Statistical Characterization and Generation of Rayleigh and Rician Fading Channel Models in OFDM

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ABSTRACT: The operation of the wireless channel is affected by fading. In this research paper, effects of fading channels on OFDM are studied. MATLAB is used to simulate wireless fading channels environment that are either based on Doppler spread or Delay Spread. This paper describes the statistical characteristics and generation of different types of fading channels. Different fading models such as Rayleigh and Rician are applicable for different types of surroundings. An attempt has been made to compare the performance of the Rayleigh and Rician fading channel models by using MATLAB simulation. We have developed algorithms for the generation of Rayleigh and Rician fading channels.

KEYWORDS: OFDM, Fading Channel, Wireless Communication, Rayleigh Fading, Rician Fading.

I. INTRODUCTION

In the communication field analysis of wireless channel is very important topic now days. An explosive development of the wireless technology has opened several new paths for its execution but some unwanted circumstances degrade the signal energy and make difficult to achieve the optimum output from the system [1]. Multi carrier modulation is the important part of the OFDM technology. Hence, data are divide to many parallel streams, which decreases bit rate. OFDM system modulates numerous subcarriers using these parallel sub streams. OFDM’s capability to send data with high speed is the important reason it is getting so popular besides robustness against Inter symbol interference (ISI). Therefore, many wireless communication standards all over the world have adopted this type modulation. OFDM model used in this research paper is shown in Fig 1. Series to parallel conversion of the input data stream results in the blocks of output symbols that we are transmitting. Assuming that N is the number of data blocks then OFDM symbols are characterized by \( X_m = (x_0, x_1, \ldots, x_{N-1}, m) T \). After that each one of the symbols modulates one subcarrier. A guard band of length \( N_{cp} \) is added to each OFDM symbol. The \( m \)th OFDM symbol is calculated by equation (1)

\[
s_m(n) = f(x) = \begin{cases} 
1 \sum_{k=0}^{N-1} x_{k,m} e^{\frac{i \pi (n-N_{cp})}{N}} & \text{for } n \in [0, N + N_{cp} - 1] \\
0 & \text{otherwise}
\end{cases}
\]
Equation (2) denotes the received signal output $r(n)$ [2]. We can notice that the received output is the addition of the AWGN(white Gaussian noise)(k) and the convolution of the channel’s impulse response $\oplus (n)$.

$$r(n) = \sum_{n=0}^{n_{max}} h(n)s(k - n) + n(k)$$ (2)

One of the most disturbing aspects in the wireless communication is fading [1]. Signal fading refers to the degradation in received output strength over a small travel distance or time interval [3]. Fading flowchart are shown in fig.2. Channel is the physical medium that is used to transmit the signal from the transmitter to the receiver. The radio link between the transmitter and receiver change from simple LOS (line-of-sight) to one that is severely obstructed by the trees, mountains, buildings, etc., and hence suffer from multipath fading [1]. However, the mobile wireless channels are very unlike from the wired channels, because of their randomness [1]. Wireless communication channels used to transmit information over long and short distances. In fixed wireless transmission, the transmitter and the receiver are static while mobile wireless transmission, the transmitter and the receiver are in motion but are moving at different speeds [4].

For most important practical channels, where signal propagation takes place in the atmosphere and close to ground, the free space transmission model is inadequate to describe the channel and guess system performance [5]. In a wireless mobile communication system, a signal can travel from transmitter to receiver over numerous reflective paths; this phenomenon is referred to as multipath transmission. There are several kinds of communication losses that are typical of the mobile wireless environment. Impairments may result mainly from multipath communication, attenuation of signal power from large objects, relative transmitter receiver motion, interference, spreading of electromagnetic power and thermal or background noise [4]. The nature of these impediments changes over time in irregular ways due to user movements causing the received signal to oscillate or vary. One of the most important methods for characterizing a fading channel is the use of a PDF (probability density function) which represents the probability density of the received signal power. The envelope of the received signal can determine the ultimate Shannon channel capacity of a fading wireless link since received power is proportional to the square of received envelope [4].

Radio-wave transmissionthrough wireless channels is a complex phenomenon characterized by various effects. A clear-cut mathematical explanation of this occurrence is either unknown or too complex for tractable communications systems analysis [6]. However, significant efforts have been devoted to the statistical modelling and characterization of these different effects.
The result is a range of relatively simple and accurate statistical models for fading channels which depend on the particular propagation environment and the underlying communication scenario [6].

II. STATISTICAL CHARACTERISTICS OF FADING CHANNELS

Statistical model of the fading channel is to Clarke’s recognition that he statistically characterized the electromagnetic field of the received signal at a moving terminal through a scattering process [7]. In Clarke’s model, there are N planewaves with random carrier phases, each coming from a random direction under the hypothesis that each planewave has the same average power [8–11].

\[ f_i = f_m \cos \theta_i = \frac{v}{\lambda} \cos \theta_i \]  

(3)

Where, \( \lambda \) is the wavelength of incident wave. Every received output signal has a different carrier frequency with small change from the centre frequency. The power spectral density of the output signal is given by equation 4. It is clear from that equation that the PDF is zero when \( |f - f_c| \geq f_m \) With center frequency \( f_c \). Spectrum is limited to \( \pm f_m \) and all the received waves have different carrier frequencies that are shifted over the multipath channel subject to various scattering components, follows the Rayleigh distribution. The power spectrum density (PSD) of the fading process is found by the Fourier transform of the autocorrelation function of the amplitude of the received signal.

\[ s_{yy}(f) = \begin{cases} \frac{\alpha_p}{4f_m} \frac{1}{\sqrt{1-\left(\frac{f-f_c}{f_m}\right)^2}} |f - f_c| & \leq f_m \\ 0 & otherwise \end{cases} \]  

(4)
Above equation referred as classical Doppler spectrum [7]. If small quantity of scattering component is powerful than most of other components, the fading process no longer follows the Rayleigh distribution. In this case, the Amplitude of the received signal follows the Rician distribution and thus, this fading process is referred to as Rician fading.

III. GENERATION OF FADING CHANNELS

In a heavily built-up urban or where there is non-line-of-sight communication between the transmitter and receiver, the objects in the environment attenuate, reflect, refract, and diffract the signal before it arrives at the receiver. This propagation environment is known as Rayleigh fading, and the Rayleigh distribution model is a specialized stochastic fading model for this type of fading environment. If there is line of sight communication between the transmitter and receiver then this propagation environment follows Rician distribution. We know that received output in the wireless channel is sum of received outputs from an infinite number of scatters. By the central limit theorem, the received output can be denoted by a Gaussian random variable.

Let X denote the amplitude of the complex Gaussian random variable $W_1 + jW_2$, such that $X = \sqrt{W_1^2 + W_2^2}$ then, note that X is a Rayleigh random variable with the following probability density function (PDF):

$$f_X(x) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}$$

Where $\sigma^2 = \mathbb{E}\{X^2\}$.

**Steps for generation of Rayleigh fading channels:**

1. First generate two random variable with zero mean and unit variance by using Matlab function “randn”. Let $Z_1$ and $Z_2$ are generated random variable.
2. Calculate $X = \sigma \sqrt{Z_1^2 + Z_2^2}$ where $Z_1 \sim \mathcal{N}(0,1)$ and $Z_2 \sim \mathcal{N}(0,1)$ are Gaussian distribution with mean equal to zero and variance equal to one.
3. Generated random variable X is known as Rayleigh with average power of $\mathbb{E}\{X^2\} = 2\sigma^2$.

Steps for generation of Rician fading channels:

1. First calculate $W_1$ and $W_2$ two Gaussian random variable with zero mean and variance of $\sigma^2$ in the non LOS environment.
2. After obtaining two random variable we calculate the amplitude of the received signal in terms of $X = C + W_1 + jW_2$ where C is known as LOS component.
3. Above calculated random variable X is known as rician random variable with the following pdf where $I_0$ is the modified Bessel function.

$$f_X(x) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} I_0 \left( \frac{xc}{\sigma^2} \right)$$

4. Then calculate Rician factor $K = \frac{\sigma^2}{2\sigma^2}$ then above equation represent in terms of the Rician K factor.
IV. ALGORITHM FOR RAYLEIGH AND RICIAN FADING CHANNEL

STEP 1: Variable Declaration

N: No of channel realizations
K_dB: Rician factor in dB
K: Rician factor
H_{RAY}: channel vector
H_{RIC}: channel vector

STEP 2: Assigning values to variable

N= 1000000
Level= 40
K_dB=[-40, 16]

STEP 3: For Rayleigh fading

1: First calculate channel vector H_{RAY}
   \[ H_{RAY} = (\text{randn}(1,N) + j \times \text{randn}(1,N))/\sqrt{2} \]
2: Then calculate absolute value of complex magnitude of channel vector by using abs function
   \[ \text{abs} (H_{RAY}(1,:)) \]
3: Then by using histogram function we store the value in count and X
   \[ \text{[count, x]} = \text{hist(abs(H_{RAY}(1,:)), level)} \]

STEP 4: For Rician fading

1: We calculate Rician fading channel for two Rician factor K_{dB_1} = -40dB and K_{dB_2} = 16dB
2: FOR I := 1 : 2
   First convert K_{dB} to K
   \[ K_I = 10^{ \frac{K_{dB_I}}{10} } \]
   Then calculate channel vector H_{RIC}
   \[ H_{RIC} = \sqrt{ \frac{k}{(K_I + 1)} } + \sqrt{ \frac{1}{(K_I + 1)} } \times H_{RAY} \]
Then calculate absolute value of complex magnitude of channel vector by using abs function

\[
\text{abs} \left( \text{H}_{\text{RIC}} \left( I, : \right) \right)
\]

Then by using histogram function we store the value in count and X

\[
\text{[count, x]} = \text{hist} \left( \text{abs} \left( \text{H}_{\text{RIC}} \left( I, : \right) \right), \text{level} \right)
\]

END

STEP5: PLOTTING

Plot the graph between X and count for both Rayleigh and Rician fading models

STEP6: LABELING

Xlabel ('x')

Ylabel ('Occurrence')

V. SIMULATION RESULTS FOR RAYLEIGH AND RICIAN FADING CHANNEL

1: Rayleigh fading channel

In Fig.3 when k is equal to zero then distribution is known as Rayleigh fading channel.
2: Rician fading channel for two Rician factor $K_{dB_1} = -40dB$ and $K_{dB_2} = 16dB$

![Graph of Rician fading channel for two Rician factor $K_{dB_1} = -40dB$ and $K_{dB_2} = 16dB$.]

From the Fig.4 when $k$ increase from $-40dB$ to $16dB$ graph tends to be the Gaussian PDF.

3: Comparison between Rayleigh, Rician at $K_{dB_1} = -40dB$ and Rician at $K_{dB_2} = 16dB$

![Graph comparing Rayleigh, Rician at $K_{dB_1} = -40dB$ and Rician at $K_{dB_2} = 16dB$.]

As we see from Fig.5 when $k$ tends to $-40dB$ then rician fading channel approaches to Rayleigh distribution.
VI. CONCLUSION

In this research paper, we have compared the Rayleigh and Rician fading channel models at different Rician factor \( k \) by using MATLAB simulation. From the simulation results, we infer that when Rician factor \( k \) is equal to zero then it is called Rayleigh channel and when \( k \) increase graph tends to be the Gaussian PDF. We have seen from the above result that losses in the Rayleigh fading channel is more than that of the Rician fading channel. Due to the presence of line-of-sight path in the Rician channel losses is less than Rayleigh fading channel. Above result also shows that the Rician distribution approaches Rayleigh distribution and Gaussian distribution when \( K_{dB} = -40dB \) and \( K_{dB} = 16dB \) respectively.

REFERENCES