjes Transform of $f(t, x)$ then

$$FS\{1\} = \frac{1}{(p-1)(is)}$$

Proof: - We have-

$$FS\{f(t, x)\} = \int_0^\infty \int_0^\infty f(t, x) e^{-ist} (x+y)^{-p} dt dx$$

If $f(t, x) = 1$ then

$$\begin{aligned} FS\{1\} &= \int_0^\infty \int_0^\infty 1 e^{-ist} (x+y)^{-p} dt dx \\ &= \int_0^\infty e^{-ist} dt \int_0^\infty (x+y)^{-p} dx \\ &= \left[\frac{e^{-ist}}{-is} \right]_0^\infty \left[\frac{(x+y)^{-p+1}}{-p+1} \right]_0^\infty \\ FS\{1\} &= \frac{1}{is} \left[\frac{y^{-p+1}}{-p+1} \right] = \frac{-i}{s} \left[\frac{y^{-p+1}}{p-1} \right] \end{aligned}$$

..... (2.1.1)

2.2 If $FS\{f(t, x)\}$ denotes generalized Fourier-Stieltjes Transform of $f(t, x)$ then

$$FS\{\delta(t-a)\delta(x-b)\} = K(a, b, s, p)$$

Proof:-

$$FS\{f(t, x)\} = \int_0^\infty \int_0^\infty f(t, x) e^{-ist} (x+y)^{-p} dt dx$$

If $f(t, x) = \delta(t-a)\delta(x-b)$ then

$$\begin{aligned} FS\{\delta(t-a)\delta(x-b)\} &= \int_0^\infty \int_0^\infty \delta(t-a)\delta(x-b) e^{-ist} (x+y)^{-p} dt dx \\ &= \int_0^\infty \int_0^\infty \delta(t-a)\delta(x-b) e^{-ist} (x+y)^{-p} dt dx \\ &= \int_0^\infty \delta(t-a) e^{-ist} dt \int_0^\infty \delta(x-b) (x+y)^{-p} dx \\ FS\{\delta(t-a)\delta(x-b)\} &= e^{-isa} (b+y)^{-p} \quad \left\{ \text{since, } \int_0^\infty \delta(t-a) \phi(t) dt = \phi(a) \right\} \end{aligned}$$

2.3 If $FS\{f(t, x)\}$ denotes generalized Fourier-Stieltjes Transform of $f(t, x)$ then-

$$FS\{tx\} = \frac{-1}{s^2(p-1)(p-2)}$$

Proof: - We have

$$FS\{f(t, x)\} = \int_0^\infty \int_0^\infty f(t, x) e^{-ist} (x+y)^{-p} dt dx$$

If $f(t, x) = tx$ then

$$\begin{aligned} FS\{tx\} &= \int_0^\infty \int_0^\infty tx e^{-ist} (x+y)^{-p} dt dx \\ &= \int_0^\infty t e^{-ist} dt \int_0^\infty x (x+y)^{-p} dx \\ &= \left[\left(t \frac{e^{-ist}}{-is} \right)_0^\infty - \int_0^\infty \frac{e^{-ist}}{-is} dt \right] \left[\left(x \frac{(x+y)^{-p+1}}{-p+1} \right)_0^\infty - \int_0^\infty \frac{(x+y)^{-p+1}}{-p+1} dx \right] \\ &= \left[0 + \frac{1}{is} \int_0^\infty e^{-ist} dt \right] \left[0 + \frac{1}{p-1} \int_0^\infty (x+y)^{-p+1} dx \right] \\ &= \frac{1}{is} \left[\frac{e^{-ist}}{-is} \right]_0^\infty \frac{1}{p-1} \left[\frac{(x+y)^{-p+2}}{-p+2} \right]_0^\infty \\ &= \left(\frac{1}{is} \right)^2 \frac{1}{p-1} \left[\frac{y^{-p+2}}{p-2} \right] \\ FS\{tx\} &= \frac{-y^{-p+2}}{s^2(p-1)(p-2)} \end{aligned}$$

..... (2.3.1)

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2.4 If $FS\{f(t, x)\}$ denotes generalized Fourier-Stieltjes Transform of $f(t, x)$ then-

$$FS\{t^n x^n\} = \frac{(n!)^2 (i)^{-(n+1)} y^{-p+(n+1)}}{s^{n+1} (p-1)(p-2)\dots(p-n)[p-(n+1)]}$$

Proof:-

$$\begin{aligned} FS\{t^n x^n\} &= \int_0^\infty \int_0^\infty t^n x^n e^{-ist} (x+y)^{-p} dt dx \\ &= \int_0^\infty t^n e^{-ist} dt \int_0^\infty x^n (x+y)^{-p} dx \\ &= \left[\left(t^n \frac{e^{-ist}}{-is} \right)_0^\infty - \int_0^\infty n t^{n-1} \frac{e^{-ist}}{-is} dt \right] \left[\left(x^n \frac{(x+y)^{-p+1}}{-p+1} \right)_0^\infty - \int_0^\infty n x^{n-1} \frac{(x+y)^{-p+1}}{-p+1} dx \right] \\ &= \left[\frac{n}{is} \int_0^\infty t^{n-1} e^{-ist} dt \right] \left[\frac{n}{p-1} \int_0^\infty x^{n-1} (x+y)^{-p+1} dx \right] \\ &= \frac{n}{is} \left[\left(t^{n-1} \frac{e^{-ist}}{-is} \right)_0^\infty - \int_0^\infty (n-1) t^{n-2} \frac{e^{-ist}}{-is} dt \right] \\ &\quad \frac{n}{p-1} \left[\left(x^{n-1} \frac{(x+y)^{-p+2}}{-p+2} \right)_0^\infty - \int_0^\infty (n-1) x^{n-2} \frac{(x+y)^{-p+2}}{-p+2} dx \right] \\ &= \frac{n}{is} \left[\frac{n-1}{is} \int_0^\infty t^{n-2} e^{-ist} dt \right] \frac{n}{p-1} \left[\frac{(n-1)}{p-2} \int_0^\infty x^{n-2} (x+y)^{-p+2} dx \right] \\ &= \frac{n(n-1)}{(is)^2} \left[\int_0^\infty t^{n-2} e^{-ist} dt \right] \frac{n}{(p-1)(p-2)} \left[\int_0^\infty x^{n-2} (x+y)^{-p+2} dx \right] \end{aligned}$$

Integrating up to n^{th} term we get-

$$\begin{aligned} &= \left[\frac{n(n-1)(n-2)\dots 3.2.1}{(i)^n (s)^n} \int_0^\infty t^{n-n} e^{-ist} dt \right] \left[\frac{n(n-1)(n-2)\dots 3.2.1}{(p-1)(p-2)\dots(p-n)} \int_0^\infty x^{n-n} (x+y)^{-p+n} dx \right] \\ &= \left[\frac{n!}{(i)^n (s)^n} \left(\frac{e^{-ist}}{-is} \right)_0^\infty \right] \left[\frac{n!}{(p-1)(p-2)\dots(p-n)} \left(\frac{(x+y)^{-p+(n+1)}}{-p+(n+1)} \right)_0^\infty \right] \\ &= \left[\frac{n!}{(i)^n (s)^n is} \right] \left[\frac{n!}{(p-1)(p-2)\dots(p-n)} \frac{y^{-p+(n+1)}}{[p-(n+1)]} \right] \\ FS\{t^n x^n\} &= \frac{(n!)^2 (i)^{-(n+1)} y^{-p+(n+1)}}{s^{n+1} (p-1)(p-2)\dots(p-n)[p-(n+1)]} \end{aligned}$$

2.5 If $FS\{f(t, x)\}$ denotes generalized Fourier-Stieltjes Transform of $f(t, x)$ then-

$$FS\{e^{at} x\} = \frac{y^{-p+2}}{(is-a)(p-2)}$$

Proof: - We have

$$\begin{aligned} FS\{e^{at} x\} &= \int_0^\infty \int_0^\infty e^{at} x e^{-ist} (x+y)^{-p} dt dx \\ &= \int_0^\infty e^{at} e^{-ist} dt \int_0^\infty x (x+y)^{-p} dx \\ &= \int_0^\infty e^{(a-is)t} dt \int_0^\infty x (x+y)^{-p} dx \\ &= \left[\frac{e^{-(is-a)t}}{-(is-a)} \right]_0^\infty \left[\left(x \frac{(x+y)^{-p+1}}{-p+1} \right)_0^\infty - \int_0^\infty \frac{(x+y)^{-p+1}}{-p+1} dx \right] \\ &= \left[\frac{1}{(is-a)} \right] \left[- \int_0^\infty \frac{(x+y)^{-p+1}}{-p+1} dx \right] \\ &= \left[\frac{1}{(is-a)} \right] \left[\frac{1}{p-1} \int_0^\infty (x+y)^{-p+1} dx \right] \\ &= \left[\frac{1}{(is-a)} \right] \left[\frac{1}{p-1} \left(\frac{(x+y)^{-p+2}}{-p+2} \right)_0^\infty \right] \\ &= \left[\frac{1}{(is-a)} \right] \left[\frac{1}{p-1} \left(\frac{y^{-p+2}}{p-2} \right) \right] \\ FS\{e^{at} x\} &= \frac{1}{(is-a)} \frac{1}{p-1} \frac{y^{-p+2}}{p-2} \end{aligned}$$

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2.6 If $FS\{f(t, x)\}$ denotes generalized Fourier-Stieltjes Transform of $f(t, x)$ then-

$$FS\{x \cos at\} = \frac{(is)(y^{-p+2})}{(s^2+a^2)(p-1)(p-2)}$$

Proof: We have

$$\begin{aligned} FS\{x \cos at\} &= \int_0^\infty \int_0^\infty x \cos at e^{-ist} (x+y)^{-p} dt dx \\ &= \int_0^\infty \cos at e^{-ist} dt \int_0^\infty x (x+y)^{-p} dx \\ &= \left[\frac{e^{-ist}}{(-is)^2+a^2} (-is \cos at + a \sin at) \right]_0^\infty \left[\frac{y^{-p+2}}{(p-1)(p-2)} \right] \quad \{\text{Since, by using the above result}\} \\ &= \left[(0) - \left(\frac{1}{s^2+a^2} (-is) \right) \right] \left[\frac{y^{-p+2}}{(p-1)(p-2)} \right] \\ FS\{x \cos at\} &= \frac{(is)(y^{-p+2})}{(s^2+a^2)(p-1)(p-2)} \end{aligned}$$

2.7 If $FS\{f(t, x)\}$ denotes generalized Fourier-Stieltjes Transform of $f(t, x)$ then-

$$FS\{x \sin at\} = \frac{(is)(y^{-p+2})}{(s^2+a^2)(p-1)(p-2)}$$

Proof:- We have

$$\begin{aligned} FS\{x \sin at\} &= \int_0^\infty \int_0^\infty x \sin at e^{-ist} (x+y)^{-p} dt dx \\ &= \int_0^\infty \sin at e^{-ist} dt \int_0^\infty x (x+y)^{-p} dx \\ &= \left[\frac{e^{-ist}}{(-is)^2+a^2} (-is \sin at - a \cos at) \right]_0^\infty \left[\frac{y^{-p+2}}{(p-1)(p-2)} \right] \quad \{\text{Since, by using the above result}\} \\ &= \left[(0) - \left(\frac{1}{s^2+a^2} (-a) \right) \right] \left[\frac{y^{-p+2}}{(p-1)(p-2)} \right] \\ FS\{x \sin at\} &= \frac{a(y^{-p+2})}{(s^2+a^2)(p-1)(p-2)} \end{aligned}$$

2.8 If $FS\{f(t, x)\}$ denotes generalized Fourier-Stieltjes Transform of $f(t, x)$ then-

$$FS\{x^n \cos at\} = \frac{(is) n! y^{-p+(n+1)}}{(s^2+a^2)(p-1)(p-2)\dots\dots(p-n)[p-(n+1)]}$$

Proof:- We have

$$\begin{aligned} FS\{x^n \cos at\} &= \int_0^\infty \int_0^\infty x^n \cos at e^{-ist} (x+y)^{-p} dt dx \\ &= \int_0^\infty \cos at e^{-ist} dt \int_0^\infty x^n (x+y)^{-p} dx \end{aligned}$$

By using result (2.4) & (2.6) we get-

$$\begin{aligned} FS\{x^n \cos at\} &= \left[\frac{is}{(s^2+a^2)} \right] \left[\frac{n!}{(p-1)(p-2)\dots\dots(p-n)} \frac{y^{-p+(n+1)}}{[p-(n+1)]} \right] \\ FS\{x^n \cos at\} &= \frac{(is) n! y^{-p+(n+1)}}{(s^2+a^2)(p-1)(p-2)\dots\dots(p-n)[p-(n+1)]} \end{aligned}$$

2.9 If $FS\{f(t, x)\}$ denotes generalized Fourier-Stieltjes Transform of $f(t, x)$ then-

$$FS\{x^n \sin at\} = \frac{a n! y^{-p+(n+1)}}{(s^2+a^2)(p-1)(p-2)\dots\dots(p-n)[p-(n+1)]}$$

Proof:- We have

$$\begin{aligned} FS\{x^n \sin at\} &= \int_0^\infty \int_0^\infty x^n \sin at e^{-ist} (x+y)^{-p} dt dx \\ &= \int_0^\infty \sin at e^{-ist} dt \int_0^\infty x^n (x+y)^{-p} dx \end{aligned}$$

By using result (2.4) & (2.7) we get-

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$$FS\{x^n \sin at\} = \left[\frac{a}{(s^2+a^2)} \right] \left[\frac{n!}{(p-1)(p-2)\dots(p-n)} \frac{y^{-p+(n+1)}}{[p-(n+1)]} \right]$$

$$FS\{x^n \sin at\} = \frac{a n! y^{-p+(n+1)}}{(s^2+a^2)(p-1)(p-2)\dots(p-n)[p-(n+1)]}$$

2.10 If $FS\{f(t, x)\}$ denotes generalized Fourier-Stieltjes Transform of $f(t, x)$ then-

$$FS\{x \cos hat\} = \frac{-is}{(s^2+a^2)} \left[\frac{y^{-p+2}}{(p-1)(p-2)} \right]$$

Proof: We have

$$FS\{x \cos hat\} = \int_0^\infty \int_0^\infty x \cos hat e^{-ist} (x+y)^{-p} dt dx$$

$$= \int_0^\infty \cos hat e^{-ist} dt \int_0^\infty x (x+y)^{-p} dx$$

By using the result (2.5) we have,

$$= \int_0^\infty \left(\frac{e^{at}+e^{-at}}{2} \right) e^{-ist} dt \left[\frac{y^{-p+2}}{(p-1)(p-2)} \right]$$

$$= \frac{1}{2} \left\{ \int_0^\infty e^{-(is-a)t} dt + \int_0^\infty e^{-(a+is)t} dt \right\} \left[\frac{y^{-p+2}}{(p-1)(p-2)} \right]$$

$$= \frac{1}{2} \left\{ \left[\frac{e^{-(is-a)t}}{-(is-a)} \right]_0^\infty + \left[\frac{e^{-(a+is)t}}{-(a+is)} \right]_0^\infty \right\} \left[\frac{y^{-p+2}}{(p-1)(p-2)} \right]$$

$$= \frac{1}{2} \left\{ \left[\frac{1}{(is-a)} \right] + \left[\frac{1}{(is+a)} \right] \right\} \left[\frac{y^{-p+2}}{(p-1)(p-2)} \right]$$

$$= \frac{1}{2} \left\{ \frac{2is}{-(s^2+a^2)} \right\} \left[\frac{y^{-p+2}}{(p-1)(p-2)} \right]$$

$$FS\{x \cos hat\} = \frac{-is}{(s^2+a^2)} \left[\frac{y^{-p+2}}{(p-1)(p-2)} \right]$$

2.11 If $FS\{f(t, x)\}$ denotes generalized Fourier-Stieltjes Transform of $f(t, x)$ then-

$$FS\{x \sin hat\} = \frac{-a}{(s^2+a^2)} \left[\frac{y^{-p+2}}{(p-1)(p-2)} \right]$$

Proof: We have

$$FS\{x \sin hat\} = \int_0^\infty \int_0^\infty x \sin hat e^{-ist} (x+y)^{-p} dt dx$$

$$= \int_0^\infty \sin hat e^{-ist} dt \int_0^\infty x (x+y)^{-p} dx$$

By using the result (2.5) we have,

$$= \int_0^\infty \left(\frac{e^{at}-e^{-at}}{2} \right) e^{-ist} dt \left[\frac{y^{-p+2}}{(p-1)(p-2)} \right]$$

$$= \frac{1}{2} \left\{ \int_0^\infty e^{-(is-a)t} dt - \int_0^\infty e^{-(a+is)t} dt \right\} \left[\frac{y^{-p+2}}{(p-1)(p-2)} \right]$$

$$= \frac{1}{2} \left\{ \left[\frac{e^{-(is-a)t}}{-(is-a)} \right]_0^\infty - \left[\frac{e^{-(a+is)t}}{-(a+is)} \right]_0^\infty \right\} \left[\frac{y^{-p+2}}{(p-1)(p-2)} \right]$$

$$= \frac{1}{2} \left\{ \left[\frac{1}{(is-a)} \right] - \left[\frac{1}{(is+a)} \right] \right\} \left[\frac{y^{-p+2}}{(p-1)(p-2)} \right]$$

$$= \frac{1}{2} \left\{ \frac{2a}{-(s^2+a^2)} \right\} \left[\frac{y^{-p+2}}{(p-1)(p-2)} \right]$$

$$FS\{x \sin hat\} = \frac{-a}{(s^2+a^2)} \left[\frac{y^{-p+2}}{(p-1)(p-2)} \right]$$

2.12 If $FS\{f(t, x)\}$ denotes generalized Fourier-Stieltjes Transform of $f(t, x)$ then-

$$FS\{f(t, x)\} \text{ If } f(t, x) = ax, \quad a < t < b, \quad 0 < x < c$$

$$= 0, \quad t > b, x > c$$

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Proof: We have

$$FS\{f(t, x)\} = \int_0^\infty \int_0^\infty f(t, x) e^{-ist} (x + y)^{-p} dt dx$$

The given function is discontinuous function. So, we have

$$\begin{aligned} &= \int_0^b \int_0^c f(t, x) e^{-ist} (x + y)^{-p} dt dx + \int_b^\infty \int_c^\infty f(t, x) e^{-ist} (x + y)^{-p} dt dx \\ &= \int_0^b \int_0^c ax e^{-ist} (x + y)^{-p} dt dx + \int_b^\infty \int_c^\infty 0 e^{-ist} (x + y)^{-p} dt dx \\ &= \int_0^b \int_0^c ax e^{-ist} (x + y)^{-p} dt dx \\ &= \int_0^b a e^{-ist} dt \int_0^c x (x + y)^{-p} dx \\ &= a \left[\frac{e^{-ist}}{-is} \right]_0^b \left[\left(x \frac{(x+y)^{-p+1}}{-p+1} \right)_0^c - \int_0^c \frac{(x+y)^{-p+1}}{-p+1} dx \right] \\ &= \frac{a}{is} [1 - e^{-isb}] \left[c \frac{(c+y)^{-p+1}}{-p+1} + \frac{1}{p-1} \int_0^c (x + y)^{-p+1} dx \right] \\ &= \frac{a}{is} [1 - e^{-isb}] \left[c \frac{(c+y)^{-p+1}}{-p+1} + \frac{1}{p-1} \left(\frac{(x+y)^{-p+2}}{-p+2} \right)_0^c \right] \\ &= \frac{a}{is} [1 - e^{-isb}] \left[c \frac{(c+y)^{-p+1}}{-p+1} + \frac{1}{p-1} \left[\frac{(c+y)^{-p+2}}{-p+2} + \frac{y^{-p+2}}{(p-2)} \right] \right] \\ &= \frac{a}{is} [1 - e^{-isb}] \left[c \frac{(c+y)^{-p+1}}{-(p-1)} + \frac{1}{(p-1)(p-2)} [-(c + y)^{-p+2} + y^{-p+2}] \right] \\ &= \frac{a}{is} [1 - e^{-isb}] \frac{1}{(p-1)} \left[-c (c + y)^{-p+1} + \frac{1}{(p-2)} [-(c + y)^{-p+2} + y^{-p+2}] \right] \\ FS\{f(t, x)\} &= \frac{a}{is} [1 - e^{-isb}] \frac{1}{(p-1)} \left[\frac{1}{(p-2)} [y^{-p+2} - (c + y)^{-p+2}] - c (c + y)^{-p+1} \right] \end{aligned}$$

$$\begin{aligned} \text{If } f(t, x) &= ax, \quad a < t < b, \quad 0 < x < c \\ &= 0, \quad t > b, x > c \end{aligned}$$

III. CONCLUSION

In the present paper, we have obtained the Fourier-Stieltjes transform of some functions.

REFERENCES

- [1] J. Berian James: The Integral Transform for you & me, Royal Observatory Edinburgh Institute for Astronomy, March 2008.
- [2] Bosi, M. and Goldberg R.E. *Introduction to Digital Audio Coding and Standards*, Boston: Kluwer Academic Publishers, (2003).
- [3] Gray, R.M. and Davisson, L.D. *An Introduction to Statistical Signal Processing*, Cambridge University Press, (2003).
- [4] Shubing Wang: Applications of Fourier Transform to Imaging Analysis, shubing@stat.wisc.edu, May 23, 2007.
- [5] Escofet, J., Millan, M.S. and Rallo, M. (2001). *Applied Optics*, Volume 40, Issue 34, pp. 6170- 6176.
- [6] Rowe, D.B., Nencka, A.S. and Hoffman, R.G. (2007). Signal and noise of Fourier reconstructed fMRI data, *Journal of Neuroscience Methods*, 159, pp. 361369.
- [7] Jack W. Silverstein: The Stieltjes Transform and its Role in Eigenvalue Behavior of Large Dimensional Random Matrices, Department of Mathematics North Carolina State University, 1982.
- [8] Romain Couillet and Merouane Debbah: Signal Processing in Large Systems: A New Paradigm , arXiv:1105.0060v3 [cs.IT] 19 Jun 2012.
- [9] Zemanian A.H.: "Generalized integral transformation", Inter science publisher, New York, 1965.
- [10] Gelfond I.M. and Shilov G.E.: "Generalized Function", Vol. II, Acad. Press, New York, 1968.
- [11] V.D. Sharma, P.D. Dolas: Generalization of Fourier-Stieltjes Transform, American Jr. of Mathematics and Sciences, Vol.2, No. 1, January 2013.
- [12] V.D. Sharma, P.D. Dolas: Operational calculus on Fourier-Stieltjes transform, Int. Jr. of Current Research, Vol. 8, Issue, 03, pp.27387-27391, March, 2016.
- [13] V.D.Sharma and A.N.Rangari: Fourier-Finite Mellin Transforms of Some Special Functions, International Journal of Advance Research in Computer Science and Management Studies, Volume 3, Issue 11, November 2015.