

Isomorphism and Anti-Isomorphism in Q-Fuzzy Translation of Q-Fuzzy Ternary Subhemiring of a Hemiring

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ABSTRACT: In this work, we presented the concept of Q-fuzzy translation of fuzzy ternary subhemiring and further, some important notions and basic algebraic properties are studied. The result of this study are an interesting and meaningful extension in fuzzy.

KEYWORDS: Q-Fuzzy Set, Hemiring, Subhemiring,, Fuzzy Subhemiring, Fuzzy Ternary Subhemiring, Q-Fuzzy Ternary Subhemiring, Q-Fuzzy Translation, Isomorphism and Anti-Isomorphism.

I. INTRODUCTION

There are several ideas of inclusive algebras generalizing an associative ring $(R; +; \cdot)$. Some of them specifically, nearrings and few sorts of semirings have been demonstrated extremely helpful. Semirings (called also half-rings) are algebras $(R; +; \cdot)$ have an indistinguishable properties from a ring with the exception of that $(R; +)$ is thought to be a semigroup rather than a commutative group. Semirings appear in a natural manner in a few applications to the theory of automata and formal languages. An algebra $(R; +; \cdot)$ is said to be a semiring if $(R; +)$ and $(R; \cdot)$ are semigroups fulfilling $a \cdot (b + c) = a \cdot b + a \cdot c$ and $(b + c) \cdot a = b \cdot a + c \cdot a$ for every one of the a, b and c in R . A semiring R is said to be a additively commutative if $a + b = b + a$ for all a and b in R . A semiring R may have an identity 1 , characterized by $1 \cdot a = a = a \cdot 1$ and a zero 0 , characterized by $0 + a = a = a + 0$ and $a \cdot 0 = 0 = 0 \cdot a$ for every one of the a in R . A Semiring R is said to be a hemiring if it is an additively commutative with zero. After the introduction of fuzzy sets by L.A.Zadeh [18], numerous researchers explored on the speculation of the idea of fuzzy sets. Selvakumaraen.N and Arjunan.K in [14] have talk about the Homomorphism in Intuitionistic L-Fuzzy structure. Likewise Giri.R.D. and Chide.B.R. in [6] have presents the structure of Ternary Hemiring. In this paper, we introduce a few Theorems in Q-Fuzzy Ternary Subhemiring of a Hemiring.

II. PRELIMINARIES

1.1. Definition: Let X be a non-empty set and Q be a non-empty set. A Q-Fuzzy subset A of X is a function $A: X \times Q \rightarrow [0, 1]$.

1.2. Definition: The union of two Q-Fuzzy subsets A and B of a set X is defined by $(A \cup B)(x, q) = \max \{A(x, q), B(x, q)\}$, for all x in X and q in Q .

International Journal of Innovative Research in Science, Engineering and Technology

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1.3.Definition: The intersection of two Q-Fuzzy subsets A and B of a set X is defined by $(A \cap B)(x, q) = \min \{A(x, q), B(x, q)\}$, for all x in X and q in Q.

1.4.Definition: Let $(R, +, \cdot)$ be a ternary hemiring. A Q-Fuzzy subset A of R is said to be a Q-Fuzzy ternary Subhemiring (QFSHR) of R if it satisfies the following conditions:

$$i) \mu_A(x + y, q) \geq \min \{ \mu_A(x, q), \mu_A(y, q) \}$$

$$ii) \mu_A(xyz, q) \geq \min \{ \mu_A(x, q), \mu_A(y, q), \mu_A(z, q) \}, \text{ for all } x, y \text{ and } z \text{ in } R \text{ and } q \text{ in } Q.$$

1.5.Definition: Let R be a Ternary hemiring. A fuzzy subset A of R is said to be a Q-Fuzzy Ternary Subhemiring of R if it fulfills the following conditions:

$$i) \mu_A(x + y, q) \geq \min \{ \mu_A(x, q), \mu_A(y, q) \}$$

$$ii) \mu_A(xyz, q) \geq \min \{ \mu_A(x, q), \mu_A(y, q), \mu_A(z, q) \}, \text{ for all } x, y \text{ and } z \text{ in } R \text{ and } q \text{ in } Q.$$

1.6.Definition: Let $(R, +, \cdot)$ be a Ternary hemiring. A Q-Fuzzy Ternary Subhemiring A of R is said to be a Q-Fuzzy Normal Ternary Subhemiring (QFNTSHR) of R if $\mu_A(xyz, q) = \mu_A(zyx, q)$, for all x, y and z in R and q in Q.

1.7.Definition: Let $(R, +, \cdot)$ and $(R^1, +, \cdot)$ be any two Ternary hemirings. Then the function $f : R \rightarrow R^1$ is called a Ternary hemiring Homomorphism if it fulfills the following axioms:

$$i) f(x + y) = f(x) + f(y)$$

$$ii) f(xyz) = f(x)f(y)f(z), \text{ for all } x, y \text{ and } z \text{ in } R.$$

1.8.Definition: Let $(R, +, \cdot)$ and $(R^1, +, \cdot)$ be any two Ternary hemirings. Then the function $f : R \rightarrow R^1$ is called a Ternary hemiring Anti-Homomorphism if it satisfies the following axioms:

$$i) f(x + y) = f(y) + f(x)$$

$$ii) f(xyz) = f(z)f(y)f(x), \text{ for all } x, y \text{ and } z \text{ in } R.$$

1.9.Definition: Let $(R, +, \cdot)$ and $(R^1, +, \cdot)$ be any two Ternary hemirings. Then the function $f : R \rightarrow R^1$ be a Ternary Hemiring Homomorphism. If f is one to one and onto, then f is called a Ternary Hemiring Isomorphism.

1.10.Definition: Let $(R, +, \cdot)$ and $(R^1, +, \cdot)$ be any two Ternary hemirings. Then the function $f : R \rightarrow R^1$ be a Ternary Hemiring Anti-Homomorphism. If f is one to one and onto, then f is called a Ternary Hemiring Anti-Isomorphism.

1.11.Definition: Let A be a Q-Fuzzy subset of X and $\alpha \in [0, 1 - \sup \{A(x, q) : x \in X, 0 < A(x, q) < 1\}]$. Then

$T = T_{\alpha}^A$ is called a Q-Fuzzy Translation of A if $T(x, q) = A(x, q) + \alpha$, for all x in X.

1.12.Example: Consider the set $X = \{0, 1, 2, 3, 4\}$ and $Q = \{q\}$. Let

$A = \{((0, q), 0.5), ((1, q), 0.4), ((2, q), 0.6), ((3, q), 0.45), ((4, q), 0.2)\}$ be a Q-Fuzzy subset of X and $\alpha = 0.25$.

The Q-Fuzzy Translation of A is

$$T = T_{0.25}^A = \{((0, q), 0.75), ((1, q), 0.65), ((2, q), 0.85), ((3, q), 0.7), ((4, q), 0.45)\}.$$

International Journal of Innovative Research in Science, Engineering and Technology

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III.ISOMORPHISM AND ANTI-ISOMORPHISM IN Q-FUZZY TRANSLATION OF Q-FUZZY TERNARY SUBHEMIRING OF A HEMIRING.

2.1.Theorem: Let $(R, +, \cdot)$ and $(R^1, +, \cdot)$ be any two Ternary Hemirings. The Q-Fuzzy normal ternary subhemiring of $f(R) = R^1$ under the anti-homomorphic preimage is a Q-Fuzzy normal ternary subhemiring of R .

Proof: Let $(R, +, \cdot)$ and $(R^1, +, \cdot)$ be any two Ternary Hemirings and $f : R \rightarrow R^1$ be an anti-homomorphism. Then $f(x+y) = f(y) + f(x)$ and $f(xyz) = f(z)f(y)f(x)$, for all x, y and z in R . Let V be a Q-Fuzzy normal ternary subhemiring of a hemiring $f(R) = R^1$ and A be an anti-homomorphic pre-image of V under f . We need to prove that A is a Q-Fuzzy normal ternary subhemiring of a hemiring R . Let x, y and z in R and q in Q , then obviously A is a Q-Fuzzy ternary subhemiring of a hemiring R , since V is a Q-Fuzzy ternary subhemiring of a hemiring R^1 . Presently, $\mu_A(xyz, q) = \mu_V(f(xyz), q)$, since $\mu_A(x, q) = \mu_V(f(x), q)$, $= \mu_V(f(z)f(y)f(x), q)$, as f is an anti-homomorphism $= \mu_V(f(x)f(y)f(z), q) = \mu_V(f(zyx), q)$, as f is an anti-homomorphism $= \mu_A(zyx, q)$, since $\mu_A(x, q) = \mu_V(f(x), q)$ which suggests that $\mu_A(xyz, q) = \mu_A(zyx, q)$, for all x, y and z in R and q in Q . Thus A is a Q-Fuzzy normal ternary subhemiring of the hemiring R .

In the following Theorem \circ is the composition operation of functions:

2.2.Theorem: Let A be a Q-Fuzzy ternary subhemiring of a hemiring H and f is a isomorphism from a hemiring R onto H . If A is a Q-Fuzzy normal ternary subhemiring of the hemiring H , then $A \circ f$ is a Q-Fuzzy normal ternary subhemiring of the hemiring R .

Proof: Let x, y and z in R and q in Q and A be a Q-Fuzzy normal ternary subhemiring of a hemiring H . After that we have, obviously $A \circ f$ is a Q-Fuzzy ternary subhemiring of the hemiring R . At the present $(\mu_{A \circ f})(xyz, q) = \mu_A(f(xyz), q) = \mu_A(f(x)f(y)f(z), q)$, as f is an isomorphism $= \mu_A(f(z)f(y)f(x), q) = \mu_A(f(zyx), q)$, as f is an isomorphism $= (\mu_{A \circ f})(zyx, q)$, which suggests that $(\mu_{A \circ f})(xyz, q) = (\mu_{A \circ f})(zyx, q)$, for all x, y and z in R and q in Q . Therefore $A \circ f$ is a Q-Fuzzy normal ternary subhemiring of the hemiring R .

2.3.Theorem: Let A be a Q-Fuzzy ternary subhemiring of a hemiring H and f is an anti-isomorphism from a hemiring R onto H . If A is a Q-Fuzzy normal ternary subhemiring of the hemiring H , then $A \circ f$ is a Q-Fuzzy normal ternary subhemiring of the hemiring R .

Proof: Let x, y and z in R and q in Q and A be a Q-Fuzzy normal ternary subhemiring of a hemiring H . Subsequently we have, obviously $A \circ f$ is a Q-Fuzzy ternary subhemiring of the hemiring R . At the present $(\mu_{A \circ f})(xyz, q) = \mu_A(f(xyz), q) = \mu_A(f(z)f(y)f(x), q)$, as f is an anti-isomorphism $= \mu_A(f(x)f(y)f(z), q) = \mu_A(f(zyx), q)$, as f is an anti-isomorphism $= (\mu_{A \circ f})(zyx, q)$, which suggests that $(\mu_{A \circ f})(xyz, q) =$

International Journal of Innovative Research in Science, Engineering and Technology

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$(\mu_A \circ f)(zyx, q)$, for all x, y and z in R and q in Q . Thus $A \circ f$ is a Q-Fuzzy normal ternary subhemiring of the hemiring R .

2.4.Theorem: If M and N are two Q-Fuzzy translation of Q-fuzzy normal ternary subhemiring A of a hemiring $(R, +, \cdot)$, then their intersection $M \cap N$ is a Q-Fuzzy translation of A .

Proof: It is insignificant.

2.5.Theorem: The intersection of a family of Q-Fuzzy translation of Q-Fuzzy normal ternary subhemiring A of a hemiring $(R, +, \cdot)$ is a Q-Fuzzy translation of A .

Proof: It is insignificant.

2.6.Theorem: If M and N are two Q-Fuzzy translation of Q-fuzzy normal ternary subhemiring A of a hemiring $(R, +, \cdot)$, then their union $M \cup N$ is a Q-Fuzzy translation of A .

Proof: It is insignificant.

2.7.Theorem: The union of a family of Q-Fuzzy translation of Q-Fuzzy normal ternary subhemiring A of a hemiring $(R, +, \cdot)$ is a Q-Fuzzy translation of A .

Proof: It is insignificant.

2.8.Theorem: Let $(R, +, \cdot)$ and $(R^1, +, \cdot)$ be any two Ternary Hemirings and Q be a non-empty set. If $f : R \rightarrow R^1$ is a homomorphism, then the Q-Fuzzy translation of a Q-Fuzzy normal ternary subhemiring A of R under the homomorphic image is a Q-Fuzzy normal ternary subhemiring of $f(R) = R^1$.

Proof: Let $(R, +, \cdot)$ and $(R^1, +, \cdot)$ be any two Ternary Hemirings and Q be a non-empty set and $f : R \rightarrow R^1$ be a homomorphism. That is $f(x + y) = f(x) + f(y)$ and $f(xyz) = f(x)f(y)f(z)$, for all x, y and z in R . Let $T = T_\alpha^A$ be the Q-Fuzzy translation of a Q-Fuzzy normal ternary subhemiring A of R and V be the homomorphic image of T under f . We have to verify that V is a Q-Fuzzy normal ternary subhemiring of R^1 . At present, for $f(x), f(y)$ and $f(z)$ in R^1 and q in Q , obviously V is a Q-Fuzzy ternary subhemiring of R^1 . We have

$$V(f(x)f(y)f(z), q) = V(f(xyz), q) \geq T(xyz, q) = A(xyz, q) + \alpha = A(zyx, q) + \alpha = T(zyx, q)$$

$$\leq V(f(zyx), q) = V(f(z)f(y)f(x), q), \quad \text{which suggests that}$$

$$V(f(x)f(y)f(z), q) = V(f(z)f(y)f(x), q), \text{ for all } f(x), f(y) \text{ and } f(z) \text{ in } R^1 \text{ and } q \text{ in } Q. \text{ Hence, } V \text{ is a Q-Fuzzy normal ternary subhemiring of the hemiring } R^1.$$

2.9.Theorem: Let $(R, +, \cdot)$ and $(R^1, +, \cdot)$ be any two Ternary Hemirings and Q be a non-empty set. If $f : R \rightarrow R^1$ is a homomorphism, then the Q-Fuzzy translation of a Q-Fuzzy normal ternary subhemiring V of $f(R) = R^1$ under the homomorphic pre-image is a Q-Fuzzy normal ternary subhemiring of R .

Proof: Let $(R, +, \cdot)$ and $(R^1, +, \cdot)$ be any two Ternary Hemirings and Q be a non-empty set and $f : R \rightarrow R^1$ be a homomorphism. That is $f(x + y) = f(x) + f(y)$ and $f(xyz) = f(x)f(y)f(z)$, for all x, y and z in R . Let $T = T_\alpha^V$ be the Q-Fuzzy translation of Q-Fuzzy normal ternary subhemiring V of R^1 and A be the homomorphic pre-

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image of T under f. We have to prove that A is a Q-Fuzzy normal ternary subhemiring of R. Let x, y and z be in R and q in Q. Evidently A is a Q-fuzzy ternary subhemiring of R, $A(xyz, q) = T(f(xyz), q) = V(f(xyz), q) + \alpha = V(f(x)f(y)f(z), q) + \alpha = V(f(z)f(y)f(x), q) + \alpha = V(f(zyx), q) + \alpha = T(f(zyx), q) = A(zyx, q)$, which implies that $A(xyz, q) = A(zyx, q)$, for all x, y and z in R and q in Q. Hence, A is a Q-Fuzzy normal ternary subhemiring of R.

2.10.Theorem: Let $(R, +, \cdot)$ and $(R^1, +, \cdot)$ be any two Ternary Hemirings and Q be a non-empty set. If $f : R \rightarrow R^1$ is an anti-homomorphism, then the Q-Fuzzy translation of a Q-Fuzzy normal ternary subhemiring A of R under the anti-homomorphic image is a Q-Fuzzy normal ternary subhemiring of R^1 .

Proof: Let $(R, +, \cdot)$ and $(R^1, +, \cdot)$ be any two Ternary Hemirings and Q be a non-empty set and $f : R \rightarrow R^1$ be an anti-homomorphism. That is $f(x+y) = f(y) + f(x)$ and $f(xyz) = f(z)f(y)f(x)$, for all x, y and z in R. Let T_α^V be the Q-Fuzzy translation of Q-Fuzzy normal ternary subhemiring A of R and V be the anti-homomorphic image of T_α^V under f. We have to prove that V is a Q-Fuzzy normal ternary subhemiring of $f(R) = R^1$. Now, for f(x), f(y) and f(z) in R^1 and q in Q, clearly V is a Q-fuzzy ternary subhemiring of R^1 . We have, $V(f(x)f(y)f(z), q) = V(f(zyx), q) \geq T(zyx, q) = A(zyx, q) + \alpha = A(xyz, q) + \alpha = T(xyz, q) \leq V(f(xyz), q) = V(f(z)f(y)f(x), q)$ which implies that $V(f(x)f(y)f(z), q) = V(f(z)f(y)f(x), q)$, for f(x), f(y) and f(z) in R^1 and q in Q. Hence, V is a Q-Fuzzy normal ternary subhemiring of the hemiring R^1 .

2.11.Theorem: Let $(R, +, \cdot)$ and $(R^1, +, \cdot)$ be any two Ternary Hemirings and Q be a non-empty set. If $f : R \rightarrow R^1$ is an anti-homomorphism, then the Q-Fuzzy translation of a Q-Fuzzy normal ternary subhemiring V of $f(R) = R^1$ under the anti-homomorphic pre-image is a Q-Fuzzy normal ternary subhemiring of R.

Proof: Let $(R, +, \cdot)$ and $(R^1, +, \cdot)$ be any two Ternary Hemirings and Q be a non-empty set and $f : R \rightarrow R^1$ be an anti-homomorphism. That is $f(x+y) = f(y) + f(x)$ and $f(xyz) = f(z)f(y)f(x)$, for all x, y and z in R. Let $T = T_\alpha^V$ be the Q-Fuzzy translation of Q-Fuzzy normal ternary subhemiring V of R^1 and A be the anti-homomorphic pre-image of T under f. We have to prove that A is a Q-Fuzzy normal ternary subhemiring of R. Let x, y and z be in R and q in Q. Then, clearly A is a Q-fuzzy ternary subhemiring of R, $A(xyz, q) = T(f(xyz), q) = V(f(xyz), q) + \alpha = V(f(z)f(y)f(x), q) + \alpha = V(f(x)f(y)f(z), q) + \alpha = V(f(zyx), q) + \alpha = T(f(zyx), q) = A(zyx, q)$, which suggests that $A(xyz, q) = A(zyx, q)$, for all x, y and z in R and q in Q. Hence, A is a Q-Fuzzy normal ternary subhemiring of R.

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IV. CONCLUSION

In the study of fuzzy algebraic system, fuzzy hemirings with some special properties always play an important role. In this paper we study those hemirings translation in Q-fuzzy.

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