A Lossless Image Compression using one-Dimensional Discrete Fractional Fourier Transform

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ABSTRACT: The study of image processing in the multimedia application becomes the very crucial part when its spectrum varying unpredictably. Processing of these images is a challenging part of computer technology; hence we are using Fourier transform to analyse the signal. But this transform is very general and failed to operate in critical conditions because of its less compactness. In this paper, a discrete fractional Fourier transform is studied as a compressing algorithm for different signals. One dimensional signal is used to evaluate the characterization of discrete fractional Fourier transform and measuring the performance of compression. Some comparisons are carried out with existing algorithms by measuring the PSNR and MSE. The simulation studies and comparison show some significant improvement in compression.

KEYWORDS: Fractional Fourier Transform(FrFT), Discrete fractional Fourier transform (DFrFT), Quantization, Mean square error (MSE), Peak signal to noise ratio(PSNR).

I. INTRODUCTION

The domestic usage of signals in each and every second in mobile and internet application required a number of data processing software and hardware technologies [1]. At the same demand for speed communicating devices and smaller in size, devices depend on very tightly with compression technologies [2-5]. The compression is a process of representation of original data with reduced number of bits. This reduction is achieved by eliminating the unwanted redundancy existed in the data. There are two types of compression; one is lossy compression in which the data could be reconstructed with minor loss, but in lossless the reconstructed data exactly resembles the original data. Both are having their own significance in their applications [6-8]. Discrete cosine transform is a very effective tool which is used in lossy compression approach of signals since it has major drawback of artifacts when more samples are used to process the signal[9,10]. Fourier transform has the problem of compactness, hence the fractional transforms are used to overcome the problem of both artifacts and compactness by using the power of fractional order[11]. In this paper, some experimental studies have been carried out to explain the compression process of DFrFT.

This paper is organized as follows: section II Describes the formal definition and background of DFrFT based compression. The compression flow diagram and proposed method described in section IV. Section V explains results and discussion followed by the conclusion in section VI.
Human visual perception based image processing become more significant in multimedia applications, an image is a cluster of pixels distributed with different frequency values. Fourier transform is a outfitted tool for such a optical applications due to its frequency analysis property of signal, but fine tuning in signal is difficult in Fourier transform. Hence a First attempt has been made by Nemieas and Almedia in 1980 by using Fractional Fourier transform in quantum mechanics and in signal processing application respectively [12,13]. The Refined DFrFT method introduced by McBride and Kerr using operational calculus in 1987[14]. Few signal processing applications uses one dimensional and two dimensional DFrFT in order to increase the performance[15-18]. After implementation of FrFT, many developments have been made by the researcher to improve the performance of FrFT. It was the Santhanam and Almedia [19] who has first reported the work in 1995. This method satisfies all properties and reduced those properties to DFT when the order is unity and approximations of the continuous FrFT are to be fulfilled by a valid discrete input and outputs. The $p^{th}$ power of Eigen values are chosen appropriately to derive the FrFT, but for DFrFT those Eigenfunctions are decentralized. But this method of computation have a constraint of signal sampling rate and computational speed was very worst[20]. In [21], DFrFT at any angle is the weighted sum of the fraction of special angles, which are multiples of $2\pi/N_2\pi/N$ for odd $N$ and $2\pi/(N+1)2\pi/(N+1)$ for even $N$ with $N$ being the number of samples. In [22] eigenvectors are obtained by sampling Hermite–Gaussian functions with a removal of error. The DFrFT functions can be derived from eigenvectors but it is not in closed form [23]. An compression algorithm proposed by Neer J and kulbir singh et.al [24] uses a two dimensional DFrFT for image and video compression. It follows segmentation based transform without improved PSNR in lossless compression, but compression rate was not improved significantly due to a quality of reconstruction. It consumes more time to process the image along rows and column and for encoding both real and imaginary parts. In order to reduce the time we implement one dimensional DFrFT based compression algorithm in this paper. DFRFT can be derived from the linear combination of identity operation, DFT, time inverse operation and IDFT [25]. The transform matrix is orthogonal apart from satisfying additivity and reversibility properties. The transform results in neither match with continuous FRFT nor exhibits the characteristics of fractionalization [26]. The approaches discussed in [27-28] involve multi-rate operations.

A closed form DFRFT [21] has been derived by simply sampling the input function $x(t)\alpha(t)$ and its transform $X_\alpha(u)\alpha(u)$ at $\Delta t$ intervals and $\Delta u$ intervals, respectively. DFRFT is written as [21] equation(1)

$$Y_\alpha(m) = \sum_{n=-N}^{N} F_\alpha(m,n)y(n) \quad (1)$$

where $y(n)y(n)$ denotes the samples of input with $n=(-N,N)n=(-N,N)$ and $Y_\alpha(m)$ denotes samples of output with $m=(-M,M)$. The normalized kernel $F_\alpha(m,n)$ is given by [21] equation(2)

$$F_\alpha(m,n) = \sqrt{\frac{\sin \alpha}{2M+1}} \times e^{\frac{j \cos \alpha m^n \Delta t}{2}} e^{\frac{j \cos \alpha n \Delta t}{2}} e^{-j 2 \pi m n} \quad (2)$$

To compute IDFRFT, DFRFT is evaluated with order $-\alpha$, with sampling interval $\Delta u$ at the input and $\Delta t$ at the output.

The concept of DFRFT computation in [21] is straightforward. There are two chirp multiplication operations and one FFT. Thus a total number of the multiplication operations required is $2P+(P/2)\log2(P)$ where $P=2M+1P=2M+1$. Though the method does not satisfy the additive property, here transform results can be altered by converting the transform results order by two chirp multiplications and one convolution operation. In comparison with results of continuous FRFT, the error involved in this method of DFRFT computation is of the order of $10^{-6}–10^{-14}$ [21].

**II. RELATED WORK**
III. PROPOSED LOSSLESS IMAGE COMPRESSION METHOD

The DFrFT is well suited for analysis of chirp signals with a suitable rotational angle. In this work, the one-dimensional DFrFT is used to compress the image by processing each column of an image with suitable quantization factor and fractional order. The Fig.1 shows the flow diagram of compression method and it is explained by steps as follows,

Step 1: Read the source image from any standard image database and calculate the size of the image.

Step 2: Each column of the sub-image is converted into an array and then apply one-dimensional discrete fractional transform with defined fractional order with selected compression percentage. The degree of data reduction as a result of the compression process is known as compression percentage. This is done to convert the gray-scale levels of pixels in spatial domain into coefficients in a transform domain.

Step 3: The Create the quantization matrix which eliminates the selective information in the matrix (means near zero values). The optimized quantized coefficients are arranged from lower-frequency to higher-frequency components and further compressed by the efficient run-length coding approach.

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Fig.1. pipeline view of the compression method.
Step 4: At decoding end, simply the contrary process of the encoder by performing the inverse 1-D discrete fractional transform. Because of orthogonality, the inverted value of ‘α’ that was used in forwarding transform with the same value can be inverted to get IDFrFT.

Here many parameters are available to measure the compression quality but most commonly used parameters compression percentage, Mean square error(MSE) and peak signal to noise ratio(PSNR) are used in this paper to analyze the proposed compression performance as,

\[
\text{Compression percentage} = \left(\frac{I_1(m,n) - I_2(m,n)}{I_2(m,n)}\right) \times 100
\]

\[
\text{MSE} = \frac{\sum_{M,N} [I_1(m,n) - I_2(m,n)]^2}{M \times N}
\]

\[
\text{PSNR} = 10 \log_{10} \left( \frac{255 \times 255}{\text{MSE}} \right)
\]

where \( I_1 \) and \( I_2 \) are original and reconstructed images respectively with \( m, n \) row and columns.

### IV. RESULTS AND DISCUSSION

The proposed compression algorithm analysed for standard test images of Lena, barbara, mandrill and pepper of dimension of 512X512. The simulations carried out for fractional order of range 0.90 to 0.99 and quantization factor also varied in range of 0.01 to 0.2.

<table>
<thead>
<tr>
<th>Compression percentage</th>
<th>( A_{\text{opt}} )</th>
<th>Quantization factor</th>
<th>MSE</th>
<th>PSNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.90</td>
<td>0.001</td>
<td>25.34</td>
<td>42.09</td>
</tr>
<tr>
<td>20</td>
<td>0.92</td>
<td>0.005</td>
<td>35.61</td>
<td>36.61</td>
</tr>
<tr>
<td>30</td>
<td>0.98</td>
<td>0.01</td>
<td>42.04</td>
<td>35.89</td>
</tr>
<tr>
<td>40</td>
<td>0.97</td>
<td>0.04</td>
<td>53.65</td>
<td>34.05</td>
</tr>
<tr>
<td>50</td>
<td>0.94</td>
<td>0.06</td>
<td>76.21</td>
<td>32.57</td>
</tr>
<tr>
<td>60</td>
<td>0.98</td>
<td>0.08</td>
<td>89.97</td>
<td>31.26</td>
</tr>
<tr>
<td>70</td>
<td>0.99</td>
<td>0.1</td>
<td>121.2</td>
<td>30.42</td>
</tr>
<tr>
<td>80</td>
<td>0.99</td>
<td>0.2</td>
<td>173.43</td>
<td>26.69</td>
</tr>
</tbody>
</table>

Table.1 shows the MSE and PSNR values of proposed method at a compression ratio of minimum 10% and maximum 75% at different quantization factor and optimal fractional orders for Lena image. Here we observe that the PSNR values is decreasing as compression percentage is increased with respect to quantization factor.
Fig. 2 shows the reconstructed Lena image at compression percentage of 20%, 40, 60 and 80% respectively. Even some loss occur in reconstructed image observable in PSNR value but for visual perception there is no such a difference found at 80% compression percentage also.

Table 2. MSE, PSNR comparison of proposed method with DFrCT and DFrST

<table>
<thead>
<tr>
<th>Test images</th>
<th>Compression percentage</th>
<th>DFrST</th>
<th>DFrCT[20]</th>
<th>DFrFT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>a_{opt}</td>
<td>MSE</td>
<td>PSNR</td>
</tr>
<tr>
<td>Lena</td>
<td>75</td>
<td>0.94</td>
<td>313.8</td>
<td>22.0</td>
</tr>
<tr>
<td>Barbara</td>
<td>75</td>
<td>0.99</td>
<td>543.04</td>
<td>19.6</td>
</tr>
<tr>
<td>Cameramen</td>
<td>75</td>
<td>0.99</td>
<td>464.30</td>
<td>20.37</td>
</tr>
<tr>
<td>Rice</td>
<td>75</td>
<td>0.92</td>
<td>161.52</td>
<td>24.95</td>
</tr>
<tr>
<td>IC</td>
<td>75</td>
<td>0.99</td>
<td>512.26</td>
<td>19.94</td>
</tr>
</tbody>
</table>

The comparative study of proposed method with other fractional transforms like Discrete fractional Sine transform (DFrST) and Discrete fractional Cosine transform (DFrCT) are tabulated in Table 2. shows that our algorithm is far better than other two method in PSNR value. Fig. 3 has some graphical study of proposed method with other art of work.
This paper introduces a lossless compression algorithm by using one-dimensional DFrFT. This process is preferably suited to operate at high compression percentages. At this high compression percentage reconstruction quality is better hence became advantages for multimedia image compression. This process has limitations, if encoding bitstreams are lost then we cannot recover the original image. Increasing in quantization factor leads to very hazy image quality, also the good quality is reconstructed at a fractional order of 0.90 to 0.99. For all these evidence the proposed method is significantly improved in compression performance. The simulated results and comparative study evidence that the compression quality was improved in terms of PSNR.

V. CONCLUSION

REFERENCES