Constraints of Leptogenesis on the Neutrino Mass Models

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ABSTRACT: The constraints imposed by leptogenesis on the validity of neutrino mass models describing the presently available neutrino mass patterns, are investigated numerically. We first parameterize the light left-handed Majorana neutrino mass matrices describing the possible patterns of neutrino masses viz, quasi-degenerate, inverted hierarchical and normal hierarchical, which obey the $\mu \leftrightarrow \tau$ symmetry. As a first test, these mass matrices predict the neutrino mass parameters and mixings consistent with neutrino oscillation data, and all the three input parameters are fixed at this stage. Next, these mass matrices are employed to estimate the baryon asymmetry of the universe, in both thermal and non-thermal leptogenesis scenario. We use the CP violating Majorana phases derived from right-handed Majorana mass matrix, and also three possible forms of Dirac neutrino mass matrices in the calculation. The overall analysis shows that normal hierarchical model appears to be the most favourable choice in nature. The present analysis though phenomenological, may serve as additional criteria to discard some of the presently available neutrino mass models. The conclusion also agrees with other criteria such as stability under quantum radiative corrections. We also observe some enhancement effects in flavour leptogenesis compared to non-flavour leptogenesis. Further enhancement from brane world cosmology may marginally modify the present finding.

KEYWORDS: Leptogenesis, baryon asymmetry, neutrino masses, Majorana right-handed neutrinos.

I. INTRODUCTION

A tiny nonzero neutrino masses are bonafide evidence for physics beyond the SM. After 20 years of intense experimental and theoretical research, we still do not know the physics that leads to nonzero neutrino masses. The mechanism behind neutrino masses is still unknown. There are many different ideas in the literature. The different models are qualitatively different; they contain distinct ingredients and make divergent predictions about the nature of the neutrino. In some scenarios, neutrinos are Dirac fermions and their masses are generated in the same way that all other fermion masses are generated. In these types of scenarios, neutrino masses are very small because neutrinos are very weakly coupled to the source of electroweak symmetry breaking, the SM Higgs field. In many other scenarios, neutrinos are Majorana fermions. This might be a consequence of a more complicated electroweak symmetry–breaking sector—more Higgs-like bosons—or maybe indicate the existence of mass scales in nature that are not related to the electroweak scale. In these types of scenarios, new particles must exist. They may be as light as $1 \text{ eV}$ or as heavy as $10^{15} \text{ GeV}$. Understanding the fate of lepton number is of the utmost importance, as it will help reveal whether neutrinos are Majorana or Dirac fermions. This, in turn, will point neutrino mass model building in different directions. The most powerful probes of lepton number conservation are searches for neutrinoless double-$\beta$ decay [1]. Precision measurements of charged-lepton properties and searches for charged-lepton flavor violation could also provide information about the origin of neutrino masses. Neutrinos and charged leptons are intimately related, and there are several scenarios in which the physics responsible for nonzero neutrino masses manifests itself in charged-lepton flavor violation [2] at experimentally accessible levels [3]. High-energy collider experiments, including those at Large hadron colliders, may also directly produced and observed the mechanism behind nonzero neutrino masses. Similarly, cosmic surveys, and other astrophysical probes, will continue to explore, indirectly, the properties of the cosmic neutrino background, and they have the potential to access information about neutrino properties that cannot be accessed in the laboratory [4].
To date, however, all positive information about nonzero neutrino masses has come from neutrino oscillations. It is highly anticipated that these will continue to provide important clues about the origin of neutrino masses and lepton flavor. A comprehensive program to explore the physics of neutrino oscillations is currently being developed [5], so the future looks very promising. Neutrino oscillations will reveal, for example, the ordering of the neutrino masses outlined in Section II, i.e., C and CP violation, as they can have an asymmetric decay to leptons ($l_L$) and Higgs particles ($\phi$), and the process occurs at different rates for particles and antiparticles. The lepton asymmetry thus generated, is then partially converted to baryon asymmetry through the non-perturbative electroweak sphaleron effects [9, 10]. In case of thermal leptogenesis, the right-handed neutrinos can be generated thermally after inflation, if their masses are comparable to or below the reheating temperature $T_R$. This allows high scale reheating temperature $T_R \geq 10^9$ GeV [11]. In non-thermal leptogenesis [12] it is also possible to produce lepton asymmetry by using a low reheating temperature, where the right-handed neutrinos are produced through the direct non-thermal decays of the inflaton fields ($\phi$). This is particularly important for super symmetric models where graviton problem [13] can be avoided, provided the reheating temperature after inflation, is bounded from above in a certain way, namely $T_R \leq (10^9 - 10^7)$ GeV.

Paper is organized as follows. Section II, describes briefly the theoretical formalism of thermal and non-thermal leptogenesis for estimating the lepton asymmetry through out-of-equilibrium decay of the heavy right-handed Majorana neutrinos. This is followed by various neutrino mass models $m_{LL}$ which obey $\mu$-symmetry given in Section III. Numerical analysis and predictions are outlined in Section IV. Finally, Section V presents conclusion.

II. THEORETICAL FORMALISM OF LEPTOGENESIS

For detailed theoretical formalism of both thermal and non-thermal leptogenesis, we refer the readers to our earlier work [14].

III. CLASSIFICATION OF NEUTRINO MASS MODELS

Realistic neutrino mass models which obey $\mu$-symmetry are constructed from their zeroth-order (completely degenerate) mass models $m^0_{LL}$ by adding a suitable perturbative term $\Delta m_{LL}$, having two additional free parameters. This approach gives a firm handle on numerical solution of the mass matrices by reducing the unknown parameters from four present in a general $\mu$-$\tau$ symmetric mass matrix, to only three in the present case. These mass matrices are again applied to the calculation for baryon asymmetry of the universe. Left-handed Majorana neutrino mass matrices which obey $\mu$-$\tau$ symmetry [15], have the following form.
which predicts an arbitrary solar mixing angle \( \tan^2 \theta_{12} = \frac{2\sqrt{2}y}{(x-z-w)} \), while the predictions on atmospheric mixing angle is maximal (\( \theta_{23} = \pi/4 \)) and that of Chooz angle is zero. In order to lift the mass degeneracy in \( m_{LL} \), we then add a suitable perturbation in the form of the parametrisation of mass matrices (with only two parameters) whereby the solar mixing is fixed at a particular value, for all possible patterns of neutrino mass models, i.e., \( m_{LL} = m_{LL}^0 + \Delta m_{LL} \). The values of three input parameters are fixed by the predictions on neutrino masses and mixings in Table 1, which are consistent with neutrino oscillation data. The following neutrino mass models are studied in the present investigation:

**Quasi-degenerate model [IA]** with \( \text{Diag}(m_{1}, m_{2}, m_{3}) \):

The zeroth-order mass matrix,

\[
m_{LL}^0 = \begin{pmatrix}
0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\
\frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2}
\end{pmatrix} m_0
\]

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\frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\
\frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2}
\end{pmatrix} m_0
\]

this leads to degenerate eigenvalues \( \text{Diag}(1, -1, 1) m_0 \), is now modified to

\[
\begin{pmatrix}
\Delta m_{12}^2 & \Delta m_{13}^2 \\
\Delta m_{23}^2 & \Delta m_{33}^2
\end{pmatrix} = \begin{pmatrix}
7.82 & 2.20 \\
7.62 & 2.49 \\
7.62 & 2.49
\end{pmatrix}
\]

Table 1 Predicted values of the solar and atmospheric neutrino mass-squared differences for \( \tan^2 \theta_{12} = 0.45 \), using \( m_{LL}^0 \) in section III.

}\[
\begin{pmatrix}
\Delta m_{12}^2 & \Delta m_{13}^2 \\
\Delta m_{23}^2 & \Delta m_{33}^2
\end{pmatrix} = \begin{pmatrix}
7.91 & 2.35 \\
8.40 & 2.03 \\
7.53 & 2.45
\end{pmatrix}
\]

\[
\begin{pmatrix}
\tan^2 \theta_{12} & \tan^2 \theta_{23} & \sin \theta_{13}
\end{pmatrix} = \begin{pmatrix}
0.45 & 1.0 & 0.0 \\
0.45 & 1.0 & 0.0 \\
0.45 & 1.0 & 0.0
\end{pmatrix}
\]
The input values of the parameters are fixed at \( \epsilon = 0.66115 \), \( \eta = 0.16535 \), \( m_0 = 0.4eV \) (for \( \tan^2 \theta_{12} = 0.45 \) we take \( a = 0.868 \) and \( b = 1.025 \)).

**Quasi-degenerate model [IB]** with Diag\((m_1,m_2,m_3)\):

\[
\begin{pmatrix}
\epsilon - 2\eta & -a\epsilon & -a\epsilon \\
-a\epsilon & 1 - b\eta & 1 - \eta \\
-a\epsilon & \frac{1}{2} - \eta & \frac{1}{2} - b\eta
\end{pmatrix} m_0
\]

which gives degenerate eigenvalues Diag\((1,1,1)\) is modified to

\[
\begin{pmatrix}
1 - \epsilon - 2\eta & a\epsilon & a\epsilon \\
a\epsilon & 1 - b\eta & -\eta \\
a\epsilon & -\eta & 1 - b\eta
\end{pmatrix} m_0
\]

with input values: \( \epsilon = 8.314 \times 10^{-5} \), \( \eta = 0.00395 \), \( m_0 = 0.4eV \) (for \( \tan^2 \theta_{12} = 0.45 \) we take \( a = 0.945 \) and \( b = 0.998 \)).

**Quasi-degenerate model [IC]** with Diag\((m_1,m_2,-m_3)\):

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix} m_0
\]

which gives degenerate eigenvalues Diag\((1,1,-1)\) are now modified to a common form,

\[
\begin{pmatrix}
\epsilon - 2\eta & -a\epsilon & -a\epsilon \\
-a\epsilon & -b\eta & 1 - b\eta \\
-a\epsilon & 1 - b\eta & -b\eta
\end{pmatrix} m_0
\]

with input values: \( \epsilon = 8.314 \times 10^{-5} \), \( \eta = 0.00395 \), \( m_0 = 0.4eV \) (for \( \tan^2 \theta_{12} = 0.45 \) we take \( a = 0.945 \) and \( b = 0.998 \)).

**Inverted Hierarchical model [II]** with Diag\((m_1,m_2,-m_3)\):

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & \frac{1}{2} & \frac{1}{2} \\
0 & \frac{1}{2} & \frac{1}{2}
\end{pmatrix} m_0
\]

which gives degenerate eigenvalues Diag\((1,1,0)\) , and
\[ m_0^L = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} m_0 \]

which gives degenerate eigenvalues \( \text{Diag}(1,-1,0) m_0 \) are now modified to a common form,

\[ m_0^L(\text{IH}) = \begin{pmatrix} 1 - 2 \epsilon & -\epsilon & -\epsilon \\ -\epsilon & 1 - \frac{1}{2} \eta & 1 + \frac{1}{2} \eta \\ -\epsilon & 1 + \frac{1}{2} \eta & 1 - \frac{1}{2} \eta \end{pmatrix} m_0. \]

Table 2: Lightest right-handed Majorana neutrino mass \( M_1 \) and values of CP asymmetry and baryon asymmetry for quasi-degenerate models (IA, IB, and IC), inverted models (IIA, IIB) and normal hierarchical models (III), with \( \tan^2 \theta_{12} = 0.45 \), using neutrino mass matrices given in the text. The entry \((m,n)\) in \( m_{\nu,\nu} \) indicates the type of Dirac neutrino mass matrix taken as charged lepton mass matrix (6,2) or up quark mass matrix (8,4) or down quark mass matrix (4,2) as explained in the text.

(a) Inverted Hierarchy with even CP parity in the first two mass eigenvalues [IIA],
\[
(m_1 = m_2, m_3): \eta/\epsilon = 1.0, \eta = 0.0048, m_0 = 0.05\text{eV}.
\]

(b) Inverted Hierarchy with odd CP parity in the first two mass eigenvalues [IIB],
\[
(m_1 = m_3 - m_2, m_3): \eta/\epsilon = 1.0, \eta = 0.6607, m_0 = 0.05\text{eV}.
\]

Normal Hierarchy model [III]

\[
m_0^L(\text{IH}) = \begin{pmatrix} 0 & -\epsilon & -\epsilon \\ -\epsilon & 1 - \epsilon & 1 + \eta \\ -\epsilon & 1 + \eta & 1 - \epsilon \end{pmatrix} m_0.
\]
with input values: \( \eta/\epsilon = 0.0, \epsilon = 0.146, m_\theta = 0.028 \text{ eV} \). The above mass matrices have the potential to decrease the solar mixing angle from the tri-bimaximal value, without sacrificing \( \mu-\tau \) symmetry, through the identification of a flavour twister term \( \eta/\epsilon \).

### IV. NUMERICAL CALCULATIONS AND RESULTS

To compute the numerical analysis, we first choose the light left-handed Majorana neutrino mass matrix \( m_\nu \) presented in section III. These mass matrices obey the \( \mu-\tau \) symmetry [15] which guarantees the deviation of solar angle from tri-bimaximal mixing [16]. The three input parameters are fixed at the stage of predictions of neutrino mass parameters and mixing given in Table 1. These results are consistent with the recent data on neutrino oscillation parameters. For the calculation of baryon asymmetry, we then translate these mass matrices to \( M_{\nu RR} \) via the inversion of the seesaw formula, \( M_{\nu RR} = - m_{LR}^T m_{\nu}^T m_{LR} \). We choose a basis \( U_R \) where \( M_{\nu RR}^\text{diag} = U_R^\dagger M_{\nu RR} U_R = \text{diag}(M_1, M_2, M_3) \) with real and positive eigenvalues. We then transform diagonal form of Dirac mass matrix,

<table>
<thead>
<tr>
<th>TYPE</th>
<th>(m,n)</th>
<th>( T_R \leq T_{\nu} \leq T_{\nu_{\text{max}}} ) (GeV)</th>
<th>( M_{\phi}^\text{min} &lt; M_{\phi} &lt; M_{\phi}^\text{max} ) (GeV)</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>IA</td>
<td>(4,2)</td>
<td>( 1.2 \times 10^6 &lt; T_R \leq 5.4 \times 10^8 )</td>
<td>( 1.1 \times 10^6 &lt; M_{\phi} \leq 4.5 \times 10^6 )</td>
<td>( 7.3 \times 10^6 &lt; M_{\phi} \leq 2.6 \times 10^6 )</td>
</tr>
<tr>
<td>IA</td>
<td>(6,2)</td>
<td>( 6.0 \times 10^9 &lt; T_R \leq 7.5 \times 10^9 )</td>
<td>( 6.4 \times 10^9 &lt; M_{\phi} \leq 7.8 \times 10^9 )</td>
<td>( 8.0 \times 10^9 &lt; M_{\phi} \leq 7.5 \times 10^9 )</td>
</tr>
<tr>
<td>IC</td>
<td>(4,2)</td>
<td>( 8.9 \times 10^3 &lt; T_R \leq 5.0 \times 10^7 )</td>
<td>( 9.0 \times 10^3 &lt; M_{\phi} \leq 6.0 \times 10^7 )</td>
<td>( 6.6 \times 10^3 &lt; M_{\phi} \leq 3.4 \times 10^7 )</td>
</tr>
<tr>
<td>IIA</td>
<td>(4,2)</td>
<td>( 1.2 \times 10^{13} &lt; T_R \leq 5.0 \times 10^{13} )</td>
<td>( 1.1 \times 10^{12} &lt; M_{\phi} \leq 2.8 \times 10^{12} )</td>
<td>( 6.5 \times 10^9 &lt; M_{\phi} \leq 1.8 \times 10^9 )</td>
</tr>
<tr>
<td>III</td>
<td>(4,2)</td>
<td>( 4.0 \times 10^6 &lt; T_R \leq 3.7 \times 10^{10} )</td>
<td>( 2.0 \times 10^6 &lt; M_{\phi} \leq 7.0 \times 10^{10} )</td>
<td>( 7.5 \times 10^6 &lt; M_{\phi} \leq 7.0 \times 10^{10} )</td>
</tr>
</tbody>
</table>

Table 3 Theoretical bound on reheating temperature \( T_R \) and inflaton masses \( M_\phi \) for non-thermal leptogenesis, for all neutrino mass models within \( U_R = 0.45 \).

\( m_{LR} = \text{diag}(\lambda^m, \lambda^n, 1) \nu \) to the \( U_R \) basis: \( m_{LR} \rightarrow m_{LR} = m_{LR} U_R Q \), where \( Q = \text{diag}(1, e^{i\alpha}, e^{i\beta}) \) is the complex matrix containing CP-violating Majorana phases derived from \( M_{\nu RR} \). Here \( \lambda \) is the Wolfenstein parameter and the choice \( (m, n) \) in \( m_{LR} \) gives the type of Dirac neutrino mass matrix. At the moment we consider phenomenologically three possible forms of Dirac neutrino mass matrix such as (i) \( (m,n) = (4,2) \) for the down-quark mass matrix, (ii) \( (6,2) \) for the charged-lepton type mass matrix, and (iii) \( (8,4) \) for up-quark type mass.
matrix. In this prime basis the Dirac neutrino Yukawa coupling becomes $h = \frac{m_1^2}{v}$ which enters in the expression of CP-asymmetry $\epsilon$, in (3) and (12). The new Yukawa coupling matrix $h$ also becomes complex, and hence the term $Im(h^\dagger h)_{ij}$ appears in lepton asymmetry $\epsilon$ gives a non-zero contribution. A straightforward simplification shows that $(h^\dagger h)_{ij}^2 = (Q_{11})^2 Q_{22}^2 R_1 + (Q_{11})^2 Q_{33}^2 R_2$ where $R_{2,3}$ are real parameters. After inserting the values of phases the above expression leads to $Im(h^\dagger h)_{ij}^2 = -[R_2 \sin(\alpha - \beta) + R_3 \sin 2\alpha]$ imparts non-zero CP asymmetry for particular choice of $(\alpha, \beta)$. 

In our numerical estimation of lepton asymmetry, we choose some arbitrary values of $\alpha$ and $\beta$ other than $\pi/2$ and 0. For example, light neutrino masses $(m_1-m_2-m_3)$ leads to $M_{\text{diag}} = diag(M_1, -M_2, M_3)$, and we thus fix the Majorana phase $Q = diag (1, e^{i\alpha}, e^{i\beta}) = diag (1, e^{i(\pi/2+n/4)}, e^{i\pi/4})$ for $\alpha = (\pi/2 + n/4)$ and $\beta = \pi/4$. 

The extra phase $\pi/2$ in $T_{R_{2,3}}$ absorbs the negative sign before heavy Majorana mass $M_{\Phi}$.

The baryons asymmetry $Y_B = \frac{n_B}{s}$ is taken as input value from WMAP observational data. Only those neutrino mass models which simultaneously satisfy the two constraints, $T_{R_{\text{max}}} > T_{R_{\text{min}}}$ and $M^{\text{max}}_{\Phi} > M^{\text{min}}_{\Phi}$, could survive in the non-thermal leptogenesis scenario. Certain inflationary models such as chaotic or natural inflation, predict the inflaton mass $M_{\Phi} \sim 10^{13}$ GeV, and from Table 3, the neutrino mass models with $(m, n)$ which are compatible with $M_{\Phi} \sim 10^{13}$ GeV, are listed as IA-(4,2), IIB-(4,2) and III-(6,2) respectively. Again in order to avoid gravitino problem [cf.13] in supersymmetric models, one has the bound on reheating temperature, $T_R \approx (10^6 - 10^7)$ GeV. This leads to we allowed models as IA-(4,2), IIB-(4,2) and III-(6,2) respectively. If we prefer the Dirac neutrino mass matrix as either charged lepton or up-quark mass matrix, then we have only one allowed model III-(6,2) in the list.

V. CONCLUSION

To conclude, we first parameterized the light left-handed Majorana neutrino mass matrices describing the possible patterns of neutrino masses viz, quasi-degenerate, inverted hierarchical and normal hierarchical, which obey the $\mu \leftrightarrow \tau$ symmetry. As a first test, these mass matrices predict the neutrino mass parameters and mixings consistent with neutrino oscillation data (Table 1), and all the three input parameters are fixed at this stage. In the next stage, these mass matrices are employed to estimate the baryon asymmetry of the universe, in both thermal and non-thermal leptogenesis scenario (Tables 2-3). We use the CP violating Majorana phases derived from right-handed Majorana mass matrix, and also three possible forms of Dirac neutrino mass matrices in the calculation. The overall analysis shows that normal hierarchical model appears to be the most favourable choice in nature. The present analysis though phenomenological, may serve as additional criteria to discard some of the presently available neutrino mass models. The conclusion also agrees with other
criteria such as stability under quantum radiative corrections. We also observe some enhancement effects in flavour leptogenesis compared to non-flavour leptogenesis. Further enhancement from brane world cosmology [17] may marginally modify the present finding.

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